PART IIA 2005 *Datasheet: Structural Data Formulae and Section Tables*

3D3: Structural materials and design

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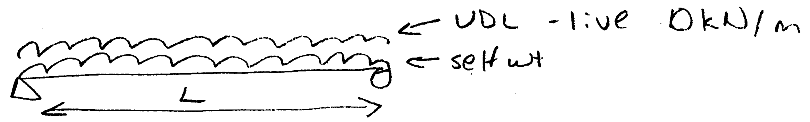
1 a)



$\gamma_{f\,DEAD} = 1.4$ Additional deflection due to live load $= \dfrac{L}{250} = \Delta\delta_L$

$\gamma_{f\,LIVE} = 1.6$

deflection constraint at midspan under live load for simply supported beam with UDL

$$\Delta\delta_L = \frac{5WL^3}{384EI}$$ (from Structures Data Book)

strength constraint

- plastic $\sigma_y Z_p = \gamma_f w \dfrac{L^2}{8}$
- elastic $\sigma_u Z_e = \gamma_f w \dfrac{L^2}{8}$

i) 406 x 140 x 46 UB

from Structures Data Book $I_{xx} = 15690\,cm^4$

$Z_p = 888\,cm^3$

from Ashby Maps $E_{steel} = 210\,000\,MPa$

$\sigma_y = 220\,MPa$

self weight load : $46\,\dfrac{kg}{m} \times 9.8\,\dfrac{m}{s^2} = 450.8\,N/m$

deflection constraint (under 10 kN/m live load)

$$\Delta\delta_L = \frac{5 \times 10 \times L^4}{384 \times 210000 \times 15690 \times 10^4} \leq \frac{L}{250}$$

$L^3 \leq 1.012 \times 10^{12}\,mm^3 \quad \therefore \quad L \leq 10.04\,m$

Strength constraint (plastic behaviour)

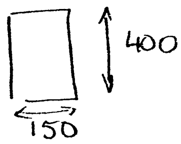
factored loads $\gamma_f w = 1.4 \times 0.451 + 1.6 \times 10 = 16.63\,\dfrac{kN}{m}$

$L^2 = \sigma_y Z_p\, 8 / \gamma_f w = 220 \times 888 \times 10^3 \times 8 / 16.63$

$L^2 = 93.98 \times 10^6\,mm^2 \quad \therefore \quad L = 9.69\,m$

strength controls $\rightarrow$ $L_{max} = 9.69\,m$

1a) ii) 400 x 150 softwood pine section



$I_{xx} = 150 \times 400^3/12 = 800 \times 10^6 \text{ mm}^4$

$Z_e = \frac{I_{xx}}{y} = 4 \times 10^6 \text{ mm}^3$

from Ashby Maps

$$\left.\begin{aligned} E_{pine} &= 9000 \text{ MPa} \\ \sigma_u &= 42 \text{ MPa} \\ \rho &= 0.53 \text{ Mg/m}^3 \end{aligned}\right\} \text{ or similar}$$

self weight load $= 0.53 \times 1000 \times 9.8 \times 0.15 \times 0.4 = 311.6$ N/m

deflection constraint

$$\Delta\delta_L = \frac{5 \times 10 \times L^4}{384 \times 9000 \times 800 \times 10^6} \leq \frac{L}{250}$$

$L^3 \leq 2.21 \times 10^{11} \text{ mm}^3 \quad \therefore \quad L \leq 6.05 \text{ m}$

strength constraint (elastic behaviour)

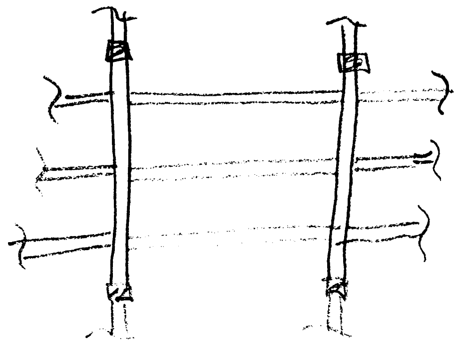
factored loads $\gamma_f w = 1.4 \times 0.312 + 1.6 \times 10 = 16.44 \frac{\text{kN}}{\text{m}}$

$L^2 = \sigma_u Z_e 8 / \gamma_f w = 42 \times 4 \times 10^6 \times 8 / 16.44$

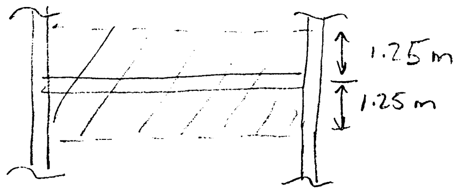
$L^2 = 81.77 \times 10^6 \text{ mm}^2 \quad \therefore \quad L = 9.04 \text{ m}$

deflection controls $\Rightarrow$ $L_{max} = 6.05$ m

1b)
(i+ii)

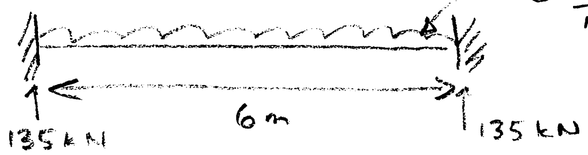


• consider a secondary beam



secondary beam - span of 6m, supporting slab on either side

$18 \frac{kN}{m^2} \times 2.5m = 45\ kN/m$

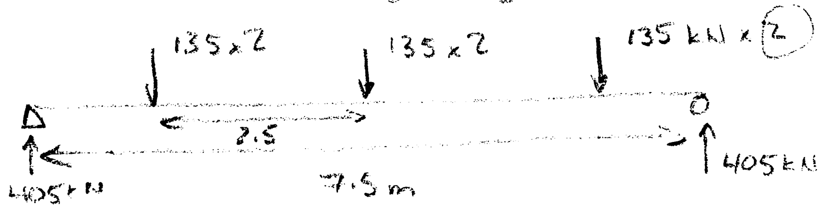


$V_{max} = 45 \times 6/2 = 135\ kN$

M_{max} (plastic) $= wL^2/16 = 45 \times 6^2/16 = 101\ kN\,m$

(although lower bound allows for any equilibrium solution so fixed end elastic solution also OK)

• primary - span of 7.5m, supporting secondary beams on both sides



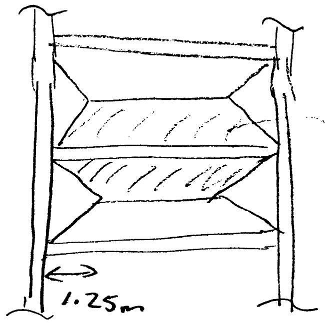
$V_{max} = 135 \times 2 \times 3/2 = 405\ kN$

M_{max} (at centre) $= 405 \times 7.5/2 - 135 \times 2 \times 2.5 = 844\ kN\,m$

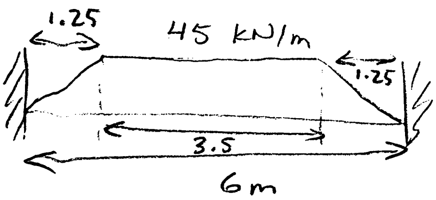
1b iii) Bookwork

In using the idea of a load path, we are assuming ductile materials so plasticity theory applies and in particular the lower bound theorem which implies that any equilibrium system (i.e. <u>any</u> load path) can be used in design. Likely materials include steel or other ductile metals & reinforced concrete (if not too heavily reinforced)

b iv)



- load on secondary beam becomes



$V_{max} = 45 \times 4.75/2 = 106.9\,kN$

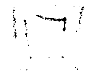
- maximum shear force in primary beam is unchanged.

COMMON MISTAKES / COMMENTS

Many students did not check <u>both</u> deflection and strength constraints in part 1(a). In part (b) answers were often out by a factor of 2 e.g. forgetting the shear force is half the total load (2x answer too high) or overlooking that the beams are loaded from both sides (2x answer too low)

2a) shape efficiency factor ϕ_f for strength in bending, using linear-elastic theory

$$\phi_f = \frac{Z}{Z_0} = \frac{6Z}{A^{3/2}}$$

i) 150 × 100 × 10 RHS  from Structures Databook
$A = 44.9\,cm^2$ $Z_e = 171\,cm^3$

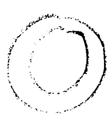
$$\phi_f = \frac{6 \times 171}{(44.9)^{3/2}} = 3.41$$

ii) 152 × 89 × 16 kg/m  from Structures Data book
$A = 20.3\,cm^2$ $Z_e = 109\,cm^3$

$$\phi_f = \frac{6 \times 109}{(20.3)^{3/2}} = 7.15$$

the I beam is optimised for bending with most of the material far away from the centroidal axis in the flanges ∴ UB more efficient although depth similar

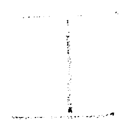
b) for simply supported beam with span 5-10 m

 CHS

difficult to support, good in torsion, not so good for bending, same properties around all axes

 RHS

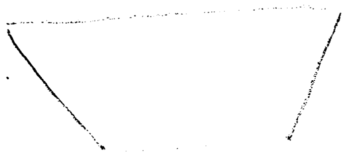
good for lateral torsional buckling, difficult to connect (but better than CHS) also OK for bending

 UB

best for bending, easy to construct, not so good for lateral torsional buckling

2b) continued

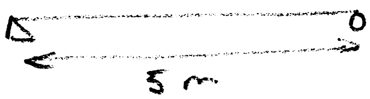
A bridge beam with depth of ~ 2m is more likely to be a box shape. It will be less prone to local buckling vis a vis a UB as held at both ends rather than having compression outstand. It will also be better torsionally. If stiffeners required, access poss. inside since depth quite large



c) 305 x 165 x 54 UB, S355 steel $\gamma_m = 1.1$

$M_p = 273$ kN m

i)



Check for elastic critical lateral torsional buckling

$$M_c = \frac{\pi}{L}\sqrt{E I_{yy}\left\{GJ + \frac{\pi^2}{L^2} E C_w\right\}}$$ (from 3D3 steel data sheet)

for 305 x 165 x 54 UB (using Structures data book (information & data provided)

$I_{yy} = 1063\ cm^4$

$G = 81$ GPa

$J = 34.8\ cm^4$

$D = 310.4$ mm

$L = 5000$ mm

$E = 210$ GPa

$C_w = D^2 I_{yy}/4$ (3D3 data sheet)

$= (310.4)^2 \times 1063 \times 10^4 / 4$

$= 2.56 \times 10^{11}\ mm^2$

$$M_c = \frac{\pi}{5000}\sqrt{210000 \times 1063 \times 10^4\left\{81000 \times 34.8 \times 10^4 + \frac{\pi^2}{5000} \times 210000 \times 2.56 \times 10^{11}\right\}}$$

$$= \frac{\pi}{5000}\sqrt{2.232 \times 10^{12}\left(2.819 \times 10^{10} + 2.123 \times 10^{10}\right)}$$

≈ 209 kN m

The associated buckling mode is lateral torsional buckling

2cii)

$M_{max} = \frac{M_c}{0.8} = \frac{209}{0.8} = 260\ kN.m = M_{cr}$ (hinge position moment for elastic buckling)

provided in question

$\lambda_{LT} = \left(\frac{M_P}{M_{cr}}\right)^{1/2} = \left(\frac{273}{260}\right)^{1/2} = 1.02$

from the chart on 3D3 data sheet the corresponding capacity reduction factor is

$\chi_{LT} = 0.66$

ciii) Need to restrain compression flange to prevent lateral torsional buckling. Could also use a beam with a bigger J. The box gives

$J \uparrow 150 \rightarrow M_c$ becomes $2364\ kN.m$

$I_{yy} \uparrow 1.5$ $\quad \therefore M_{cr} = 2364/0.8 = 2955\ kNm$

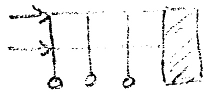
$\lambda_{LT} = \left(\frac{273}{2955}\right)^{1/2} = 0.30$ and $\chi_{LT} = 0.95$

Hence the box is much more effective. However now need to consider if thin webs need stiffening.

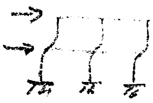
COMMON MISTAKES / COMMENTS

Common mistakes in 2a) were to use ϕ_e instead of ϕ_f and or Z_p instead of Z_e. Part b and ci) were answered well. In cii) a large number of students simply said M_{cr} 0.8 M_p which is incorrect.

3a) i) In a braced column, the lateral loads are carried through the structure to a rigid support and the top and bottom of the column are unable to deflect horizontally e.g.



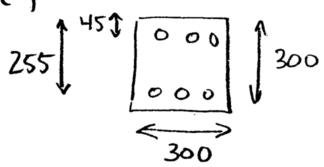
In an unbraced column, the ends of the column may deflect laterally as the horizontal forces are transmitted through shear e.g.



ii) In a stocky column, no additional stresses due to the deformation of the column need to be taken into account - the pure axial capacity can be reached.

In a slender column, the pure axial capacity is reduced to reflect $P.\delta$ effects and imperfections. Slenderness ratios are typically used in design to determine if a column is stocky or slender.

b) i)



$f_{cu} = 25\,MPa$ $\quad f_y = 460\,MPa$

$\gamma_c = 1.5$ $\quad \gamma_s = 1.15$

$\varepsilon_{cu} = 0.0035$ $\quad \varepsilon_y = 0.002$

$E_s = 210\,GPa$

In compression - need to carry 2000kN

$$N_u = A_c \frac{0.6 f_{cu}}{\gamma_c} + \frac{A_s f_y}{\gamma_s} \quad \therefore \quad A_s = \frac{\gamma_s}{f_y}\left(N_u - A_c \frac{0.6 f_{cu}}{\gamma_c}\right)$$

$$A_s = \frac{1.15}{460}\left(2000 \times 10^3 - 300^2 \times 0.6 \times 25 / 1.5\right)$$

$$= 2750\ mm^2$$

In tension - need to carry 1000 kN, assume only the steel carries the tensile load

$$N_u = A_s f_y / \gamma_s \quad \therefore \quad A_s = \gamma_s N_u / f_y$$

$$A_s = 1.15 \times 1000 \times 10^3 / 460 = 2500\ mm^2$$

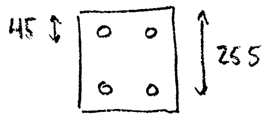
3bi) continued

Compression requirement controls → need $A_s = 2750\,mm^2$

6 No 25ϕ bars → $A_s = 2945\,mm^2$ OR

4 No 32ϕ bars → $A_s = 3217\,mm^2$

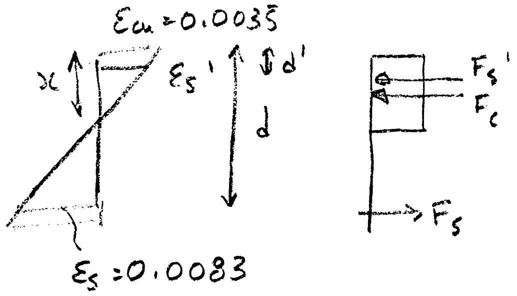
3bii)



Given $P_u = 2200\,kN$ (axial)

$P_u = 486\,kN$, $M_u = 169\,kNm$ } balanced

for $\varepsilon_s = 0.0083$ and $\varepsilon_{cu} = 0.0035$ (concrete crushing)



$A_{sc} = A_{ST} = 1608\,mm^2$ (32ϕ bars)

$\varepsilon_s > \varepsilon_y$ ∴ yielded

strain compatability

$$\frac{\varepsilon_{cu} + \varepsilon_s}{d} = \frac{\varepsilon_{cu}}{x} \rightarrow x = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_s} d = \frac{0.0035}{0.0035 + 0.0083}(255)$$

$$= 75.6\,mm$$

Check strain in top steel

$$\frac{\varepsilon_s'}{x - d'} = \frac{\varepsilon_{cu}}{x} \Rightarrow \varepsilon_s' = \frac{\varepsilon_{cu}(x - d')}{x} = \frac{0.0035\,(75.6 - 45)}{75.6}$$

$$= 0.00142 < \varepsilon_y$$ NOT yielded

Calculate axial force

$$N_u = \frac{0.6 f_{cu} b x}{\gamma_c} + \frac{\varepsilon_s' E_s A_{sc}}{\gamma_s} - \frac{f_y A_{ST}}{\gamma_s}$$

$$= \frac{0.6 \times 25 \times 300 \times 75.6}{1.5} + \frac{0.00142 \times 210000 \times 1608}{1.15} - \frac{460 \times 1608}{1.15}$$

$$= 226800 + 416961 - 643200 = 561\,N \sim 0$$ (almost pure bending)

Calculate moment → moments about centre of section

$$M_u = F_c (h/2 - x/2) + F_s'(h/2 - d') + F_s(d - h/2)$$

$$= 226800\,(150 - 75.6/2) + 416961\,(150 - 45) + 643200\,(255 - 150)$$

$$= 136.9\,kNm$$

3b ii) continued

Since $N_u \sim 0$, this is almost the case of pure bending.

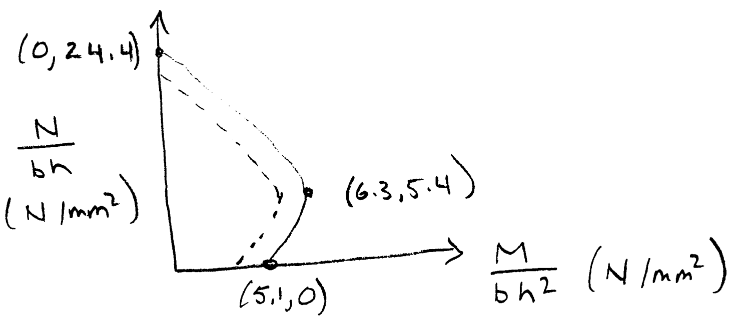
b iii)

	N/bh	M/bh^2
pure axial	24.4	0
balanced	5.4	6.3
pure moment	0	5.1
	(N/mm^2)	(N/mm^2)

$\frac{N}{bh} = \frac{2200 \times 10^3}{300 \times 300}$ N/mm^2

$\frac{N}{bh} = \frac{486 \times 10^3}{300 \times 300}$ $\frac{M}{bh^2} = \frac{169 \times 10^6}{300^3}$

$\frac{M}{bh^2} = \frac{137 \times 10^6}{300^3}$ N/mm^2



Detailed reasoning

If f_{cu} increases and b & h increase but ρ decreases

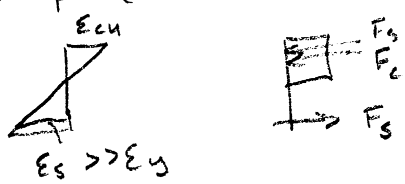
- consider pure axial case

$$N_u = \underbrace{\frac{0.6 f_{cu} bh}{\gamma_c}}_{\text{increases}} + \underbrace{\frac{A_s f_y}{\gamma_s}}_{\text{doesn't change}} = \frac{0.6 \times 30 \times 400^2}{1.5} + \frac{1608 \times 2 \times 460}{1.15}$$

$$= 3206 \text{ kN}$$

$$\frac{N_u}{bh} = \frac{3206 \times 10^3}{400 \times 400} = 20 \frac{N}{mm^2} \rightarrow \text{smaller}$$

- In pure moment case



$F_s = A_s f_y \rightarrow$ doesn't change

the compressive forces F_c + F_s' still need to balance same tensile force F_s → there will be a small change in x but not a great deal. Taking moments about concrete compressive force (& neglecting compressive steel contribution) - **NOTE** pure moment case - gives

$$M_u \sim \frac{A_s f_y}{\gamma_s}(d - x/2) \approx \frac{1608 \times 460}{1.15}(340 - 75.6/2) \approx 195 \text{ kNm}$$

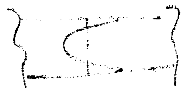
$M/bh^2 = 195 \times 10^6 / (400)^3 = 3.01$ ∴ diagram contracts (see dotted line)

COMMON MISTAKES / COMMENTS

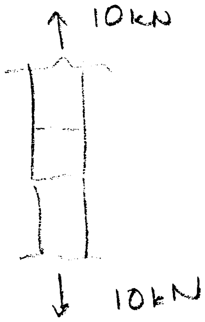
There were many difficulties with part 3b(ii). The strain in the tension steel is 0.0083 which is greater than ε_y, this means the stress is then σ_y NOT $E_s \times 0.0083$. It was also necessary to check the strain in the compression steel to confirm if it had yielded (it did not). Students who approached this question by considering the strain profile and the stress profile did well. Students who randomly chose reinforced concrete formulae from the 3D3 data sheet did not.

4a). - timber → brittle & elastic in tension, some ductility in compression which is reflected in plasticity approach in joint design, fibre direction will influence behaviour, material is anisotropic

- advanced composites → composite consisting of fibres & matrix, again brittle elastic in tension and fibre buckling in compression which will depend on fibre & matrix design, properties can be designed to suit particular requirements elastic design, although each lamina may be anisotropic, symmetric, balanced laminates result in quasi-isotropic behaviour
- toughened glass → failure in tension controlled by fast fracture, compression considered as elastic, brittle material but toughened glass is prestressed to improve tensile performance



b) i)



75 × 220 C24
Service class 2, permanent loading
∴ $k_{mod} = 0.6$
$\gamma_s = 1.15$, $\gamma_m = 1.3$
predrilled holes

for bolts in predrilled holes

$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm²

for C24 $\rho_k = 350$ kg/m
8mm ϕ bolts $d = 8$ mm

$\therefore f_{h,0,k} = 0.082(1 - 0.01 \times 8)350$
$= 26.4$ N/mm²

4bi) continued

$$f_{h,0,d} = f_{h,0,k} \, k_{mod} / \gamma_m$$

$$= 26.4 \times 0.6 / 1.3 = 12.2 \text{ N/mm}^2$$

(for material 1 and 2)

for 8 mm ϕ 4.6 bolts with $f_{uk} = 400$ MPa

$$M_{y,d} = \frac{0.8 f_{u,k} d^3}{6 \gamma_m} = \frac{0.8 \times 400 \times 8^3}{6 \times 1.15}$$

$$= 23745 \text{ N.mm}$$

Check failure modes c and f using datasheet expressions

$t_2 = t_1$ and $f_{h,1,d} = f_{h,2,d}$ $\therefore \beta = 1$

Mode c

$$R_d = \frac{f_{h,1,d} t_1 d}{1+\beta} \left[\sqrt{\beta + 2\beta^2 \left[1 + \frac{t_2}{t_1} + \left(\frac{t_2}{t_1}\right)^2 \right] + \beta^3 \left(\frac{t_2}{t_1}\right)^2} - \beta \left(1 + \frac{t_2}{t_1}\right) \right]$$

$$= \frac{f_{h,1,d} t_1 d}{2} \left[\sqrt{8} - 2 \right]$$

$$= \frac{12.2 \times 75 \times 8}{2} \left[\sqrt{8} - 2 \right] = 3032 \text{ N}$$

Mode f

$$R_d = 1.1 \sqrt{\frac{2\beta}{1+\beta}} \sqrt{2 M_{yd} f_{h,1,d} d}$$

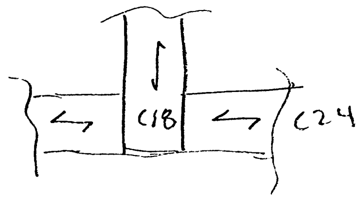
$$= 1.1 \sqrt{2 M_{yd} f_{h,1,d} d}$$

$$= 1.1 \sqrt{2 \times 23745 \times 12.2 \times 8} = 2368 \text{ N}$$

mode f controls $\therefore$ require $\frac{10000}{2368} = 4.2$ bolts

so 5 bolts are required

4 b ii)



$\boxed{\text{Bolted case}}$

<u>for C18</u> loading ∥ to grain $\rho_k = 320\,kg/m^3$, $d = 8mm$

$f_{h,0,k} = 0.082\,(1 - 0.01 \times 8) \times 320 = 24.1\ N/mm^2$

$f_{h,0,d} = 24.1 \times 0.6 / 1.3 = 11.1\ N/mm^2$

<u>for C24</u> loading ⊥ to grain ($f_{h,0,k}$ as in part b(i))

$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} = \frac{f_{h,0,k}}{k_{90}} \quad \text{for } \alpha = 90°$$

$k_{90} = 1.35 + 0.015\,d$ for softwood

$= 1.35 + 0.015 \times 8 = 1.47$

$\therefore f_{h,90,d} = f_{h,0,d} / k_{90}$

$= 12.2 / 1.47 = 8.3\ N/mm^2$

<u>Mode a</u> (failure in C18)

$R_d = f_{h,1,d}\, t_1\, d = 11.1 \times 75 \times 8 = 6660\ N$

<u>Mode b</u> (failure in C24)

$R_d = f_{h,2,d}\, t_2\, d = 8.3 \times 75 \times 8 = 4980\ N$

(same as data sheet expression with $\beta = f_{h,2,d} / f_{h,1,d}$)

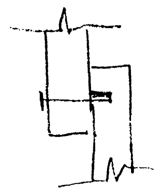
$\therefore$ mode b controls

$\boxed{\text{Nails}}$. there is no grain angle reduction for small diameter fasteners so for nails with d = 6mm

$f_{h,1,d}$ (C18) $= 11.4\ N/mm^2$

$f_{h,2,d}$ (C24) $= 12.5\ N/mm^2$

4 b ii) continued



embedment in C24 is only 50 mm

$\therefore$ use $t_2 = 50$ mm

Mode a (failure in C18)

$R_d = 11.4 \times 75 \times 6 = 5130$ N

Mode b (failure in C24)

$R_d = 12.5 \times 50 \times 6 = 3750$ N

$\therefore$ mode b still controls

c) In timber, the joint design is an example of the upper bound theory of plasticity where the timber in compression and the steel fastener are assumed to be rigid-plastic materials. However the bolts or nails etc are relatively small diameter rods and thus the possibility of a plastic hinge forming in the fastener must be considered. In addition, the fasteners tend to have fairly small heads and can rotate. Thus a number of possible collapse mechanisms are checked and the lowest upper bound selected

In steel joint design the steel sections tend to be fairly thin, the bolt diameters are fairly large and fairly solid nuts and bolts are used. Thus the possibility of a hinge forming in the bolt is unlikely. In practice therefore only modes a & b are checked (bearing failure) and that the bolt can sustain the necessary forces in shear

4c) continued

Additional factors to consider in timber joint design is the grain direction, service class & length of loading. Steel in contrast is isotropic and not susceptible to properties changing with moisture.

COMMON MISTAKES / COMMENTS

Part 4a) was generally answered well. In Part 4bi) a number of students simply got confused about the joint design process and the notation. The key things that were overlooked in bii) was the dependency of the bolt embedding strength on the grain direction and the need to use a reduced thickness if a fastener is only partly embedded. In c, few students seemed to realise that timber joint design is an example of the application of the upper bound theory of plasticity and why we check different mechanisms in timber & steel.

JL May 2005