ENGINEERING TRIPOS PART IIA

Friday 29 April 2005 9 to 12

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*The **Compressible Flow Data Book** is provided for use with this paper.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 Figure 1 shows the position - time diagram for a piston impulsively started in an open-ended tube. The air in the tube is initially at rest at an ambient temperature and pressure of 288 K and 10^5 Pa. Initially the piston is at rest. At time $t = 0$ the piston velocity rises instantaneously to 220 ms^{-1}. The piston velocity then remains constant.

(a) In a frame of reference moving with the shock wave, express the ratio of densities on either side of the shock as a function of the piston and shock velocities. [20%]

(b) Using the result of (a) and the normal shock tables, show that the velocity of the shock is approximately 497 ms^{-1}. Calculate the static temperature and pressure of the air in region 1. [40%]

(c) As the shock wave reaches the open end of the tube a left running expansion wave is formed. Calculate the velocity and static temperature of the gas in region 2. You may make use of the Riemann invariant for a left running wave : [20%]

$$V + \frac{2a}{\gamma - 1}$$

where all symbols have their usual meaning.

(d) At time T the front of the expansion wave contacts the piston face. Calculate the location of the piston in the tube, as a percentage of the tube length L, at time T. [20%]

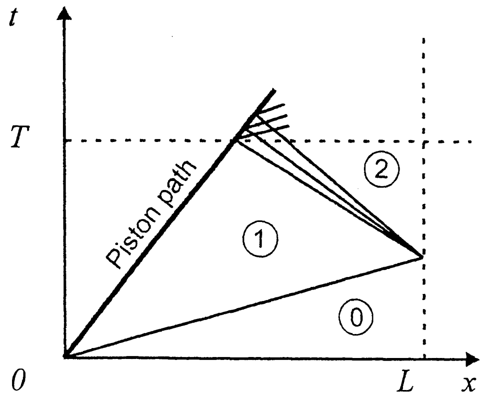

Fig 1.

2 (a) A convergent nozzle is connected to a reservoir filled with air at 3 times atmospheric pressure. A pipe with a friction coefficient, C_f, is connected to the end of the nozzle and discharges to the atmosphere. Draw a temperature - entropy diagram (T - s) showing the variation of flow conditions through the nozzle and pipe for a number of values of friction coefficient. You may treat the flow throughout as adiabatic and the nozzle flow as frictionless. What is the structure of the flow for the two extreme cases of a frictionless pipe and a pipe with very high friction coefficient? [40%]

(b) A pipe with an unknown friction coefficient is connected to the exit of the convergent nozzle from part (a). The pipe length is 1m and the pipe diameter is 0.01m. The pressure in the reservoir is lowered until the static pressure at the exit plane of the pipe is equal to the local atmospheric pressure. The reservoir pressure is then 2.5 times atmospheric pressure. Calculate the Mach number at the pipe inlet plane and the friction coefficient, C_f, of the pipe. [35%]

(c) In an alternative device, pipes are connected to the exit of a smooth (frictionless) convergent-divergent nozzle with an exit Mach number of 2.0. The nozzle is supplied with air from a large reservoir. During testing the exit of each pipe remains choked. Explain how the location of the shock wave within the pipe varies with the friction coefficient of the pipe. [25%]

3 (a) Sketch the various flow regimes which may occur inside a converging-diverging nozzle as the pressure at the outlet is lowered relative to that at the inlet. Show annotated plots of Mach number and static pressure along the nozzle. Assume that the flow may be treated as adiabatic and inviscid. [30%]

(b) The inlet of a small convergent-divergent nozzle is connected to a 2×10^5 Pa reservoir of air. The nozzle is designed to expand the exit gas isentropically to atmospheric pressure at 16km above sea level. Calculate the ratio of exit area to throat area of the nozzle. If the nozzle has a throat diameter of 0.01m calculate its thrust. [35%]

(c) It is decided that the nozzle in part (b) may also be of use at sea level. The nozzle inlet is connected to the same 2×10^5 Pa reservoir. Show that when operating at sea level a shock occurs in the nozzle. Determine the Mach number upstream of the shock. Calculate the thrust of the nozzle. [35%]

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4 An aerofoil is to be designed to operate at transonic flow velocities.

(a) Show that the surface pressure coefficient can be expressed as

$$c_p = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right).$$

All symbols have their usual meaning. [25%]

(b) Hence derive an equation for the critical pressure coefficient c_p^* as a function of the free-stream Mach number M_∞. [35%]

(c) The aerofoil is found to have a minimum pressure coefficient of -1.0 at incompressible flow velocities. Estimate its critical Mach number by trial and error or using a graphical solution. What assumptions have you made in the calculation? [40%]

5 A flat-plate aerofoil with a chord length of 2 m is immersed into a supersonic flow with a free stream Mach number of 2.5 and a static pressure of 100 kPa. Its angle of attack is 8°.

(a) Sketch the expected flowfield. [20%]

(b) Calculate the expected lift and drag per unit span as accurately as possible. What assumptions have you made? [30%]

(c) Determine the lift and drag coefficients. [20%]

(d) How do the results of (a)-(c) differ from those you might expect at low (incompressible) velocities assuming:

(i) Inviscid flow?

(ii) Viscous flow? [20%]

(e) Where on the aerofoil might you expect separation to occur at supersonic speeds? [10%]

6 (a) Describe the circumstances in which a method based on a velocity potential can be used to calculate a fluid flow. What fundamental physical equation is solved when this method is used? [20%]

(b) It is intended to solve for the steady incompressible flow of fluid along a one-dimensional duct of width h, as illustrated in Fig. 2, using a velocity potential method. Show that the resulting equation can be written as

$$\frac{d^2\phi}{dx^2} = -\frac{1}{h}\frac{dh}{dx}\frac{d\phi}{dx}$$

where ϕ is the velocity potential. [25%]

(c) Express this equation as a central difference finite difference equation in ϕ using the notation $\beta = \frac{\Delta x}{2h}\frac{dh}{dx}$, where Δx is the grid spacing. Explain how the resulting equation could be solved numerically using a relaxation procedure. [30%]

(d) By considering a saw-tooth perturbation in ϕ expressed as $\phi_i = \varepsilon(-1)^i$, where i is the space index, calculate the maximum stable relaxation factor for this type of perturbation. [15%]

(e) Consider a saw-tooth perturbation with twice the above wavelength expressed as

$\phi_i = 0$ if i is even
$\phi_i = -\varepsilon(-1)^{0.5(i+1)}$ if i is odd.

By considering the change in the maximum difference in ϕ between two grid points over one iteration determine the range of values of β for which this disturbance is less stable than the short wavelength one. [10%]

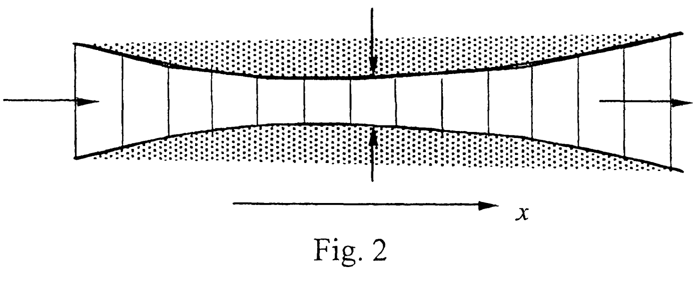

Fig. 2

7 The differential form of the Euler equations for one-dimensional flow in a constant area duct is:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial (\rho V)}{\partial x}$$

$$\frac{\partial (\rho V)}{\partial t} = -\frac{\partial (p + \rho V^2)}{\partial x}$$

$$\frac{\partial (\rho (e + 0.5V^2))}{\partial t} = -\frac{\partial (\rho V h_o)}{\partial x}$$

All symbols have their usual meaning.

(a) Show how these equations may be applied to a control volume, such as ABCD in Fig 3, to obtain a finite volume formulation of the flow along a duct of varying area. Write down the resulting equations and describe how they may be solved using a time marching method. [40%]

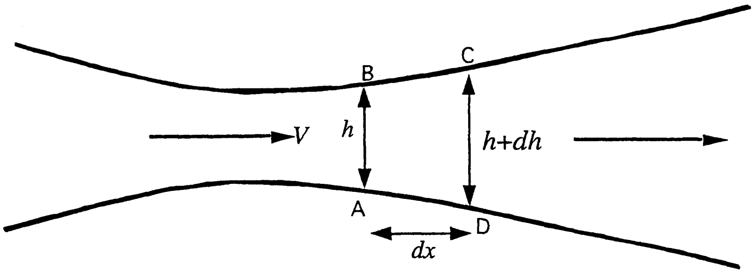

Fig. 3

(b) For small changes, the energy equation may be replaced by the characteristic relationship

$$dp = \pm \rho \ a \ dV$$

where a is the speed of sound and the + sign applies to right running waves and the − sign to left running waves. Show how this may be combined with the differential form of the continuity and momentum equations, given above, to obtain an equation of the form

$$\frac{\partial V}{\partial t} = C \frac{\partial V}{\partial x}$$

Obtain an expression for C and suggest a finite difference formulation for this equation. [60%]

8 (a) Give the fundamental fluid dynamic reason why there are many more compressor stages than turbine stages in a simple turbojet engine. [10%]

(b) An axial flow compressor, working with air, has an overall stagnation pressure ratio of 10, a total-to-total isentropic efficiency of 0.86 and an inlet stagnation temperature of 300K. All stages of the compressor have the same blade speed $U = 250\ \mathrm{ms}^{-1}$, and the same stage loading coefficient $\psi = \Delta h_0 / U^2 = 0.35$. Calculate the number of stages required, rounded to the nearest whole number. [20%]

(c) If the axial velocity through the compressor of (b) is constant at $125\ \mathrm{ms}^{-1}$ and there is no swirl component of velocity at entry and exit from each stage, sketch the velocity triangles for a stage and calculate the flow angles at entry and exit to each blade row. [40%]

(d) If the blade speed and axial velocity of the compressor of (b) could be increased up to the point where the relative Mach number at entry to the first rotor was 0.9, with all the velocity triangles remaining similar to the solution of (b), calculate the new blade speed. How many stages would then be required for the compressor? [30%]

END OF PAPER

Module 3A3

Answer sheet

PART IIA 2005

3A3 Fluid mechanics II (double module)

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Q1 a) $\rho_0/\rho_1 = (V_s - V_p)/V_s$

b) 372.7 K, $2.3 \cdot 10^5$ Pa

c) 440 $\frac{m}{s}$, 293 K

d) 0.76 L

Q2 b) 0.0025

Q3 b) 2.89, 21.6 N

c) M = 2.35, 4.88 N

Q4 c) 0.605

Q5 b) 216 kN, 30 kN

c) 0.25, 0.034

Q6 d) $R < 1$

e) more unstable when $\beta < -2$

Q7 b) $C = -(v \mp a)$

Q8 b) 15

d) 13

No changes needed to paper