PART IIA 2005
3D5: Environment engineering I
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ENGINEERING TRIPOS

Datasheet: Environmental Engineering I Data Sheet

3D5 ENVIRONMENTAL ENGINEERING I.

Q1. a) Distrib %'s: 4, 15, 30, 25, 15, 11 for 3hr intervals
Cumulative 4, 19, 49, 74, 89, 100
Plot at ½ times: t = 1·5, 4·5, 7·5, 10·5, 13·5, 16·5 hrs.
Connect by smooth curve (see graph)
Lag by 1 hour
Subtract. i.e. measure difference between S-curves at new ½ time pts,
(i.e. t = 0·5, 1·5, 2·5, ... hrs ...)

From graph, these are
1, 2, 3, 4, 6, 8, 10, 12, 10, 9, 8, 7, 6, 5, 4, 3, 2
check $\Sigma = 100$ ✓

Connect by smooth curve → hydrograph shape.

Because these are at ½ time points, then by piecewise linear approx these are the distribution percentages.

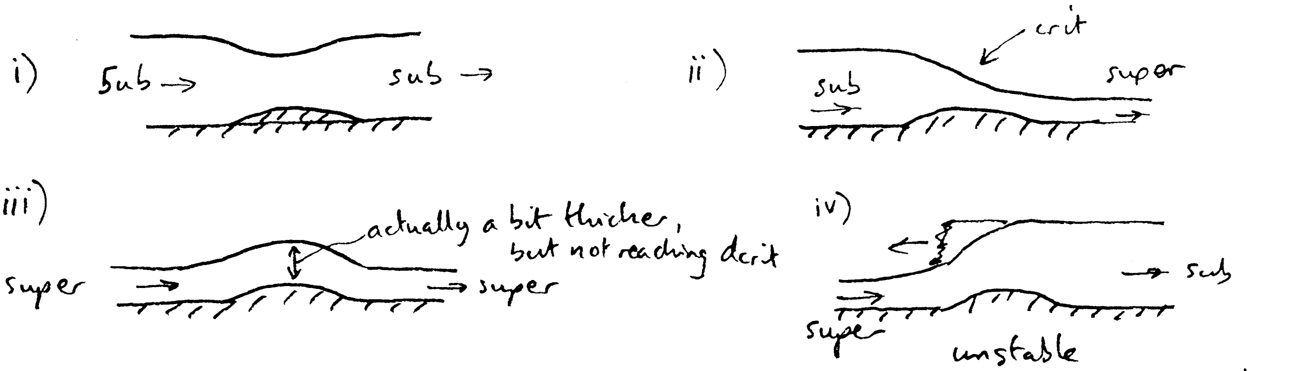
b)

$$\left.\begin{array}{ll}\text{Subcritical} & Fr < 1 \\ \text{Critical} & Fr = 1 \\ \text{Supercritical} & Fr > 1\end{array}\right\}$$

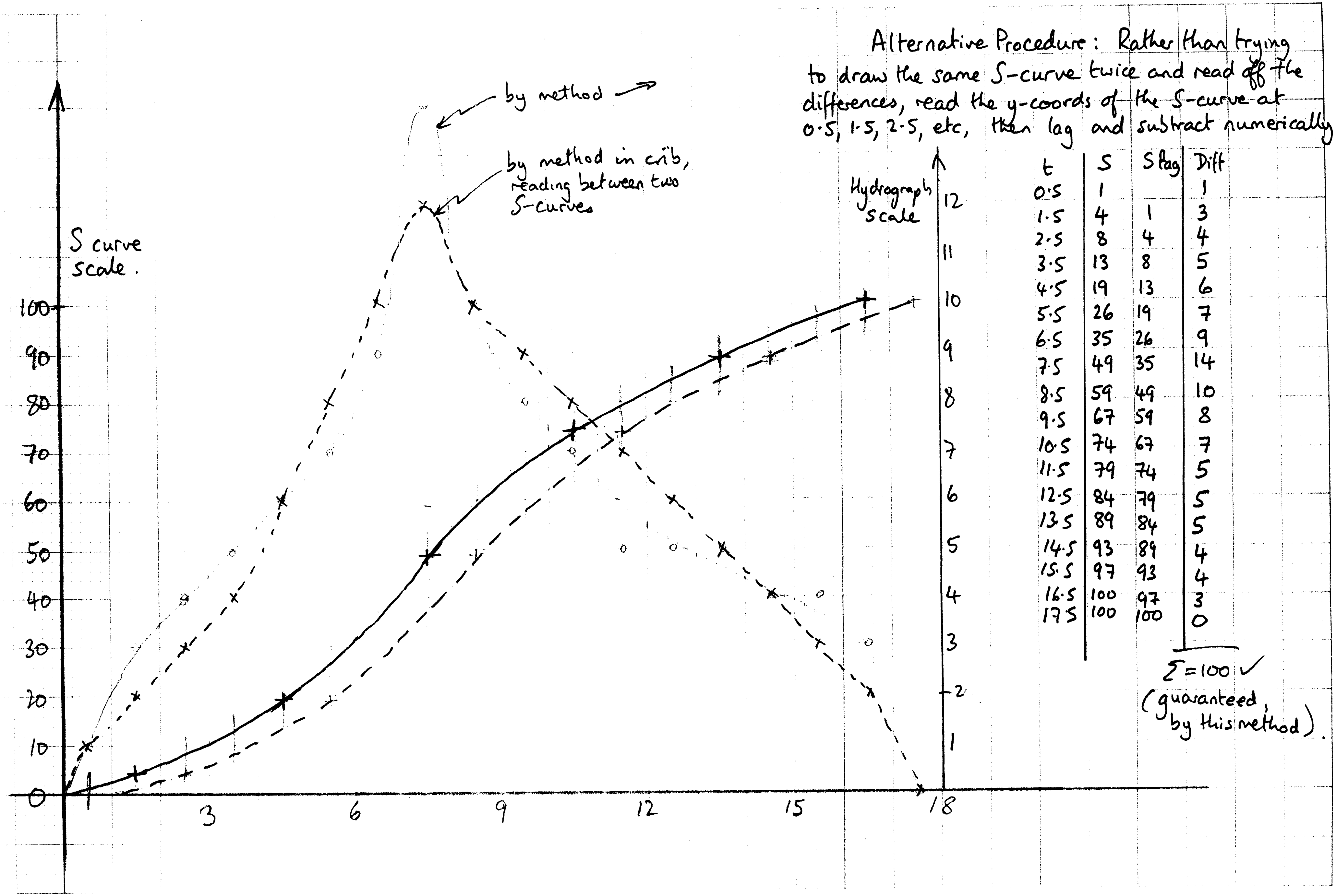
Froude No $Fr \equiv \dfrac{U}{\sqrt{gd}} = \dfrac{U}{c}$

where U = velocity of flow relative to bed
c = velocity of propogation of small disturbances relative to flow

– hence, for example, small disturbances cannot travel upstream (relative to bed) for supercritical flow.



unstable
– hydraulic jump travels upstream until it stabilises.



Alternative Procedure: Rather than trying to draw the same S-curve twice and read off the differences, read the y-coords of the S-curve at 0·5, 1·5, 2·5, etc, then lag and subtract numerically.

t	S	S lag	Diff
0·5	1		1
1·5	4	1	3
2·5	8	4	4
3·5	13	8	5
4·5	19	13	6
5·5	26	19	7
6·5	35	26	9
7·5	49	35	14
8·5	59	49	10
9·5	67	59	8
10·5	74	67	7
11·5	79	74	5
12·5	84	79	5
13·5	89	84	5
14·5	93	89	4
15·5	97	93	4
16·5	100	97	3
17·5	100	100	0

$\Sigma = 100$ ✓ (guaranteed, by this method).

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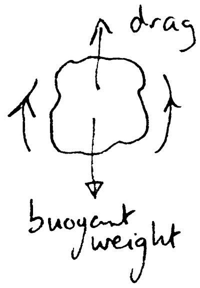
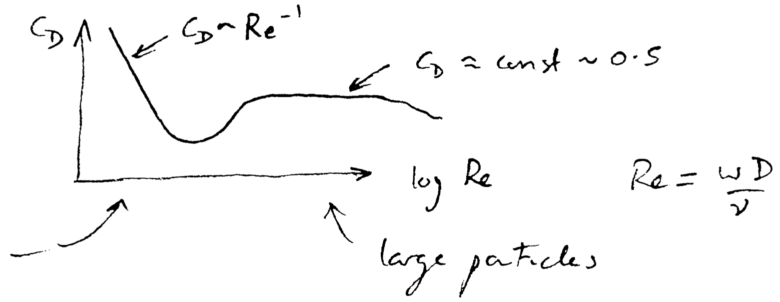
Q1 (c). Terminal velocity. No accel. Equate forces.



$$\underbrace{\frac{\pi D^3}{6}(\rho_s-\rho)g}_{\text{submerged weight}} = \underbrace{C_D \tfrac{1}{2}\rho w^2 \frac{\pi D^2}{4}}_{\text{drag}} \qquad (1)$$

but C_D



$Re = \frac{wD}{\nu}$

Small particles: viscous drag dominates. no flow separation
Low Re

$$C_D \propto \frac{1}{Re} \propto \frac{1}{wD}$$

$$\therefore \quad D^3 \propto \frac{1}{wD} w^2 D^2 \qquad \text{from (1)}$$

$$\therefore \quad w \propto D^2 \qquad \text{QED.}$$

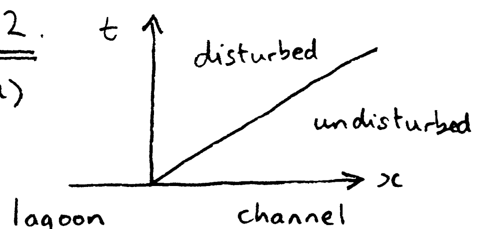
Large particles: form (or pressure) drag dominates. flow separation
High Re

$$C_D \simeq \text{const}$$

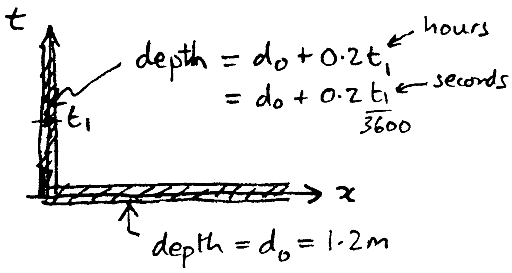
$$\therefore \quad D^3 \propto w^2 D^2 \qquad \text{from (1)}$$

$$\therefore \quad w \propto D^{1/2}$$

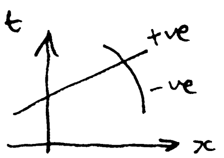
Q2.
a)



boundary conditions



From data book
$u - 2c = k_1$ const on any −ve charac. (1)
$u + 2c = k_2$ const on any +ve charac. (2)



Any −ve charac can be traced back to undisturbed region where $u = u_0$ and $c = c_0 = \sqrt{g d_0}$

$\therefore \quad u - 2c = k_1 = u_0 - 2c_0$ <u>everywhere</u> in RH quadrant.

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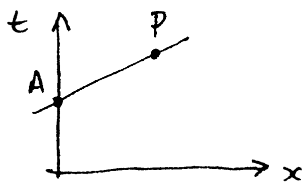
Q2 (a) cont'd.



On +ve charac AP, for any P on AP

$u_A + 2c_A = u_p + 2c_p = k_2$ from (2)

but $u_p - 2c_p = k_1$ from (1)

Adding $\Rightarrow$ $u_p = (k_1 + k_2)/2$ = const on AP

Subtracting $c_p = (k_2 - k_1)/4$ = const on AP

$\therefore \dfrac{1}{\text{slope of +ve charac}} = \left(\dfrac{dx}{dt}\right)_{\substack{AP \\ \text{at } P}} = u_p + c_p$ = const + const = const

$\therefore$ +ve charac.s are STRAIGHT lines.

So $\left(\dfrac{dx}{dt}\right)_{\substack{AP \\ \text{at } P}} = u_p + c_p = u_p - 2c_p + 3c_p$

$= u_0 - 2c_0 + 3c_p$ from (1)

$= u_0 - 2\sqrt{gd_0} + 3c_p$

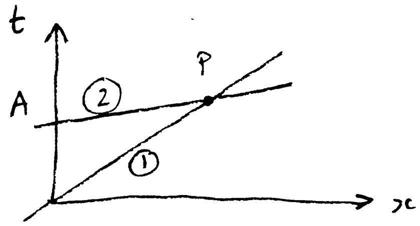
But c_p = const on AP $= c_A = \sqrt{gd_A}$

$\therefore \dfrac{1}{\text{+ve slope}} = u_0 - 2\sqrt{gd_0} + 3\sqrt{gd_A}$

$\text{slope} = \dfrac{dt}{dx} = \dfrac{1}{dx/dt} = \left[u_0 - 2\sqrt{gd_0} + 3\sqrt{gd_A}\right]^{-1}$

with $d_A = d_0 + 0.2\dfrac{t_1}{3600} = d_0 + \dfrac{t_1}{18 \times 10^3}$ QED

2.b)



Undisturbed bdy (1)

$\dfrac{dx}{dt} = u_0 + \sqrt{gd_0}$

$u_0 = -0.5$ m/s

$\sqrt{gd_0} = \sqrt{9.81(1.2)} = 3.43$ m/s

$\therefore \dfrac{dx}{dt} = -0.5 + 3.43 = 2.93$ m/s

$\left(\dfrac{dt}{dx}\right)_1 = \dfrac{1}{2.93} = 0.341$ s/m.

On AP

$d_A = 1.2 + 0.2 = 1.4$ m

$\sqrt{gd_A} = \sqrt{9.81(1.4)} = 3.71$ m/s

$\dfrac{dx}{dt} = -0.5 - 2(3.43) + 3(3.71) = 3.76$ m/s

$\left(\dfrac{dt}{dx}\right)_2 = 0.266$ s/m.

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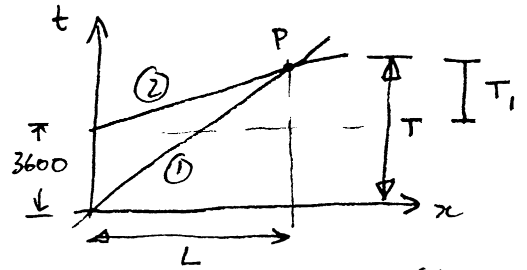
Q2(b) cont'd



$T = 3600 + T_1$

$T = L\left(\frac{dt}{dx}\right)_1 \qquad T_1 = L\left(\frac{dt}{dx}\right)_2 \qquad \therefore \; L(0.341) = 3600 + L(0.266)$

$\therefore \quad L = 3600/(0.341 - 0.266) = 48 \times 10^3\,\text{m}$

$T = L(0.341) = 16.4 \times 10^3$ seconds $=$ **4.55 hrs**

At intersection point (and beyond) Method of Characteristics no longer applies (– it predicts two values of depth at P). Will get a hydraulic jump, bore, surge here (if one has not occurred sooner, (which it probably has)). Momentum and continuity considerations may help track further evolution of the surge as it progresses into the undisturbed region.

Q3 $\quad \frac{1}{\sqrt{\lambda}} = -2\log\left[\frac{k}{3.7d} + \frac{2.51}{Re\sqrt{\lambda}}\right]$

Darcy: $h_f = \frac{\lambda L v^2}{2gD}$

$\Rightarrow \sqrt{\lambda} = \frac{\sqrt{2gDS}}{v}$ with $S = \frac{h_f}{L}$ hydraulic grad.

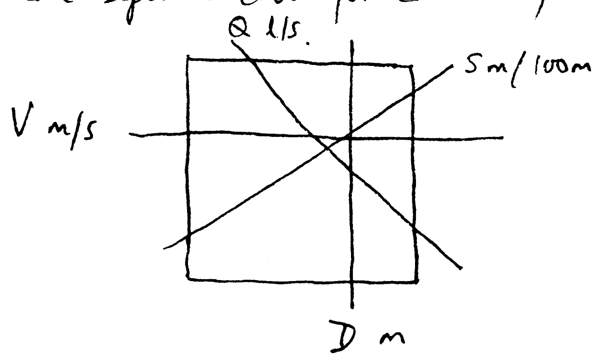
$\therefore \quad \frac{v}{\sqrt{2gDS}} = -2\log\left[\frac{k}{3.7D} + \frac{2.51\,v}{Re\sqrt{2gDS}}\right] \qquad Re = vD/\nu$

$v = -2\sqrt{2gDS}\,\log\left[\frac{k}{3.7D} + \frac{2.51\,\nu}{D\sqrt{2gDS}}\right]$

Five variables v, D, S, ν, k $\qquad$ (also $Q = \frac{\pi D^2}{4}.v$)

Assume Temp = 15°C ⇒ kinematic viscosity constant.

Create separate chart for each k, relating v, D, S (and Q).



If any two are known, the other two may be found from intersection point.

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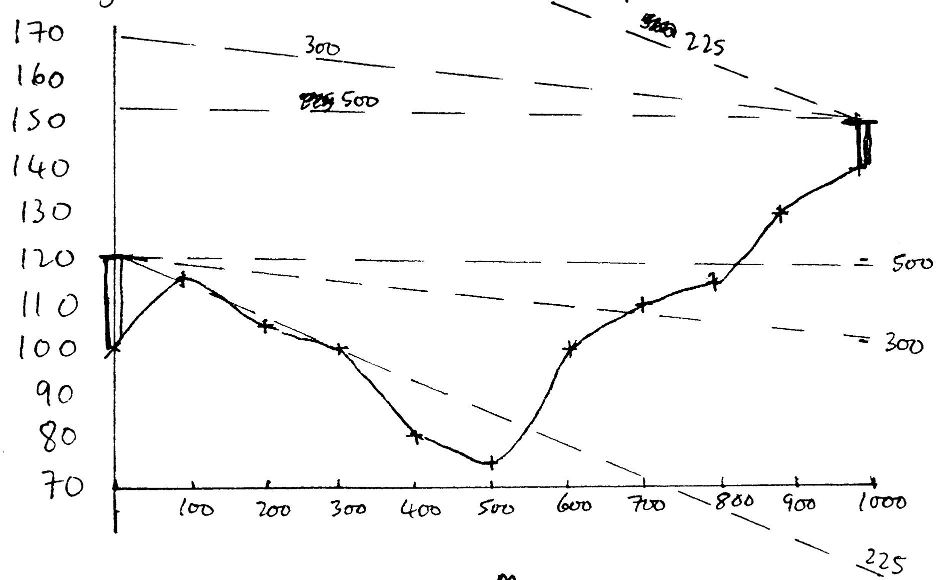
Q3 cont'd.

b) Longitudinal section. Draw at compressed horiz. scale.



Available pipes (dia, mm) Headloss m/100m at Q = 200 l/s

~~500~~ 225 mm ϕ	7 m/100m
~~300~~ 300 mm ϕ	1·8 m/100m
~~225~~ 500 mm ϕ	0·15 m/100m.

Draw on hydraulic grade lines from either end.

Difference between hydraulic grade lines = (150m − 120m) + head loss over 1km

500 mm ⇒ 30 + 1·5 = 31·5 m
300 mm ⇒ 30 + 18 = 48 m
225 mm ⇒ 30 + 70 = 100 m

Booster pump gives 200 l/s for 59 m head, so pick 300 mm dia pipe, and put pump at 600m point.

Reason for 300 mm ϕ:

i) should really draw pipeline characteristic on (H, Q) diagram for each diameter, and then pick the diam. whose characteristic ~~cha~~ crosses the pump characteristic nearest the required flow rate.

ii) can see 225 mm ϕ gives negative pressures near 100m mark – unless one pumps from right next to treatment works – but that would give huge pressures (~ 100 m head) in the pipeline, and 100 mm head seems to be beyond capabilities of the pump anyway.

iii) The 500 mm ϕ is an option. Could have gravity flow to 800m then a 31·5m lift by the pump.

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Q3 cont'd.

→ BUT

it seems more sensible to have a 300 mm ϕ pipe to flow under gravity to near the 600 m mark, and then have a 48 m lift there.

Why 600 m? If pick 500 m ⇒ large pressures beyond pump
If pick 700 m ⇒ negative pressures before pump.

So, pick pump speed to give 200 l/s at 48 m head.

i) Pump characteristic is plotted on graph paper
ii) Pipeline characteristic is sketched on graph paper.
There is no need to calculate it. We already know that
- it passes through point O ($Q=0$ with static head = 30 m)
- it passes through req'd point A ($Q = 200$ l/s with head = 48 m
(= $\underset{\text{static}}{30\text{m}} + \underset{\text{friction}}{18\text{m}}$)

Now pick any point Q_1, H_1 on pump characteristic at $N_1 = 1000$ rpm.

If pump runs at N_2 r.p.m. then the points

$$Q_2 = \frac{N_2}{N_1} Q_1 \quad \text{and} \quad H_2 = \frac{N_2^2}{N_1^2} H_1$$

will lie on pump characteristic curve for N_2 r.p.m.

So, want to find N_2 such that $(Q_2, H_2) =$ point A_{Design} $= (200\,\text{l/s}, 48\,\text{m})$.

Starting at any Q_1, H_1 on N_1 curve, and letting N_2 be a variable defines a curve

$$(Q_2, H_2) = (Q_2(N_2), H_2(N_2))$$
$$= ((N_2/N_1) Q_1 , (N_2^2/N_1^2) H_1)$$
$$= (x, y)$$

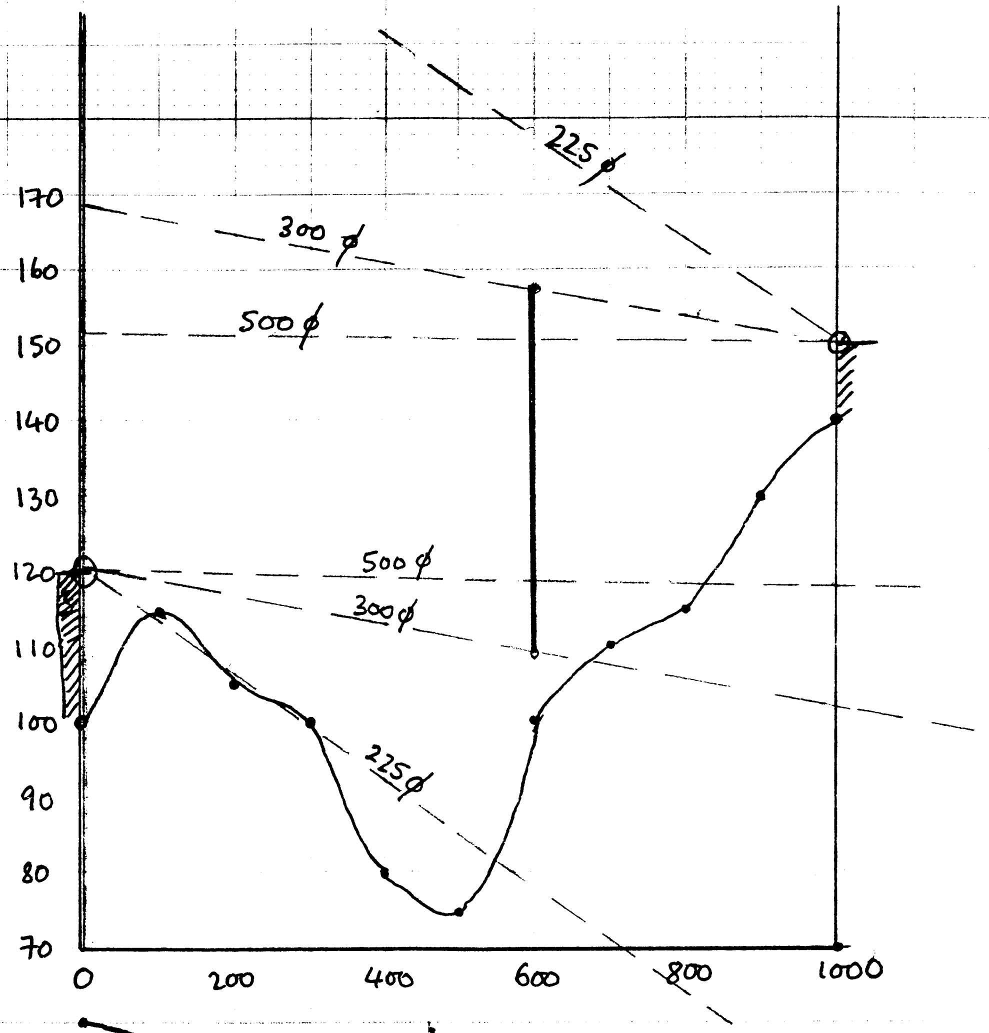
So $$x^2 = \frac{N_2^2}{N_1^2} Q_1^2 = \frac{y}{H_1} Q_1^2 \quad \therefore \quad y = \left(\frac{H_1}{Q_1^2}\right) x^2$$

← known, if Q_1, H_1 are known.

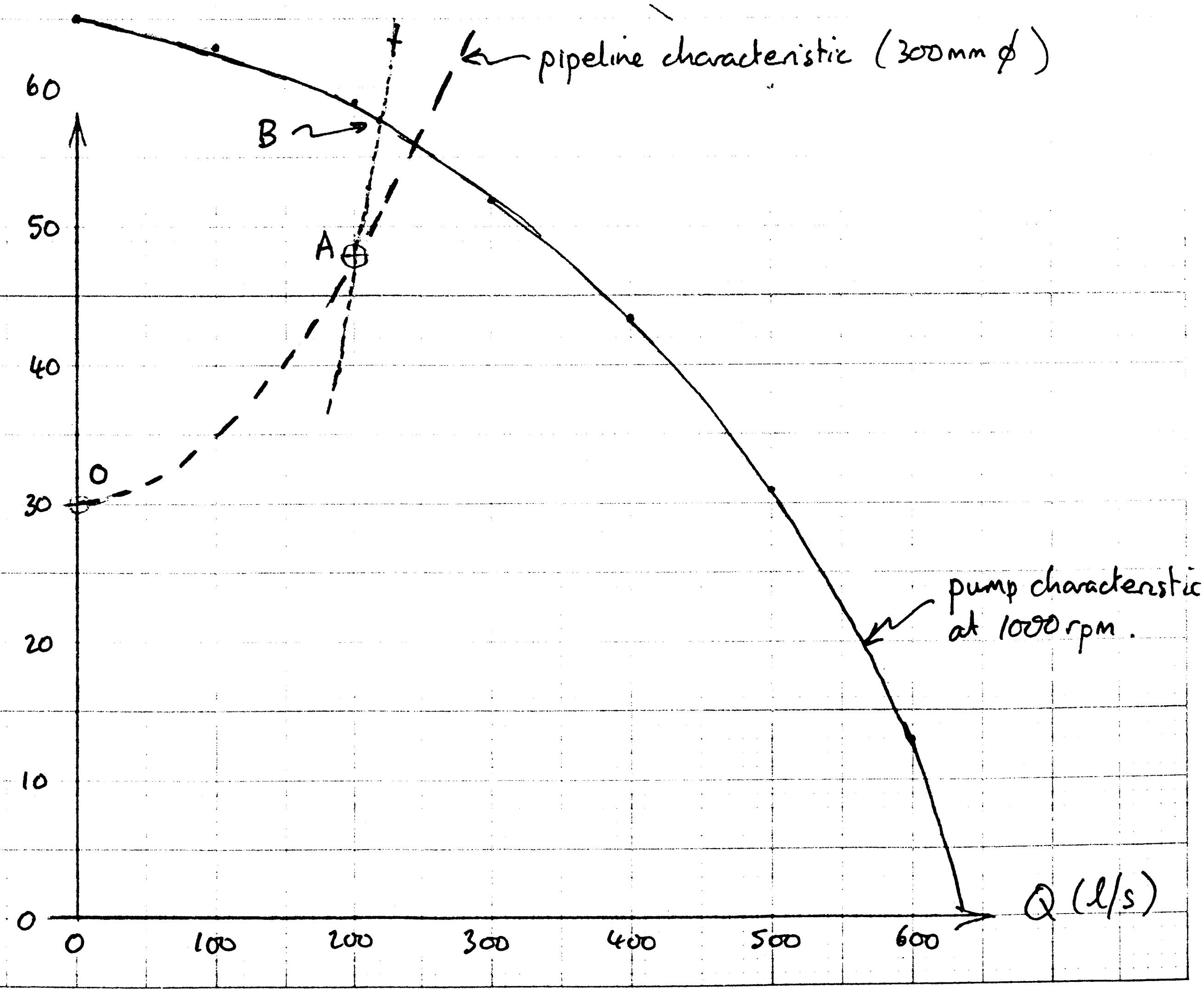
i.e. the set of points "generated" by Q_1, H_1 lie on a parabola.
We want that parabola to pass through A.

$y = kx^2$ with $y = 48$ m, $x = 200$ l/s

$$\therefore \quad k = \frac{48}{(200)^2} = 0.0012$$



225 ϕ
300 ϕ
500 ϕ
500 ϕ
300 ϕ
225 ϕ
170
160
150
140
130
120
110
100
90
80
70
0
200
400
600
800
1000



pipeline characteristic (300mm ϕ)
B
A
O
pump characteristic at 1000 rpm.
60
50
40
30
20
10
0
0
100
200
300
400
500
600
Q (l/s)

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Q3 b) cont'd.

Draw the parabola $H = kQ^2$ above the point A until it intersects pump 1000 rpm curve.

Q	200	210	220	230
$kQ^2 = H$	48	52.9	58.0	63.5

This crosses $N_1 = 1000$ rpm pump curve at B, where $Q_1 \simeq 220$ l/s, $H_1 = 58.0$ m.

Can thus solve for $N_2 = N_1 \dfrac{Q_2}{Q_1} = (1000)\left(\dfrac{200}{220}\right) = 909.1$ rpm

Check $N_2 = N_1 \sqrt{\dfrac{H_2}{H_1}} = (1000)\left(\dfrac{48}{58}\right)^{1/2} = 909.7$ rpm ✓

∴ Run pump at 910 rpm.

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Q4. a) $u_* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$ $\quad \tau_0 = \rho g R S \quad u_* = \sqrt{gRS}$

Wide channel $R \approx d \Rightarrow u_* \approx \sqrt{gdS} = \sqrt{9.81(1)(0.001)} \approx 0.1\,m/s.$

$\frac{u_* k_s}{\nu} = \frac{(0.1)(0.01)}{10^{-6}} = 1000 > 70 \quad \therefore$ hydraulically rough.

b) Use $u = u_*\, 2.5 \log\left(\frac{30.2y}{k_s}\right)$ for all heights (although only really applies to overlap layer).

y	0.01	0.1	0.2	0.5	1.0 (m)
u	0.85	1.43	1.6	1.83	2.0 (m/s)

c) $W = (56\times10^4)(0.00021)^2(1.65) = 0.0407$

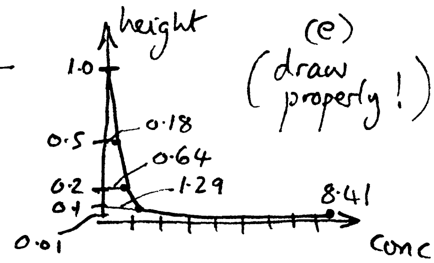
$\frac{W}{Ku_*} = \frac{0.0407}{(0.4)(0.1)} = 1.02 \approx 1.0$

d) $\frac{C}{C_a} = \left[\left(\frac{d-y}{y}\right)\left(\frac{a}{d-a}\right)\right]^1$ $\qquad C_a = 9.9\ kg/m^3$ at $a = 0.01\,m$

$\therefore C = (9.9)\left(\frac{0.01}{0.99}\right)\left(\frac{1-y}{y}\right) = 0.1\left(\frac{1-y}{y}\right)$

y	0.01	0.1	0.2	0.5	1.0
C	9.9	0.9	0.4	0.1	0
u	0.85	1.43	1.6	1.83	2.0
uC	8.41	1.29	0.64	0.18	0

(e) (draw properly!)



(f) i) $\int_{0.01}^{1} uC\,dy = 0.5\left(\frac{0.183}{2}\right) + 0.3\left(\frac{0.64+0.183}{2}\right) + 0.1\left(\frac{1.29+0.64}{2}\right)$

$+ \left(\frac{8.41+1.29}{2}\right)(0.09) = \underline{0.7\ kg/ms}$

ii) $\int_b^d uC\,dy = 11.6\,u_* C_b\, b\left[I_1 \log Ad + I_2\right]$

$b/d = \frac{0.01}{1} = 0.01 \qquad I_1 = 0.788 \qquad I_2 = 2.107$

$A = \frac{30.2}{k_s} = 3020 \Rightarrow \int_b^d uC\,dy = (11.6)(0.1)(9.9)(0.01)\left[0.788 \log 3020 + 2.107\right]$

$= \underline{0.97\ kg\ m^{-1}\,s^{-1}}$