

ENGINEERING TRIPOS PART IIA

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Wednesday 11 May 2005 2.30 to 4

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Module 3B4

ELECTRIC DRIVE SYSTEMS

*Answer no more than **three** questions*

*All questions carry the same number of marks*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin*

*There are no attachments*

**You may not start to read the  
questions printed on the subsequent  
pages of this question paper until  
instructed that you may do so by the  
Invigilator**

(TURN OVER

1 (a) A designer wishes to construct a permanent magnet motor. He has four materials available as shown in Table 1 below. Assuming that each material has a linear magnetic characteristic plot the data given and calculate the relative permeability  $\mu_r$  for  $\text{Sm}_2\text{Co}_{17}$  and one other material type. [20%]

	Alnico	Ceramic	$\text{Sm}_2\text{Co}_{17}$	NdFeB
Br(T)	1.32	0.39	1.06	1.21
Hc(AT/m)	56000	176000	815000	840000

Table 1 – Magnet Properties

(b) It is known that the maximum energy product for each material occurs when  $H=H_c/2$ . Starting from an expression for  $B$  in terms of  $H$  derived from your graphs, show that this is true and calculate  $(BH)_{max}$  for the materials selected in part (a). [30%]

(c) Assume that the magnet material used is  $\text{Sm}_2\text{Co}_{17}$ , that it is in a magnetic circuit where the iron is of sufficiently high permeability to be ignored, and the cross-sectional area of the gap is the same as the magnet. Use Ampere's law to show that at the maximum energy product the ideal length of the gap is approximately equal to that of the magnet. [30%]

(d) Magnets are expensive and the designer wants to use less  $\text{Sm}_2\text{Co}_{17}$  than that required to produce the maximum energy product. What will be the effect on the **total** energy product of:

(i) halving the cross-sectional area of the magnet while keeping all other elements the same; [10%]

(ii) halving the length of the magnet while keeping all other elements the same. [10%]

2 (a) Describe the difference between a salient pole synchronous machine and a reluctance machine. [10%]

(b) Draw the phasor diagram for a reluctance machine (assume that the stator has negligible resistance). By considering the output power for the machine derive an expression for the torque delivered. [35%]

(c) When the machine is manufactured a mistake is made and the machine is wound with the wrong wire, this means that the stator resistance can be no longer counted as negligible. Assuming that the direct reactance  $X_{ds} = 2 \Omega$  per phase, the quadrature reactance  $X_{qs} = 1 \Omega$  per phase and that  $R$  the stator resistance is  $1 \Omega$ :

(i) draw a new phasor diagram; [20%]

(ii) calculate the output power of a 6.6 kV, star-connected, 50 Hz machine if the load angle is  $30^\circ$ . [35%]

(TURN OVER

- 3 (a) Fig. 1 shows the per-phase equivalent circuit of an induction motor. Copy the diagram and label all the elements, identifying which belong to the stator and which belong to the rotor. [30%]

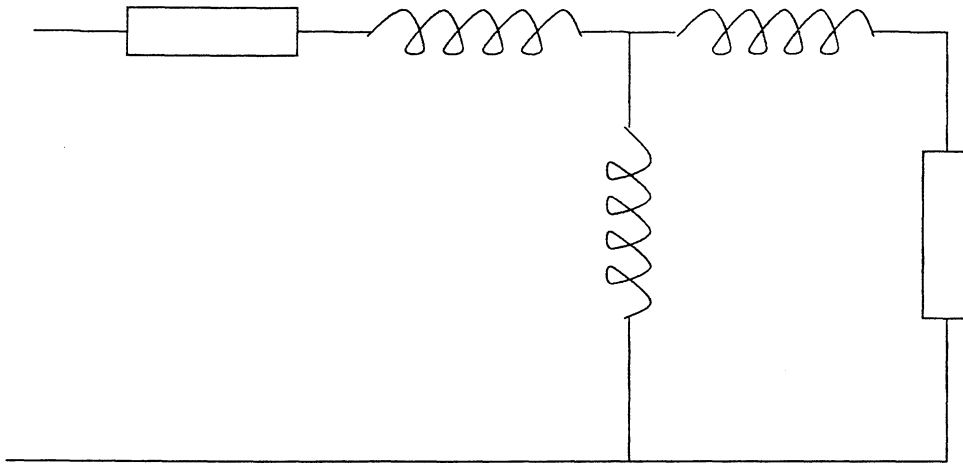


Fig.1 Per-phase induction motor equivalent circuit.

- (b) Starting from the equation

$$P_{\text{gap}} = P_{\text{mech}} + P_{\text{losses}},$$

where  $P$  is the power, derive an equation for the torque developed and for the variation of torque with speed making the usual assumptions. [30%]

- (c) With reference to the equation derived in part (b) explain why it is desirable to operate this type of motor with a constant ratio of voltage to frequency. [15%]

- (d) It is desired to operate the motor at higher than base speed. If the maximum voltage is limited, explain how this is done and describe what happens to the torque as the speed is increased. Sketch the torque speed curve in this region. [25%]

4 A single phase A.C. motor, (mechanical power output) 1 kW, 240 V, 50 Hz, 4 poles, is run with the following duty cycle:

Run for 10 minutes at half power and full speed during which time the temperature rises from ambient (20 °C) to 70 °C,

Rest for 2 minutes during which time the temperature falls to 40 °C,

Run for 3 minutes at full power and full speed during which time the temperature rises to 80 °C.

(a) Calculate the Specific Heat Capacity  $C$ , assuming that the dissipation coefficient  $k$  is equal to  $1.1 \text{ WK}^{-1}$ . [25%]

(b) Draw and carefully label the equivalent circuit for the motor, including both the forward and backward components. [25%]

(c) Paying careful attention to your diagram of the equivalent circuit derive an expression for the power dissipated in the windings. You may assume that the magnetising reactance is large and that the stator resistance and reactance and the rotor reactance are negligible:

(i) evaluate the expression for the first ten minutes of the duty cycle; [25%]

(ii) calculate the rated slip at full power. [25%]

**END OF PAPER**

Principal Assessor: Dr T A Coombs

1.

$$\mu_0 := 4 \cdot \pi \cdot 10^{-7} \frac{\text{henry}}{\text{m}}$$

$$i := 1..4$$

$$\text{Alnico} = 1$$

$$\text{Ceramic} = 2$$

$$\text{SmCo} = 3$$

$$\text{NdFeB} = 4$$

$$B_{r_i} :=$$

$$H_{c_i} :=$$

1.32 · tesla
0.39 · tesla
1.06 · tesla
1.21 · tesla

56000 · $\frac{\text{amp}}{\text{m}}$
176000 · $\frac{\text{amp}}{\text{m}}$
815000 · $\frac{\text{amp}}{\text{m}}$
840000 · $\frac{\text{amp}}{\text{m}}$

$$\mu_{r_i} = \frac{B_{r_i}}{\mu_0 \cdot H_{c_i}}$$

$$BH_{\max_i} = \frac{B_{r_i} \cdot H_{c_i}}{4}$$

a)

0	
18.758	Alnico
1.763	Ceramic
1.035	SmCo
1.146	NdFeB

b)

0	
1.848 · 10 <sup>4</sup>	Alnico
1.716 · 10 <sup>4</sup>	Ceramic
2.16 · 10 <sup>5</sup>	SmCo
2.541 · 10 <sup>5</sup>	NdFeB

· m<sup>-3</sup> · joule

c)

d) i) 4/9 of original energy product ii) 4/9 of original energy product

2.

$$\delta := 30 \cdot \text{deg} \quad X_{qs} := 1 \cdot \text{ohm}$$

$$X_{ds} := 2 \cdot \text{ohm}$$

$$R := 1 \cdot \text{ohm}$$

$$\text{voltage} := \frac{6.6 \cdot 10^3}{3^{0.5}} \cdot \text{volt}$$

$$\text{Power} := 3 \cdot \text{voltage}^2 \cdot \left[ \frac{\sin(\delta) \cdot \cos(\delta) - \frac{R}{X_{ds}} \cdot (\cos(\delta))^2}{X_{qs} - \frac{R^2}{X_{ds}}} - \frac{\sin(\delta) \cdot \cos(\delta) - \frac{R}{X_{qs}} \cdot (\sin(\delta))^2}{X_{ds} - \frac{R^2}{X_{qs}}} \right]$$

Power = -8.233 · 10<sup>6</sup> · watt

3.

$$4. \quad i = 1..3 \quad \text{start} = 1 \quad \text{end} = 2 \quad \text{ambient} = 193 \cdot \text{K}$$

$$\text{period}_i := \text{temperature\_above\_ambient}_{\text{start},i} \quad \text{temperature\_above\_ambient}_{\text{end},i} :=$$

10·60·sec
2·60·sec
3·60·sec

0·K
50·K
20·K

50·K
20·K
60·K

$$\tau := \frac{\text{period}_2}{\ln \left( \frac{\text{temperature\_above\_ambient}_{\text{end},2}}{\text{temperature\_above\_ambient}_{\text{start},2}} \right)}$$

$$\tau = 130.963 \cdot \text{sec} \quad k := 1.1 \cdot \frac{\text{watt}}{\text{K}}$$

$$C := \tau \cdot k$$

$$C = 144.059 \cdot \text{joule} \cdot \text{K}^{-1}$$

c)

i) at half-power (steady state power dissipation)

$$T_{\text{infinity}} := \frac{\text{temperature\_above\_ambient}_{\text{end},1} + \text{ambient}}{1 - e^{-\left(\frac{\text{period}_1}{\tau}\right)}}$$

$$T_{\text{infinity}} = 243.517 \cdot \text{K}$$

$$P_{\text{diss}} := k \cdot (T_{\text{infinity}} - \text{ambient})$$

$$P_{\text{diss}} = 55.569 \cdot \text{watt}$$

ii) at full-power (steady state power dissipation)

$$T_{\text{infinity}} := \frac{\text{temperature\_above\_ambient}_{\text{end},3} + \text{ambient}}{1 - e^{-\left(\frac{\text{period}_3}{\tau}\right)}}$$

$$T_{\text{infinity}} = 273.319 \cdot \text{K}$$

$$P_{\text{diss}} := k \cdot (T_{\text{infinity}} - \text{ambient})$$

$$P_{\text{diss}} = 88.351 \cdot \text{watt}$$

$$P_{\text{mech}} := 1000 \cdot \text{watt}$$

$$P_{\text{mech}} = P_{\text{diss}} \cdot \frac{(1 - s)^2}{s(2 - s)}$$

$$s := \frac{1}{(2 \cdot (P_{\text{mech}} + P_{\text{diss}}))} \cdot \left( 2 \cdot P_{\text{mech}} + 2 \cdot P_{\text{diss}} + 2 \cdot \sqrt{P_{\text{mech}} \cdot (P_{\text{mech}} + P_{\text{diss}})} \right)$$

$$s := \frac{1}{(2 \cdot (P_{\text{mech}} + P_{\text{diss}}))} \cdot \left( 2 \cdot P_{\text{mech}} + 2 \cdot P_{\text{diss}} - 2 \cdot \sqrt{P_{\text{mech}} \cdot (P_{\text{mech}} + P_{\text{diss}})} \right)$$

$$s = \begin{pmatrix} 1.959 \\ 0.041 \end{pmatrix}$$