1

3F4 Data Transmission, May 2005 - Solutions

1. (a) To match the PSD of the transmitted signal to suit the frequency response of the channel. For example, many channels have a poor response at d.c. and at low frequencies owing to the use of a.c. coupling. Also a low-pass channel limits the ability of the channel to carry high frequencies.

To permit self-synchronisation, i.e., there should be sufficient information in the transitions of the transmitted signal to allow timing regeneration to be performed at the receiver.

[5]

(b) Now,

$$S_x(\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} R(m)\, e^{j m \omega T_s}$$

where $R(m) = E[a_k a_{k+m}]$

Now need to calculate $R(m)$ for the polar line coding scheme

m	b_k		b_{k+m}	a_k	a_{k+m}	R_i	p_i	$R(m) = \sum R_i p_i$
0	0		0	-1	-1	1	0.4	1
	1		1	1	1	1	0.6	
1	0		0	-1	-1	1	$(0.4)^2 = 0.16$	0.04
	0		1	-1	1	-1	$0.4 \times 0.6 = 0.24$	
	1		0	1	-1	-1	$0.4 \times 0.6 = 0.24$	
	1		1	1	1	1	$(0.6)^2 = 0.36$	
2	0	(0)	0	-1	-1	1	0.064	0.04
	0	(1)	0	-1	-1	1	0.096	
	0	(0)	1	-1	1	-1	0.096	
	0	(1)	1	-1	1	-1	0.144	
	1	(0)	0	1	-1	-1	0.096	
	1	(1)	0	1	-1	-1	0.144	
	1	(0)	1	1	1	1	0.144	
	1	(1)	1	1	1	1	0.216	

Same result for $m \geq 2$, i.e., $R(m) = 0.04$.

Clearly $R(m)$ is finite over a ∞ range of m, so to permit a more meaningful physical interpretation to be made, $R(m)$ will be expressed as a sum of two parts, i.e.,

$$S_x(\omega) = \frac{1}{T_s}\left\{ \dots + 0.04e^{-j2\omega T_s} + 0.04e^{-j\omega T_s} + 1 + 0.04e^{j\omega T_s} + 0.04e^{j2\omega T_s} + \dots \right\}$$

$$= \frac{1}{T_s}\left\{ \dots + 0.04e^{-j2\omega T_s} + 0.04e^{-j\omega T_s} + 0.04 + 0.04e^{j\omega T_s} + 0.04e^{j2\omega T_s} + \dots \right\}$$

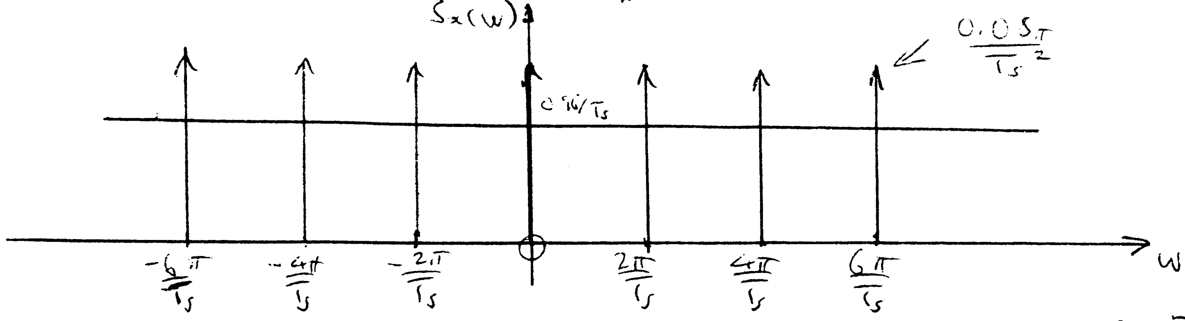
$$+ \frac{1}{T_s} \times 0.96$$

$$= \frac{1}{T_s}\left[0.04 \sum_{m=-\infty}^{\infty} e^{-jm\omega T_s} + 0.96 \right]$$

Substitute the sum of exponentials as a series of impulses in the frequency domain,

$$S_x(\omega) = \frac{1}{T_s}\left[0.04 \, \frac{2\pi}{T_s} \sum_{m=-\infty}^{\infty} \delta\left(\omega - m\frac{2\pi}{T_s}\right) + 0.96 \right]$$

$$S_x(\omega) = \frac{0.96}{T_s} + \frac{0.08\pi}{T_s^2} \sum_{m=-\infty}^{\infty} \delta\left(\omega - m\frac{2\pi}{T_s}\right)$$



[10]

(c) The transmitted power spectrum is given by

$$S_y(\omega) = S_x(\omega) \, |H(\omega)|^2$$

Need to work out $H(\omega)$ – use E+I Data Book,

(3)

$$H(\omega) = ab \operatorname{sinc}\left(\frac{\omega b}{2}\right)$$

where from Fig. 1, $a = 1$, $b = T_s$, so

$$H(\omega) = T_s \operatorname{sinc}\left(\frac{\omega T_s}{2}\right)$$

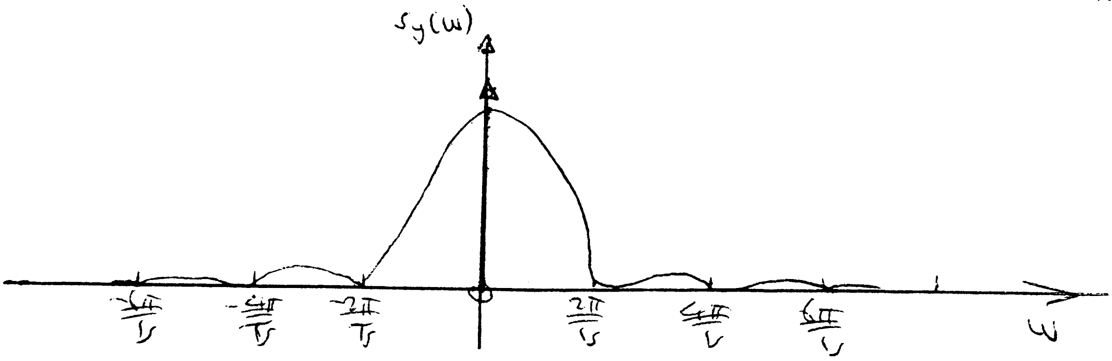
So,

$$S_y(\omega) = \left(\frac{0.08\pi}{T_s^2}\sum_{m=-\infty}^{\infty}\delta\left(\omega - m\frac{2\pi}{T_s}\right) + \frac{0.96}{T_s}\right)\left|T_s \operatorname{sinc}\left(\frac{\omega T_s}{2}\right)\right|^2$$

$$= \left(0.08\pi\sum_{m=-\infty}^{\infty}\delta\left(\omega - m\frac{2\pi}{T_s}\right) + 0.96\,T_s\right)\left|\operatorname{sinc}\left(\frac{\omega T_s}{2}\right)\right|^2$$

$$= 0.08\,\delta(\omega) + 0.96\,T_s\left|\operatorname{sinc}\left(\frac{\omega T_s}{2}\right)\right|^2$$

since sinc function $= 0$ at multiples of $\frac{2\pi}{T_s}$.



[5]

2 (a) Equalisers are filters which process the received signal to reduce intersymbol interference (ISI) at the input of the data slicer.

A zero-forcing equaliser forces the pulse response to zero at all times except for the response to symbol n at time instant n, i.e., the equaliser output is the unit pulse δ_n in response to p_n. Note that the z-transform of δ_n is equal to 1 and so,

$P(z)\, H_E(z) = 1$. Consequently the equaliser response is,

$$H_E(z) = \frac{1}{P(z)}$$

where $P(z)$ is the z-transform of the pulse response and $H_E(z)$ is the z-transform of the equaliser response.

In the frequency domain,

$$H_E(e^{j\omega T}) = \frac{1}{P(e^{j\omega T})}$$

Thus at frequencies where $P(e^{j\omega T})$ is small large noise amplification will occur,

This will make the bit error rate (BER) worse.

The MMSE equaliser overcomes this problem to some extent by taking into account the noise. That is, the ISI supression properties of the equaliser are reduced in a trade-off for less noise amplification. Unfortunately the optimum design now depends on the quantity of noise present. In the absence of noise, the MMSE and zero-forcing equaliser designs are identical.

[6]

(b) $$P(z) = 0.8 - 0.6z^{-1} + 0.1z^{-2}$$

For a ZF design,

$$H_E(z) = \frac{1}{P(z)}$$

$$= \frac{1}{0.8 - 0.6z^{-1} + 0.1z^{-2}}$$

$$= \frac{1}{0.8(1 - 0.75z^{-1} + 0.125z^{-2})}$$

$$= 1.25 \times \frac{1}{(1 - 0.75z^{-1} - 0.125z^{-2})}$$

An IIR filter has the response

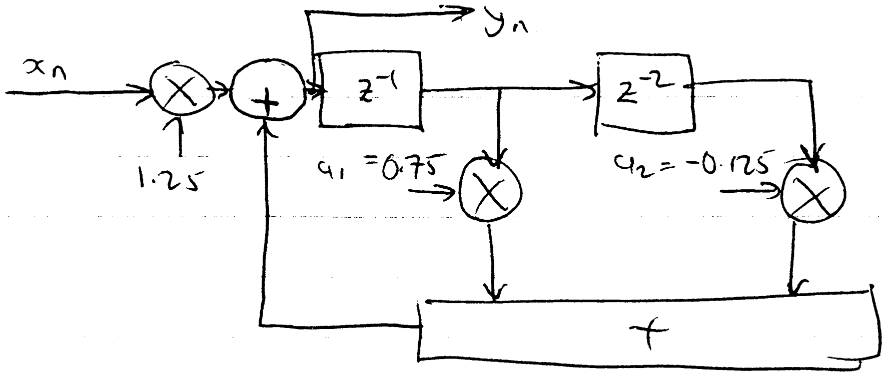
$$H(z) = \frac{1}{1 - z^{-1}a_1 - z^{-2}a^2 - z^{-3}a_3 - \ldots}$$

so, $a_1 = 0.75$ and $a_2 = -0.125$.

where,

$$H_E(z) = 1.25\,H(z)$$

yielding the following block diagram,



IIR filters are difficult to deal with, i.e.,

- stability is not guaranteed
- adaptation methods are difficult to derive
- their recursive nature makes them prone to numerical instability.

[6]

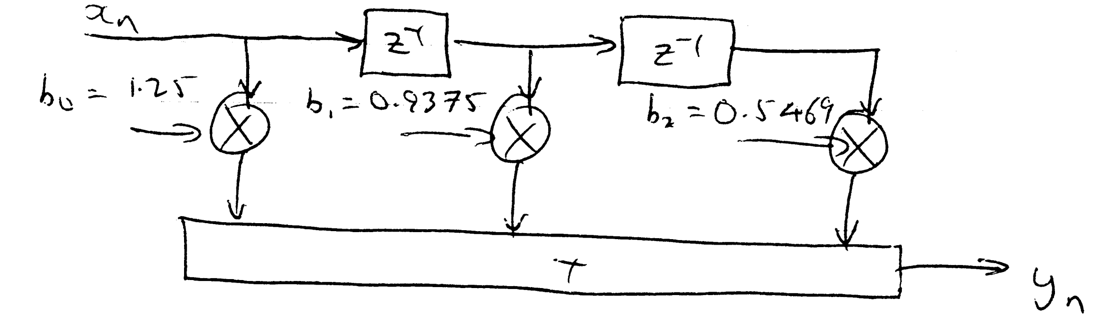
(c) We have $H_E(z) = \frac{1}{P(z)}$ so we can get $H_E(z)$ by performing polynomial division.

$$
\begin{array}{r|l}
 & 1.25 + 0.9375z^{-1} + 0.546875z^{-2} \\
\hline
0.8 - 0.6z^{-1} + 0.1z^{-2} & 1 \\
 & 1 - 0.75z^{-1} + 0.125z^{-2} \\
\cline{2-2}
 & 0.75z^{-1} - 0.125z^{-2} \\
 & 0.75z^{-1} - 0.5625z^{-2} + 0.09375z^{-2} \\
\cline{2-2}
 & 0.4375z^{-2} - 0.09375z^{-3} \\
 & 0.4375z^{-2} - 0.328125z^{-3} + 0.0546875z^{-4} \\
\cline{2-2}
 & 0.234375z^{-3} - 0.0546875z^{-4}
\end{array}
$$

so,

$$H_E(z) = 1.25 + 0.9375z^{-1} + 0.546875z^{-2}$$
$$\equiv b_0 + b_1z^{-1} + b_2z^{-2}$$
$$\therefore \quad b_0 = 1.25, \quad b_1 = 0.9375 \text{ and } b_2 = 0.5469$$

7



Without equalisation:
For a binary uni-polar scheme the minimum opening for a '1' is

$$0.8 - 0.6 = 0.2$$

and the minimum opening for a '0' is

$$0 + 0.1 = 0.1$$

So the worst case eye opening is

$$h_{min} = 0.2 - 0.1 = 0.1$$

Now,

$$BER = Q\left(\frac{h_{min}}{2\sigma_v}\right) = Q\left(\frac{0.1}{2 \times 0.05}\right)$$

$$= Q(1) = 0.1587.$$

With equalisation:
Need to calculate the 'residuals' and the noise power.
The equaliser output is given by

$$Y(z) = P(z)\, H_E(z)$$

$$= (p_0 + p_1 z^{-1} + p_2 z^{-2})(b_0 + b_1 z^{-1} + b_2 z^{-2})$$

$$= p_0 b_0 + (p_0 b_1 + p_1 b_0) z^{-1} + (p_0 b_2 + p_1 b_1 + p_2 b_0) z^{-2} + (p_1 b_2 + p_2 b_1) z^{-3} + p_2 b_2 z^{-4}$$

$$= 0.8 \times 1.25 + (0.8 \times 0.9375 - 0.6 \times 1.25) z^{-1} + (0.8 \times 0.5469 - 0.6 \times 0.9375 + 0.1 \times 1.25) z^{-2} + (-0.6 \times 0.5469 + 0.1 \times 0.9375) z^{-3} + 0.1 \times 0.5469 z^{-4}$$

$$= 1 \underbrace{- 0.2344 z^{-3} + 0.05469 z^{-4}}_{\text{'residuals'}}$$

8

Note they are equal to the negative of the remainder of the polynomial division.

So,

worst case opening for a '1' $= 1 - 0.2344 = 0.7656$

" " " " '0' $= 0 + 0.05469 = 0.05469$

$\therefore$ minimum eye opening, $h_{min} = 0.7656 - 0.05469$

$= 0.7109$.

rms noise,

$$\sigma_w = \sigma_v \sqrt{b_0^2 + b_1^2 + b_2^2}$$

$$= 0.05 \sqrt{1.25^2 + 0.9375^2 + 0.5469^2}$$

$$= 0.05 \times 1.66$$

$$= 0.0828$$

So,

$$BER = Q\left(\frac{h_{min}}{2\sigma_w}\right) = Q\left(\frac{0.7109}{2 \times 0.0828}\right)$$

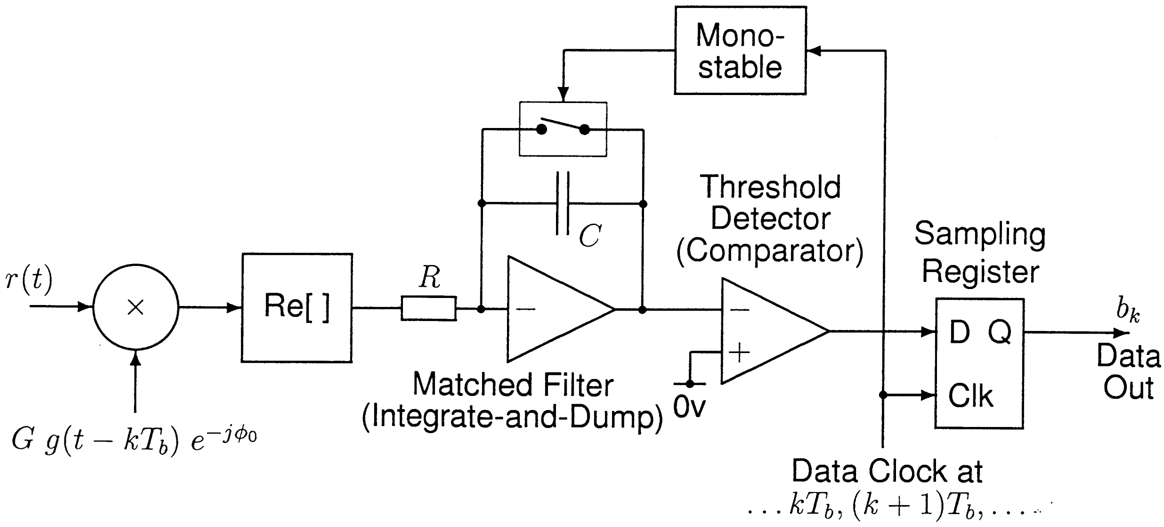
$$= Q(4.294)$$

$$= 8.78 \times 10^{-6}$$

[8]

(9)

3. (a) **BPSK Demodulator**



$$y(k) = G \int_{kT_b}^{(k+1)T_b} Re[r(t)\ g(t-kT_b)e^{-j\phi_0}]dt$$

For an optimum demodulator the decision for the k^{th} bit is based on whether $y(k)$ is positive or negative. G is an arbitrary positive gain constant which does not affect the polarity of $y(k)$ or the demodulator decision.

Working from the left in the figure, the multiplier calculates $r(t)\ G\ g(t-kT_b)\ e^{-j\phi_0}$. The next box takes the real part of the result (the $Re[.]$ operation). To work correctly, the demodulator must know the reference phase ϕ_0 of the received phasors. This is normally extracted by a phase-locked loop (see part (d) of this question).

The integration in the equation is performed by the op. amp. circuit (Matched Filter). The correct limits for the integration are achieved by discharging the capacitor at the start of each bit period (at $t = kT_b$ using a short pulse from the monostable), and by sampling the polarity of the integrator output at the end of each bit period (at $t = (k+1)T_b$) just before the monostable discharges the integrator for the next bit. $y(k)$ is the signal on the op. amp. output at the end of the bit period.

The threshold detector determines the polarity of $y(k)$ and the sampling register samples this decision at the end of each bit period, when the signal-to-noise ratio is greatest. [25%]

(b) **Quadrature Demodulator:**

If $s(t) = Re[r(t)\, e^{j\omega_C t}]$ and $r(t) = [u(t) + j\, v(t)]\, e^{j\phi_0}$ then:

$$s(t) = Re[\, [u(t) + j\, v(t)]\, e^{j(\omega_C t + \phi_0)}\,] = u(t)\cos(\omega_C t + \phi_0) - v(t)\sin(\omega_C t + \phi_0)$$

The quadrature demodulator obtains $i(t)$ and $q(t)$, from the modulated signal $s(t)$ as follows.

From multiplier 1:

$$\begin{aligned} i'(t) &= s(t) \times 2\cos(\omega_C t + \phi_1) \\ &= [u(t)\cos(\omega_C t + \phi_0) - v(t)\sin(\omega_C t + \phi_0)] \times 2\cos(\omega_C t + \phi_1) \\ &= 2u(t)\cos(\omega_C t + \phi_0)\cos(\omega_C t + \phi_1) - 2v(t)\sin(\omega_C t + \phi_0)\cos(\omega_C t + \phi_1) \\ &= u(t)\cos(\phi_0 - \phi_1) + u(t)\cos(2\omega_C t + \phi_0 + \phi_1) \\ &\quad - v(t)\sin(\phi_0 - \phi_1) - v(t)\sin(2\omega_C t + \phi_0 + \phi_1) \end{aligned}$$

The two terms modulated onto carriers at $2\omega_C$ are rejected by lowpass filter 1, so the output of the filter is:

$$i(t) = u(t)\cos(\phi_0 - \phi_1) - v(t)\sin(\phi_0 - \phi_1)$$

Similarly from multiplier 2:

$$\begin{aligned} q'(t) &= s(t) \times [-2\sin(\omega_C t + \phi_1)] \\ &= [u(t)\cos(\omega_C t + \phi_0) - v(t)\sin(\omega_C t + \phi_0)] \times [-2\sin(\omega_C t + \phi_1)] \\ &= -2u(t)\cos(\omega_C t + \phi_0)\sin(\omega_C t + \phi_1) + 2v(t)\sin(\omega_C t + \phi_0)\sin(\omega_C t + \phi_1) \\ &= u(t)\sin(\phi_0 - \phi_1) - u(t)\sin(2\omega_C t + \phi_0 + \phi_1) \\ &\quad + v(t)\cos(\phi_0 - \phi_1) - v(t)\cos(2\omega_C t + \phi_0 + \phi_1) \end{aligned}$$

and the output of lowpass filter 2 is:

$$q(t) = u(t)\sin(\phi_0 - \phi_1) + v(t)\cos(\phi_0 - \phi_1)$$

(c) **Practical BPSK receiver**

Since $g(t - kT_b)$ is constant over the entire bit period, it can be incorparted as part of the arbitrary gain constant G. Hence the first two blocks just need to calculate $Re[r(t)\,.\,G\, e^{-j\phi_0}]$.

If $r(t) = [u(t) + j\, v(t)]\, e^{j\phi_0}$, then

$$Re[r(t)\,.\,G\, e^{-j\phi_0}] = G\, u(t)$$

We see from the first result in part (b) that $i(t) = u(t)$ if $\phi_1 = \phi_0$. Hence the upper output of the quadrature demodulator can provide the equivalent signal to the output of the second block in Fig. 1 of the question. Hence the quadrature demodulator can replace the first two blocks of Fig. 1 when the system input is $s(t)$ rather than $r(t)$.

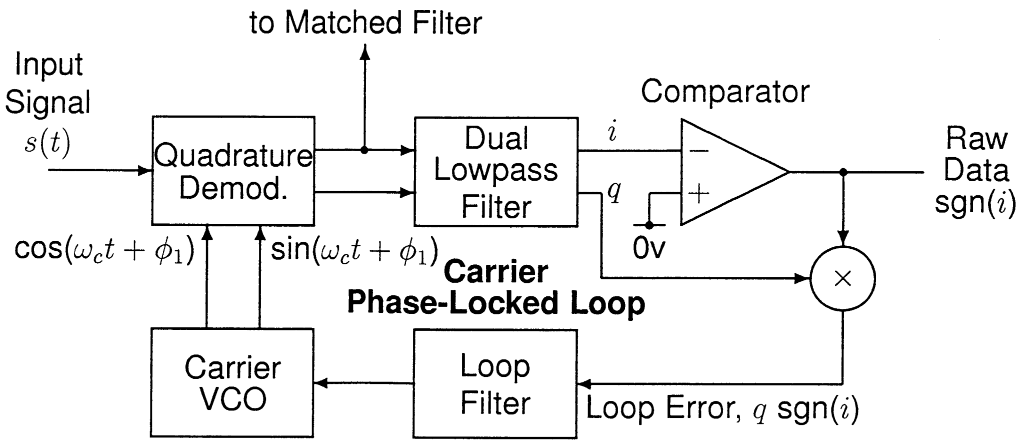
It is desirable that $\phi_1 = \phi_0$ in order that the upper output of the quadrature demodulator is given by:

$$i(t) = u(t)\cos(\phi_0 - \phi_1) - v(t)\sin(\phi_0 - \phi_1) = u(t) \quad \text{if } \phi_1 = \phi_0$$

(11)

Hence $i(t)$ responds only to $u(t)$ and not to $v(t)$. If we assume that ϕ_0 represents the phase shift between the modulator and demodulator due to the path delay, then $u(t)$ is the inphase component at the modulator, which, for a BPSK signal, is normally the signal-dependent component that conveys all the signal information. Thus we get the optimum signal-to-noise ratio at the detector.

(d) **Phase-Locked Loop**



The block diagram of a suitable PLL is shown above. The loop error signal is q sgn(i) in order to produce a characteristic which repeats at multiples of π. This allows the loop to lock up at the positive zero-crossings of the error characteristic such that the phase error, $\phi_1 - \phi_0$, between $s(t)$ and the Carrier VCO (voltage controlled oscillator) is either 0 or π. Thus the Carrier VCO will either be in phase or in antiphase with the carrier of $s(t)$, and the $i(t)$ signal will be a smoothed version of the original data or its complement. Differential decoding (not shown) must be used to eliminate this ambiguity.

4. (a) **Digital vs. Analogue transmission**

Digital is preferred to Analogue because:

- It is less susceptible to cumulative degradations.
  Effects of additive noise can be eliminated at regular intervals in a digital comms link by threshold detection and error correction coding.
- Ultimate noise levels are determined by the analogue/digital conversions and not by the channel, giving much better dynamic range.
- It can be made more secure by encryption.
- It can be multiplexed using frequency-division, time-division or spread-spectrum multiple access *(FDMA, TDMA or SSMA)*; whereas FDMA is the only sensible method for analogue signals.
- Digital links can handle a wide range of source material (multi-media).

The extra bandwidth of digital systems can be minimised by use of compression at the source encoding stage (eg MP3 for audio and MPEG-2 for video), and also by employing bandwidth efficient modulation methods, such as 64-QAM, which transmit a number of bits (6 in the case of 64-QAM) of information per symbol. In general the bandwidth of a modulation scheme is proportional to its symbol rate, not its bit rate.

(b) **A bandwidth efficient modulation scheme**

M^2-QAM (quadrature amplitude modulation) is the general class of bandwidth efficient modulation schemes. It employs M levels of modulation on each of two quadrature carriers, and thus is able to convey $m = \log_2 M$ bits on each carrier per modulation symbol. Hence each symbol conveys $2m$ bits of data. Thus the symbol rate is $1/2m$ of the bit rate and the bandwidth required is $1/2m$ of that of a binary modulation scheme such as BPSK.

The bandwidth required is proportional to the symbol rate, because the autocorrelation function (ACF) of QAM is a triangular pulse whose width equals twice the symbol period and is independent of the number of modulation levels. Hence the width of the power spectrum (the Fourier transform of the ACF) is proportional to the symbol rate.

Below (from the lecture notes) is shown the constellation of 16-QAM, which has 4-level modulation on each of the two quadrature components. This conveys 4 bits per symbol. The arrows show the modulation states and the dotted lines show the decision boundaries for an optimal detector.

A more complex scheme is 64-QAM which has 8-level modulation on each component and conveys 6 bits per symbol. The number of bits per symbol cannot be increased much above 6, because the system loses immunity to noise when the transmitted states get too close together.

13

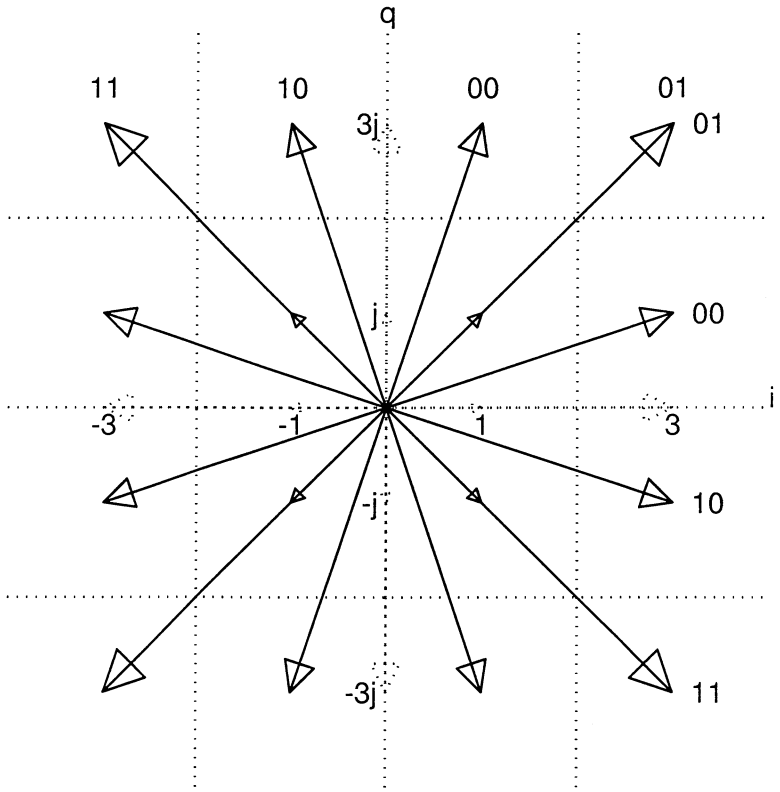


Fig 5.2: 16-QAM phasor diagram

(c) **Overcoming multi-path ISI:**

Problems due to multi-path delay differences being greater than the symbol period can be overcome by employing orthogonal frequency division multiplexing (OFDM), as well as a bandwidth efficient modulation method such as 64-QAM.

The aim of OFDM is to demultiplex the high-speed bit stream into N streams, each at $1/N$ of the original rate, which are then modulated onto N separate carrier waves. Typically $N \approx 1000$ to 2000.

The inverse FFT is used to put QAM data on each of N carriers, spaced by $1/T$ Hz, where T is the IFFT block period. Each carrier is an IFFT basis function which is multiplied by the modulation phasor. In this way the carriers are orthogonal to each other and may be demodulated by an equivalent FFT process without mutual interference at the receiver. The mutual orthogonality of the IFFT basis functions, means that there should be no interference between each modulated carrier and its neighbours. Orthogonality is not affected by the modulation process, because the modulation rate is no faster than once per FFT block period, so each modulated carrier is a pure tone for the duration of the block period T.

The OFDM signal has much improved resilience to typical multipath delays because of the much lower modulation rate on each carrier, compared with a single carrier system operating at the same overall bit rate. The use of a Guard Band between each IFFT block allows for variations in path delay up to the duration of the Guard Band, before any signal degradation occurs.

(d) Design of a DVB system

In order to tolerate path delay differences of up to $10\mu s$, the guard band period ΔT must be at least $10\mu s$ also. This is so that the phase and amplitude transitions, which occur on each tone due to the modulation, do not occur during the analysis period. This preserves the orthogonality of the tones at the FFT input.

To maintain full bandwidth efficiency the FFT analysis period T should be much longer than the guard band – say ten times the guard band $= 100\mu s$. (This is purely an example; many other choices for this parameter exist, such as $T = 224\mu s$ used for DVB in the UK.)

Hence the tone spacing of the FFT will be $1/T = 10$ kHz.

If 64-QAM is used as the modulation method, then each tone carries 6 bits per symbol at a symbol rate of $1/(T + \Delta T) = 9.09$ ksym/s.

Hence the bit rate of each tone is $6 \times 9.09 = 54.54$ kbit/s.

Therefore the number of tones required for a total bit rate of 40 Mbit/s is

$$N = 40\,.\,10^6 \,/\, (54.54\,.\,10^3) = 734.$$

Typically some extra tones will be needed to provide phase and amplitude references across the band, increasing N to about 800 tones. The bandwidth occupied by this signal will then be $N\times$ (tone spacing) $= 800 \times 10^4 = 8\,.\,10^6$ Hz $= 8$ MHz.

Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2005 - Comments

1. Generally well answered. Some candidates confused line coding with forward error correction (FEC) coding in part (a). A number of candidates made errors when calculating the power spectrum in part (b).

2. This question was in general answered very well. In part (c) some errors were made when calculating the min eye opening and neglecting to take into account noise power enhancement with the equaliser in place.

3. This question was answered very poorly. Part (a) was generally answered quite well. Given the bookwork nature of part (b) the number of errors was surprising. Relatively few candidates appreciated what was required in part (c).

4. This question was answered reasonably well. Most difficulties were experienced with part (d) where the assumptions made and the design rational was generally not explained very clearly.