ENGINEERING TRIPOS PART IIA

Thursday 28th April 2005 9 - 12

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (4 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 3

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1 (a) Derive an expression for the effective pressure force experienced by a small cubic fluid particle moving in a pressure gradient $\frac{dp}{ds}$, where s is the coordinate in flow direction. [20%]

(b) By applying Newton's Second law to the fluid particle of Part (a), express its acceleration as a function of $\frac{dp}{ds}$. [20%]

(c) Using the result of Part (b), derive Bernoulli's equation. List the necessary assumptions. [40%]

(d) How would you include viscous forces, for example if the fluid particle were inside the boundary layer region, in the result of Part (b)? [20%]

2 A Pitot-tube for measuring the velocity of a fluid is to be modelled by a three-dimensional point source of strength m in a uniform flow with velocity U.

(a) Write down an expression for the distance, s, from the source to the upstream stagnation point in terms of U and m. [30%]

(b) By using continuity, or otherwise, write down an expression for the diameter, D, of the Pitot-tube far downstream. [30%]

(c) A static pressure tapping is to be placed on the side of the tube so that the velocity can be measured using the difference between the stagnation pressure (at the nose) and the static pressure at this location. Work out how many tube diameters (D) downstream from the nose to install this static pressure tapping such that the error in measured velocity is less than 1%. For this estimate you do not need to find the functional form of the surface. Assume that at this distance downstream the probe is cylindrical with diameter D and the velocity induced by the source is parallel to the surface. Also assume that the point is far enough back from the surface that the angle between the line from the origin to that point and the x-axis is small enough so that $\sin\theta \approx 0$ and $\cos\theta \approx 1$. [40%]

3 (a) Explain how an aircraft wing gains circulation when starting from stationary in still air at an airport and accelerating up to take-off speed. What happens when it lands and comes to rest? [40%]

(b) An aircraft weighing 4 x 10^5 kg is flying at 200 ms^{-1} in steady level flight and is a long way from the ground. Each of the two wings is 30 m in span and the lift distribution may be considered uniform over each wing. The tail-plane of the aircraft is located 30 m behind the wing and at the same horizontal level. The tail-plane is a symmetric aerofoil. Estimate the angle, relative to the horizontal, that the tailplane be placed so as to give zero lift. Assume a very simple horseshoe vortex model of the lifting wing. [30%]

(c) The same aircraft as in part (b) is now flown within 20 m of the ground, still in level flight. Assuming that the tailplane is oriented at the angle to horizontal calculated in (b) (to give zero lift when far from the ground). Estimate its angle of attack to the oncoming air and comment on its lift in this situation. Again you may assume a very simple horseshoe vortex model. [30%]

4 (a) Write down the complex potential for a source, of strength m, located at $z \equiv x + iy = -b$ and a sink, of strength $-m$, located at $+b$ with a uniform flow, of speed, U from left to right as shown in Fig. 1. [10%]

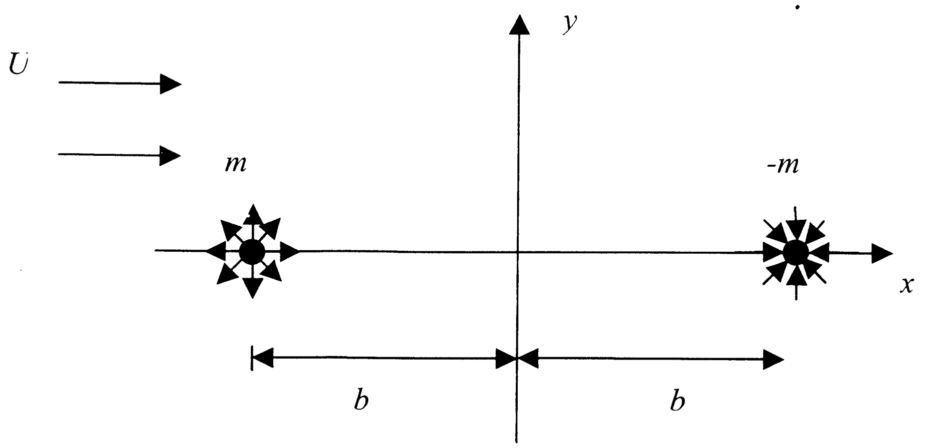

Fig. 1

(b) Locate the stagnation points and hence find the length l in the x-direction of the body formed by the closed streamline which passes through the stagnation points. [15%]

(c) The sink and the source are now moved together and their strength is increased such that mb=constant. Consider the limiting case as $b \to 0$.

(i) What is the length between the stagnation points in this limit (along the x-axis)? [15%]

(ii) Write down the complex potential for this flow (as $b \to 0$). You may use the fact that the Taylor series for
$ln(1+x) = x +$ higher order terms, and
$ln(1-x) = -x +$ higher order terms about $x = 0$. [20%]

(iii) Write an equation for the streamline that passes through the stagnation points in terms of z. [20%]

(iv) Sketch the flow and comment on the type of real flow that this might be used to model. Where and how would the real flow and the ideal flow differ? [20%]

5 A large flat plate in an initially still fluid of density ρ and viscosity μ starts from rest and begins to oscillate in simple harmonic motion in its own plane with a frequency n and a maximum velocity U_o.

(a) Sketch the velocity profiles in the fluid for different times after the plate begins to move. [15%]

(b) Starting from the Navier-Stokes equation

$$\rho \frac{D\tilde{u}}{Dt} = -\nabla p + \mu \nabla^2 \tilde{u}$$

show that the motion in the fluid can be described by the simpler equation

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} \equiv \nu \frac{\partial^2 u}{\partial y^2}$$

where y is the co-ordinate normal to the wall. [20%]

(c) Explain why, after the plate has been oscillating for some time, the velocity at some point, a distance y from the wall, will undergo simple harmonic motion. Sketch how the amplitude of the motion will vary with y. [15%]

(d) Denote the scale over which the amplitude reduces towards zero by the "penetration distance" δ. By using scaling arguments determine how δ will vary with ν and n. Explain, in physical terms, your results. [20%]

(e) The equation describing the temperature variation due to unsteady conduction, in one dimension, in a solid is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$$

and the solution, for a sinusoidal variation of temperature at the solid boundary is $T(y,t) = T_o e^{-ky} \cos(nt - ky)$ where T_o is the amplitude of the surface temperature variation and k is a parameter to be determined.

Write down the solution for the velocity $u(y,t)$ for the problem in (c) above. [15%]

(f) Determine the parameter k and explain its significance. [15%]

6 (a) What is a "boundary layer"? [20%]

(b) Describe the similarities of and differences between laminar and turbulent boundary layers. [20%]

(c) What are the similarities of and differences between momentum transfer and heat transfer in the context of boundary layers? [20%]

(d) In a gas the transfer of heat and mass are very similar. Why is this? [20%]

(e) An experimental technique to determine heat transfer coefficients is to cover a surface with solid naphthalene. The naphthalene sublimates (becomes a gas) when air is blown over it. By determining the weight loss of naphthalene over a period of time the average heat transfer coefficient can be determined.

The sublimation of naphthalene is "driven" by the difference of the naphthalene vapour density (ρ_s) at the surface and the value in the free stream (which is equal to zero). In fact, the mass flux per unit area of the surface is given by

$$F = StU(\rho_s - \rho_{\text{free-stream}}).$$

Determine the average rate of naphthalene mass removal expected from a 1 metre long plate in a flow of $8\ \mathrm{ms^{-1}}$ when the naphthalene vapour density at the plate surface is $10^{-4}\ \mathrm{kg\ m^{-3}}$. [20%]

For a boundary layer growing over a heated flat plate in a zero pressure gradient flow the heat transfer coefficient is given by the Stanton number

$$St = \frac{h}{\rho c_p U} = 0.037 \quad \mathrm{Re}_x^{-0.2}.$$

7 (a) For a finite wing in steady, level flight, the local circulation, angle of attack and downwash can be related by the following expression

$$\Gamma(z) = \frac{1}{2} V\, c\, a_o \left(\alpha - \frac{w(z)}{V} \right)$$

where z is the spanwise co-ordinate, c the chord, α the angle of attack and w the local downwash. Describe the derivation of this expression, making any assumptions clear. What is the approximate value of a_o? [40%]

(b) If now the aeroplane starts to roll at constant rate $\dot{p}$, show how the above expression is modified. [30%]

(c) The wing is designed to have elliptic loading; what will be the effect of the rolling on the drag coefficient and why? [30%]

8 It is proposed that aeroplanes could fly together in formation like birds do.

(a) Explain how this might be expected to reduce the overall drag. [30%]

(b) Why might these reductions not be realised in practice? [40%]

(c) Consider a simple horse-shoe vortex model of five identical aeroplanes flying in formation as sketched in Figure 2. Estimate the mean drag reduction per aeroplane compared to isolated flight. [30%]

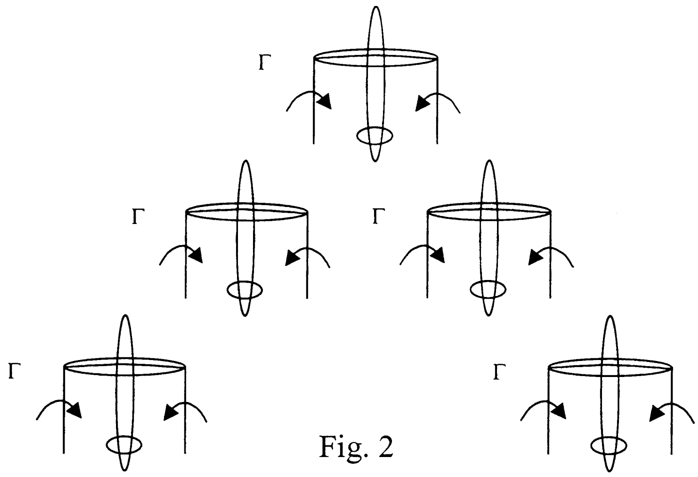

Fig. 2

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