UNIVERSITY OF
CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Saturday, 30 April 2005 9 to 12

Module 3A5

ENERGY AND POWER GENERATION

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: Chart (one page)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER.

1 (a) (i) Consider the relationship $h = h(T, p)$ for a simple gaseous substance. By using Maxwell's relation

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p ,$$

show that the change in enthalpy for this substance along an isotherm is given by

$$dh = dp\left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\} .$$

[15%]

(ii) The (p, v, T) equation of state for this substance is given by

$$v = \frac{RT}{p} - \frac{C}{T^3},$$

where C and R are constants. Obtain expressions for the changes in enthalpy and entropy during an isothermal process from state 1 to state 2. [20%]

(b) Steam at 8 MPa and 500 °C enters the first stage of a two-stage steam turbine. It leaves at 2 MPa and 350 °C. Before the steam enters the second stage, it is reheated at constant pressure to a temperature of 500 °C from a source at 800 °C. The steam leaves the second stage as saturated vapour at 30 kPa. The shaft power is 5 MW. The environment is at 25 °C.

(i) Find the mass flow rate of the steam. What is the maximum possible power output from each stage of the turbine for the given entry and exit conditions to each stage? [20%]

(ii) Calculate the lost power due to irreversibility in the turbines. What is the rate of exergy transfer due to the reheating? [15%]

(iii) Find the maximum possible total power output from the steam as it passes between the entry conditions to the first turbine and the exit conditions of the second turbine. [15%]

(iv) What is the lost power under the circumstances in (b)(iii)? Why is this lost power different to that obtained in (b)(ii)? [15%]

You may use the following properties of steam.

Condition	Specific Enthalpy (kJ kg^{-1})	Specific Entropy (kJ kg^{-1} K^{-1})
Stage 1 Inlet	3399.5	6.727
Stage 1 Exit	3137.7	6.958
Stage 2 Inlet	3468.2	7.434
Stage 2 Exit	2624.5	7.767

2 (a) Sketch how the temperature in the heat recovery steam generator (HRSG) of a combined-cycle gas turbine (CCGT) power plant varies with the amount of heat transferred. Explain the significance of the *pinch point*. [25%]

(b) The temperature at the exit of the gas turbine in a combined-cycle gas turbine power plant is 580 °C. The exhaust gases may be treated as a perfect gas with a specific heat capacity at constant pressure of 1.15 kJ kg^{-1} K^{-1}. The condenser temperature is 30 °C. The temperature and pressure at entry to the steam turbine are 550 °C and 40 bar. The temperature difference at the pinch point is 25 °C. The feed pump work and the pressure losses in the HRSG are negligible.

(i) Determine the ratio of the mass flow rate of steam to the mass flow rate of gas. [10%]

(ii) Determine the temperature of the gas leaving the HRSG. Comment on the value you obtain. [20%]

(c) The environment is at the same temperature as the condenser. What is the maximum possible work output from the steam per unit mass of the gas? How much work is lost in the HRSG per unit mass of the gas? Explain how this could be reduced. [45%]

You may use the following properties of steam.

Condition	Specific Enthalpy (kJ kg^{-1})	Specific Entropy (kJ kg^{-1} K^{-1})	Temperature (°C)
HRSG Inlet	125.7	0.437	
HRSG Exit	3560.3	7.235	
Pinch Point	1087.5		250.35

3 (a) Comment on the options available to reduce CO_2 emissions from electrical power generation. [30%]

(b) A simple conventional steam cycle has a boiler pressure of 100 bar. The condenser pressure is 0.04 bar. The turbine entry temperature is 550 °C. The plant has been modified so that there is a single direct contact feed heater that produces saturated water at 2 bar. There are no pressure losses in the feed heater. The dryness fraction at the exit of the turbine is 0.85 and the expansion line of the turbine is straight on the enthalpy-entropy chart. The power of the feed pump can be neglected. The power output of the plant is 500 MW.

(i) Sketch the layout and the temperature-entropy diagram of the above cycle. [20%]

(ii) Find the ratio of the mass flow rate of the bled steam to the mass flow rate through the boiler. Sufficient accuracy may be obtained by using the enthalpy-entropy chart for the expansion through the turbine. [10%]

(iii) Determine the mass flow rate through the boiler. [15%]

(iv) What is the efficiency of the cycle with and without feed heating? [10%]

(v) Calculate the lost power due to irreversibilities in the direct contact feed heater assuming that the ambient temperature is equal to the condenser temperature. Express this as a percentage of the rate of heat transfer in the boiler. [15%]

You may use the following properties of steam. Use the attached Chart for other properties you may need.

Condition	Specific Enthalpy (kJ kg^{-1})	Specific Entropy (kJ kg^{-1} K^{-1})	Temperature (°C)
100 bar, 550 °C	3502	6.758	
Saturated Liquid at 0.04 bar	121.4	0.422	28.96
Saturated Vapour at 0.04 bar	2553.7	8.473	28.96
Saturated Liquid at 2 bar	504.7	1.530	

4 (a) The power output from a Radioisotope Thermal Generator (RTG) is given by $P = P_0 \exp(-\lambda t)$, where P_0 is the initial power and λ is the decay constant.

(i) Define the half-life $T_{1/2}$ of the radioactive source and show how it is related to the decay constant. [10%]

(ii) Show that the mean power output from the RTG over the N half-lives is

$$\frac{P_0}{N \ln 2}\left[1 - \frac{1}{2^N}\right].$$

[15%]

(b) Polonium metal is used as the radioactive power source for the batteries used in a space vehicle. The decay reaction is

$$^{210}\mathrm{Po} \rightarrow {}^{206}\mathrm{Pb} + {}^{4}\mathrm{He} + \gamma \quad .$$

Given that the atomic masses of polonium-210, lead-206 and helium-4 in atomic mass units are 209.9829 u, 205.9744 u and 4.0026 u respectively, determine in Joules the energy released per decay reaction. You may assume that the energy equivalent of 1 u is 931 MeV and that $1\,\mathrm{MeV} = 1.6 \times 10^{-13}$ J. [10%]

(c) Explain why an α-emitter, such as polonium-210, is a good choice of power source in an RTG, and list three other considerations when choosing an isotopic power source. [15%]

(d) The half-life of polonium-210 is 138.4 days and the energy of the characteristic γ-ray released in its decay is 0.802 MeV. The space vehicle's mission is to last three polonium-210 half-lives.

(i) Calculate the minimum initial power of the RTG if it is still to be producing 250 W at the end of the mission. [10%]

(ii) Hence, assuming all the energy released except that associated with the γ-rays is recoverable, calculate the mass of polonium-210 required. You can take the mean molar mass of polonium-210 to be $210\ \mathrm{kg\,kmol^{-1}}$. [25%]

(e) Calculate the expansion volume needed in the RTG to ensure that the pressure of the helium gas generated in the decay process does not exceed 100 bar at 250 °C. [15%]

5 (a) Briefly describe what we mean by *lean flammability limit.* [30%]

(b) An approximate theory for the lean flammability limit states that it occurs when the flame temperature T_f becomes less than 1600 K.

A burner for methane (CH_4) is constructed to burn very lean mixtures by preheating the reactants as shown in Fig. 1. The fresh reactants at an equivalence ratio ϕ enter a heat exchanger at a temperature $T_{in} = 300\,\mathrm{K}$. The heat is transferred to the reactants from the hot flame products flowing on the other side of the heat exchanger. Immediately before the flame zone, the reactants reach a temperature T_R. The flame products flow out of the burner at a temperature T_{out} of 1000 K. Assume that the specific heat capacity at constant pressure of the reactants and products is constant and equal to 1.2 kJ $\mathrm{kg}^{-1}\,\mathrm{K}^{-1}$. The lower calorific value of methane is 50 MJ kg^{-1}. Calculate the leanest possible equivalence ratio of burner operation. [50%]

(c) Despite the low flame temperature in the burner of Fig. 1, finite amounts of NO are measured at the exit. Briefly discuss the origin of this NO. [20%]

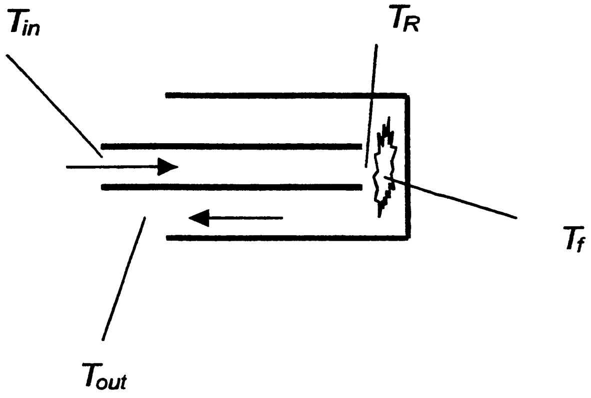

Fig. 1

6 A small-scale hydrogen generator is used to produce hydrogen from fossil fuels for use in a fuel cell. The hydrogen generator is assumed to be a partial oxidation combustor operating at 1600 K and an equivalence ratio of 2.5 with $C_{10}H_{20}$ as fuel and atmospheric air as the oxidizer. The products leaving the combustion chamber are in thermodynamic equilibrium.

(a) Would you expect thermodynamic equilibrium everywhere inside the combustion chamber? Explain your answer. [15%]

(b) Calculate the volume fractions of gases leaving the combustor, assuming that the only species present are N_2, CO_2, CO, H_2 and H_2O. [60%]

(c) It is suggested that the conversion efficiency to hydrogen will increase if steam is added to the reactants. Is this suggestion qualitatively correct? Briefly discuss your answer. [10%]

(d) Does the pressure affect the composition of the products in either of the above cases? Explain your answer. [15%]

7 A conventional 4-stroke spark ignition engine has a compression ratio r of 9. After the evaporation of the fuel and mixing of the air, fuel and residual gases is complete, the initial pressure p_1 is 0.9 bar and temperature T_1 is 310 K. The gravimetric air-fuel ratio (AFR) is 14.7. The total mass of residual gas can be neglected for the purposes of the AFR calculation. The mixture can be considered to be perfect, with a specific heat capacity at constant volume c_v of 0.75 kJ kg^{-1} K^{-1}. The ratio of the specific heat capacities γ is 1.4. The lower calorific value of the fuel Q_f is 44 MJ kg^{-1}.

(a) Sketch the cycle on a pressure-volume diagram, including the pumping loop. [10%]

(b) By neglecting the work done in the pumping loop and assuming that the engine is represented by the ideal constant volume cycle, show that the indicated gross efficiency is given by

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

[15%]

(c) By neglecting the work done in the pumping loop and assuming that the engine is represented by the ideal constant volume cycle, show that the specific gross work per unit displacement volume (*imep*) of the cycle is given by

$$imep = p_1\left(\frac{m_f Q_f}{m c_v T_1}\right)\left(\frac{1}{\gamma-1}\right)\left(\frac{r}{r-1}\right)\left(1-\frac{1}{r^{\gamma-1}}\right)$$

where T_1 is the initial charge temperature, p_1 is the initial charge pressure, m is the initial charge mass and m_f is the mass of fuel. [35%]

(d) Calculate the indicated gross efficiency and *imep* for the above ideal constant volume cycle. [10%]

(e) Explain how engine speed affects the indicated mean effective pressure, the brake mean effective pressure, the friction mean effective pressure, the specific fuel consumption and the engine power in Compression-Ignition and Spark-Ignition engines. [30%]

8 (a) (i) Show that the gross indicated efficiency for the ideal limited-pressure cycle, which is often used to model the CI engine, is

$$\eta_{lp} = 1 - \left(\frac{1}{r^{\gamma-1}}\right)\left(\frac{\alpha\beta^{\gamma} - 1}{(\alpha - 1) + \gamma\alpha(\beta - 1)}\right)$$

where r is the compression ratio, γ is the ratio of the specific heat capacities, α is the pressure ratio during the constant volume part of the heat release and β is the volumetric expansion ratio during the constant pressure part of the heat release. [40%]

(ii) Explain why CI engines are more efficient than SI engines in practice. [10%]

(b) (i) Show that the residual gas fraction is given by

$$x_r = \frac{1}{r}\left(\frac{p_e}{p_i}\right)^{1/\gamma}\left(1 + \frac{q^*}{r^{\gamma-1}}\right)^{-1/\gamma}$$

for an ideal constant volume cycle, and

$$x_r = \frac{1}{r}\left(\frac{p_e}{p_i}\right)^{1/\gamma}\left(1 + \frac{q^*}{\gamma\, r^{\gamma-1}}\right)^{-1}$$

for an ideal constant pressure cycle, where p_e and p_i are the exhaust and intake manifold pressures, respectively. The compression ratio is r and

$$q^* = \frac{m_f Q_f}{m c_v T_1}$$

where Q_f is the lower calorific value of the fuel, m_f is the mass of fuel, m is the mass of the charge, c_v is the specific heat capacity at constant volume and T_1 is the initial charge temperature. [30%]

(ii) Based on the equations above and your understanding of IC engine operation, explain under what conditions and for which type of engine the residual gas fraction is likely to be the highest. [10%]

(cont.

(c) What are the main differences between the ideal and actual cycles for both CI and SI engines? [10%]

END OF PAPER

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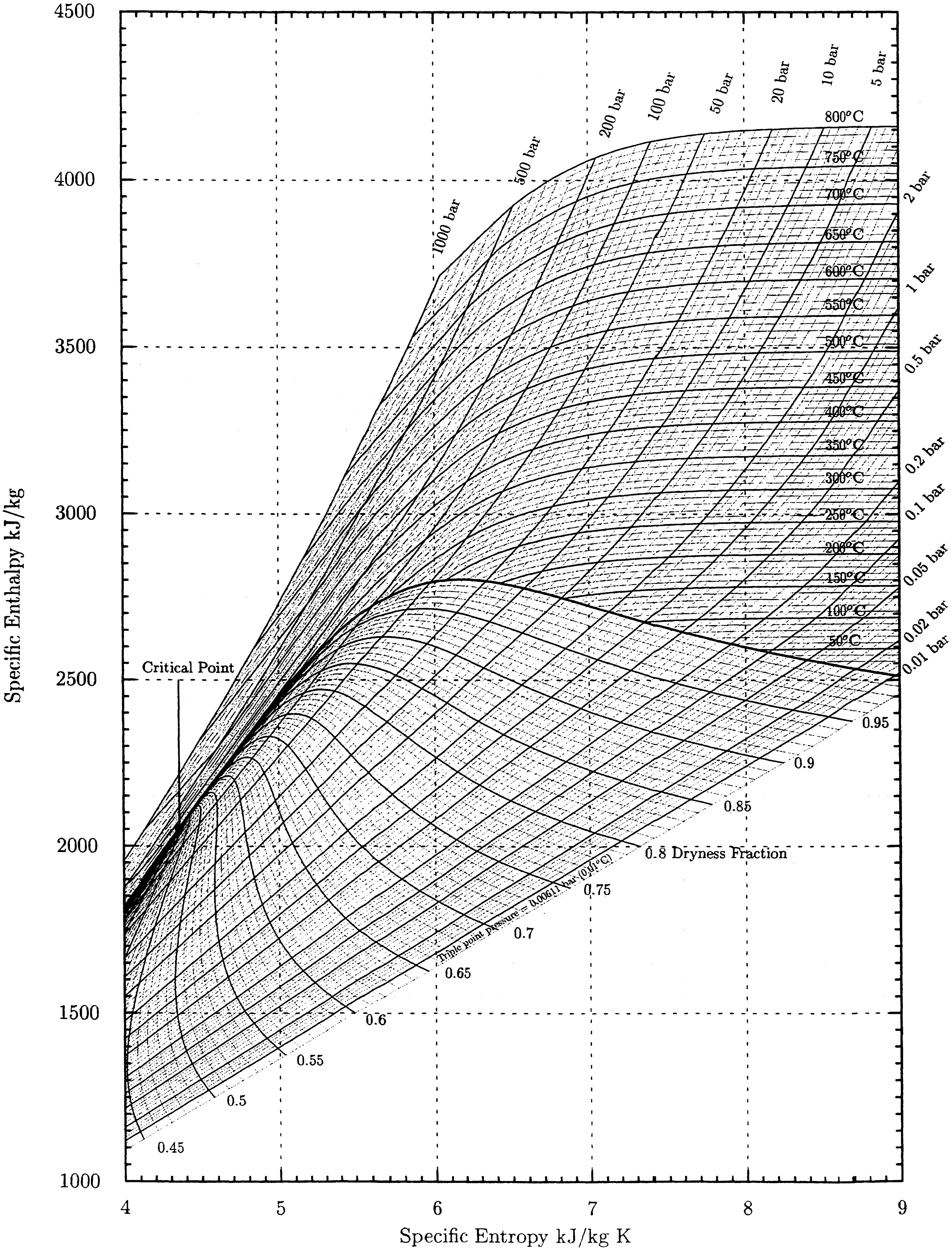
Enthalpy-Entropy Diagram for Steam

Plotted from the IAPWS equations http://www.iapws.org (Duncan A. Simpson (2002))



PART IIA 2005

3A5 Energy and power generation (double module)

Principal Assessor: **Prof H P Hodson/ Dr N Swaminathan**

Answers

1. (a)(ii) $h_2 - h_1 = -\frac{4C}{T^3}(p_2 - p_1)$, $s_2 - s_1 = -R\ln\left(\frac{p_2}{p_1}\right) - \frac{3C}{T^4}(p_2 - p_1)$

   (b)(i) 4.523 kg/s, 1495.4 kw, 4264.7 kw

   (ii) 760.1 kw, 1079.6 kw

   (iii) 5.986 Mw

   (iv) 986 kw - high because of exergy loss during heat addition

2 (b)(i) 0.142

   (ii) 156.85°C

   (c) 194.8 kJ/kg 53.18 kJ/kg - use multiple pressure levels

3 (b)(ii) 0.1495

   (iii) 404 kg/s

   (iv) 41.3% with feed heating and 38.8% without feed heating

   (v) 1.374 Mw 1.1%

4 (a)(i) $T_{1/2} = \frac{\ln 2}{\lambda}$

   (b) 8.789×10^{-13} J

   (d)(i) 2.0 kw

   (ii) 0.016 kg

   (e) $2.904\times10^{-5}\text{m}^3$

5 (b) 0.293

6 (b) $X_{CO_2} = 0.0127, X_{CO} = 0.2221, X_{H_2O} = 0.0342, X_{H_2} = 0.2006, X_{N_2} = 0.5302$

7 (d) 58.5%, 17.84 MPa

SWAMINATHAN.
26/May/2005