ENGINEERING TRIPOS PART IIA

Friday 13 May 2005 2.30 to 4.00

Module 3C4

MACHINE DESIGN - TRANSMISSIONS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheet (10 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Discuss the considerations affecting the design of high speed cams. [20%]

(b) A high speed cam rotating at an angular velocity ω is required to give a lift L over a cam rotation of 40° with the follower stationary at the beginning and end of the lift. (Another part of the cam profile lowers the follower by L.) A general power-law cam profile of the form

$$y = \frac{L}{2}\left(\frac{\theta}{20}\right)^n, \text{ for } 0° \le \theta \le 20°,$$

$$y = L - \frac{L}{2}\left(\frac{40-\theta}{20}\right)^n, \text{ for } 20° < \theta \le 40°,$$

is proposed, where y is the cam profile, θ is the cam rotation angle in degrees and n is an integer constant.

(i) For $n = 2$ show that the cam profile corresponds to constant acceleration and deceleration phases of magnitude $81L\omega^2/\pi^2$. Explain why this profile minimizes the maximum acceleration imposed for the given lift and duration. [30%]

(ii) For the constant acceleration cam with $n = 2$ outlined in part (i) above, derive an expression in terms of L and ω for the acceleration required if the lift can start early and finish late, with lift errors at 0° and 40° of $0.02L$. [25%]

(iii) Investigate and discuss how the choice of n affects the suitability of the cam profile for high speed use. [25%]

2 (a) Consider an epicyclic gear with annulus and sun tooth numbers A and S.

(i) Derive an expression for the ratio of the torques acting on the shafts connected to the annulus and the sun wheel. Explain why this ratio is independent of the speeds of the epicyclic components. You may use the epicyclic speed equation given on the datasheet without proof. [15%]

(ii) Derive an expression for the ratio of the power flows *into* the sun and annulus components, where the planet carrier rotates at half the speed of the annulus and in the same direction. [15%]

(b) Figure 1 shows the characteristics of a converter-coupling idealized using bi-linear functions. The ratio of output to input torques T_o/T_i and the input torque coefficient C_{Ti} are given as functions of the ratio of output to input speeds ω_o/ω_i. A definition of C_{Ti} is given in the figure, with D being the diameter of the converter-coupling and ρ the density of the fluid.

(i) Explain the form of torque ratio characteristic in Fig. 1, making suitable reference to the way in which the converter-coupling works. [20%]

(ii) The converter-coupling is driven by a motor of constant torque $T_i = 0.2\rho D^5$. Sketch carefully the variation of converter coupling efficiency with input speed ω_i, marking salient points. [50%]

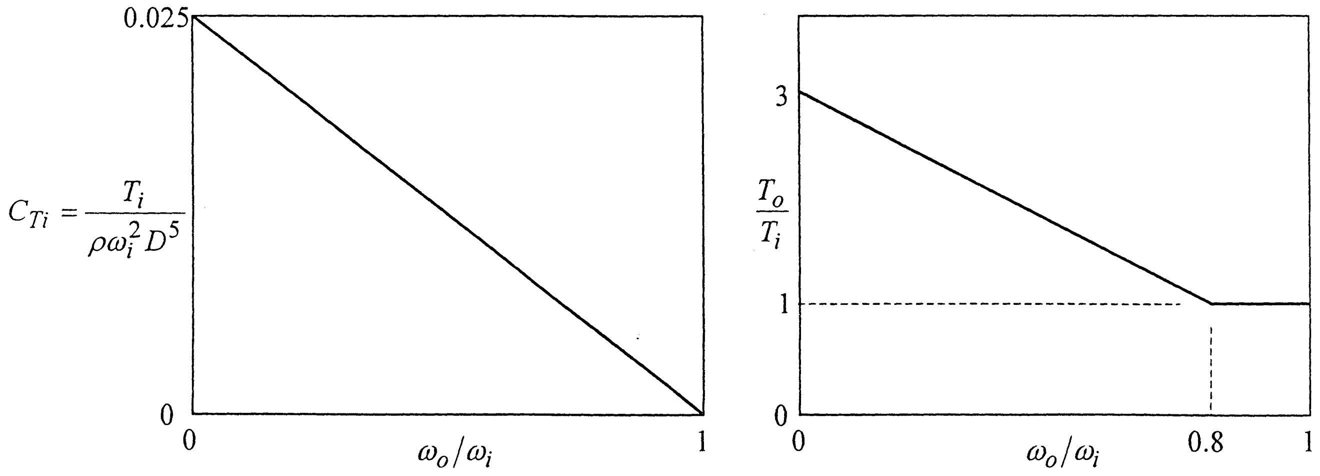


Fig. 1

3 Figure 2 shows a constant speed power source driving a load inertia J. The constant speed of the power source is ω_1. The torque is transmitted through a series combination of a permanently engaged friction clutch and a speed ratio $G = \omega_2/\omega_3$. The clutch has no inertia so that $T_1 = T_2$. Difference in speed between the power source and the load is accommodated by slip in the clutch. The maximum torque transmitted by the clutch is T_μ, so that if $|T_1| < T_\mu$ then $\omega_1 = \omega_2$, but if $\omega_1 \neq \omega_2$ then $|T_1| = T_\mu$.

(a) If $\omega_3 = 0$ at time $t = 0$, show that the clutch stops slipping when

$$t = \frac{\omega_1 J}{G^2 T_\mu}$$

and find the energy dissipated in the clutch during this time interval as a fraction of total energy taken from the power source. [30%]

(b) The fixed speed ratio is replaced with an epicyclic gear that enables the speed ratio to be switched between two values G_1 and G_2. Figure 3 shows how the speed of the load changes with time when the initial speed is zero and $G_1 > G_2$. G_1 is selected first. When the clutch first stops slipping, at time t_1, G_2 is selected. The clutch next stops slipping at time t_2.

(i) Show that $t_2 = \dfrac{\omega_1 J}{T_\mu}\left(\dfrac{1}{G_2^2} - \dfrac{1}{G_1 G_2} + \dfrac{1}{G_1^2}\right)$. [30%]

(ii) Find an expression for the energy dissipated in the clutch between $t = 0$ and $t = t_2$. [20%]

(iii) If G_1=2 and G_2=1, calculate the energy dissipated as a fraction of the total energy taken from the power source. Explain qualitatively why the fraction of energy dissipated in the clutch is less than in part (a), and suggest how further reduction in energy dissipation could be achieved. [20%]

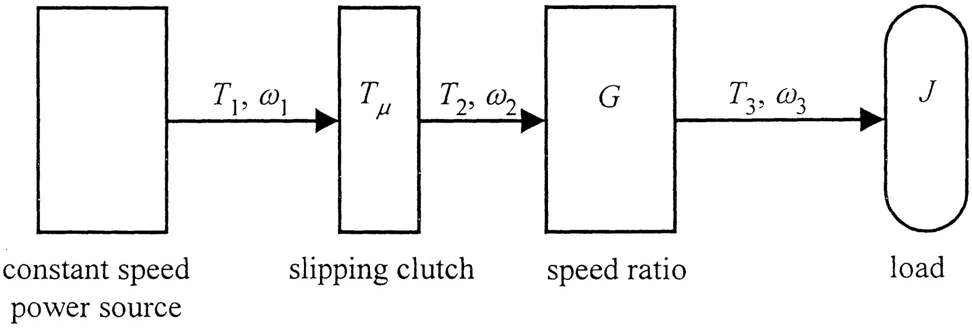


Fig. 2

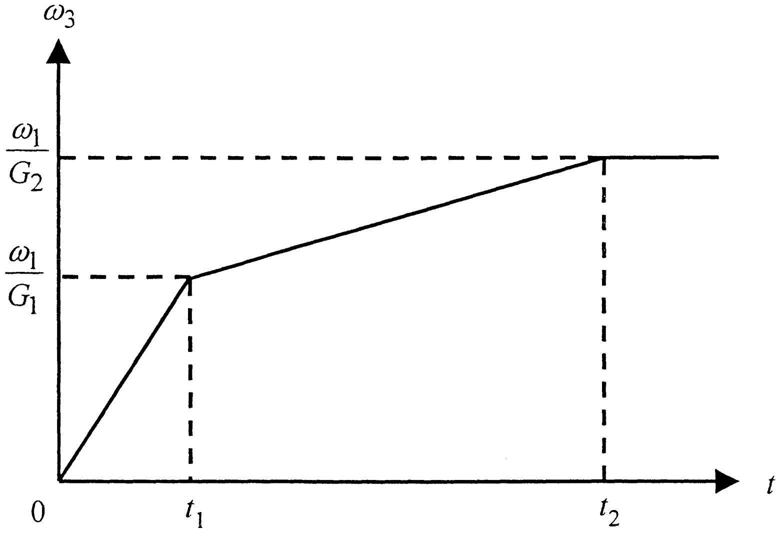


Fig. 3

(TURN OVER

4 (a) Briefly explain the operating principle of a parallel hybrid drive. Describe two different devices suitable for energy storage in a parallel hybrid drive. [15%]

(b) A load has a torque-speed characteristic $T = \omega c$ where T is torque, ω is speed and c is a constant. The load is driven with a duty cycle consisting of constant acceleration for τ seconds from zero speed to speed Ω, followed by constant deceleration for a further τ seconds to zero speed. Calculate the peak power required and show that the mean power is $\Omega^2 c/3$. [30%]

(c) The load is driven by a hybrid drive consisting of a constant power source (providing the mean power $\Omega^2 c/3$) and an energy storage device. If there are no losses in the storage device, derive expressions for:

(i) the maximum power flows into and out of the storage device;

(ii) the energy required in the storage device at the beginning of the duty cycle to ensure that the energy store is not depleted;

(iii) the maximum energy in the storage device during the duty cycle. [40%]

(d) If the conversions of energy into and out of the storage device each have an efficiency η, find an approximate expression for the increase in mean power required from the constant power source. [15%]

END OF PAPER

ENGINEERING TRIPOS Part IIA

Modules 3C3 and 3C4 Data Sheet

HYDRODYNAMIC LUBRICATION

Viscosity: temperature and pressure effects

Vogel formula $\eta = \eta_0 \exp\left\{\dfrac{b}{T + T_c}\right\}$

Barus equation $\eta = \eta_0 \exp\{\alpha p\}$

Roelands equation $\eta = \eta_0 \exp\left\{[9.67 + \ln \eta_0]\left[\left(1 + \dfrac{p}{p_0 *}\right)^{\beta} - 1\right]\right\}$

Viscous pressure flow

Rate of flow q_x per unit width of fluid of viscosity η down a channel of height h due to pressure gradient, $q_x = -\dfrac{h^3}{12\eta}\dfrac{dp}{dx}$

Reynolds' Equation for a steady configuration

1–D flow: $$\frac{dp}{dx} = 12\eta\overline{U}\left\{\frac{h - h^*}{h^3}\right\}$$

$\overline{U}$ is the entraining velocity so that $|\overline{U}h^*|$ is flow per unit width through the contact.

2–D flow: $$\frac{\partial}{\partial x}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right\} = 12\overline{U}\frac{\partial h}{\partial x}$$

Hydrodynamic lubrication of discs

$\dfrac{h}{R} = C\dfrac{\eta\overline{U}}{W'}$ where R is the reduced or effective radius and W' the load per unit length

$C_{min} = 4.00$ for half Sommerfeld boundary conditions

$C_{min} = 4.89$ for half Reynolds' boundary conditions

ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

Effective curvature $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where R_1, R_2 are the radii of curvature of the two bodies (convex positive).

Contact modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

where E_1, E_2 and ν_1, ν_2 are Young's moduli and Poisson's ratios.

	Line contact (width $2b$; load W' per unit length)	Circular contact (diameter $2a$; load W)
Semi contact width or contact radius	$b = 2\left\{\frac{W'R}{\pi E^*}\right\}^{1/2}$	$a = \left\{\frac{3WR}{4E^*}\right\}^{1/3}$
Maximum contact pressure ("Hertz stress")	$p_0 = \left\{\frac{W'E^*}{\pi R}\right\}^{1/2}$	$p_0 = \frac{1}{\pi}\left\{\frac{6WE^{*2}}{R^2}\right\}^{1/3}$
Approach of centres	$\delta = \frac{2W'}{\pi}\left[\frac{1-\nu_1^2}{E_1}\left\{\ln\left(\frac{4R_1}{b}\right) - \frac{1}{2}\right\} + \frac{1-\nu_2^2}{E_2}\left\{\ln\left(\frac{4R_2}{b}\right) - \frac{1}{2}\right\}\right]$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{W^2}{E^{*2}R}\right\}^{1/3}$
Mean contact pressure	$\bar{p} = \frac{W'}{2b} = \frac{\pi}{4}p_0$	$\bar{p} = \frac{W}{\pi a^2} = \frac{2}{3}p_0$
Maximum shear stress	$\tau_{\max} = 0.300p_0$ at ($x = 0$, $z = 0.79b$)	$\tau_{\max} = 0.310p_0$ at ($r = 0$, $z = 0.48a$) for $\nu = 0.3$
Maximum tensile stress	zero	$\frac{1}{3}(1-2\nu)p_0$ at ($r = a$, $z = 0$)

Mildly elliptical contacts

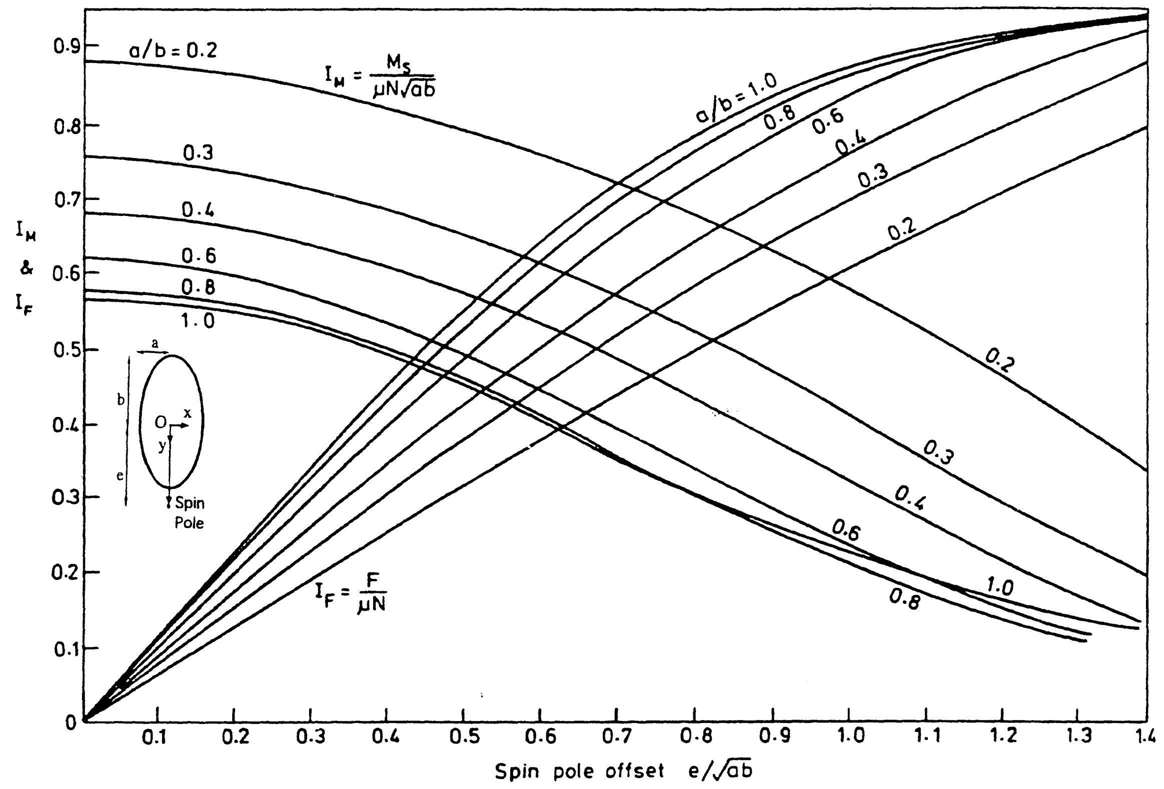
If the gap at zero load is $h = \frac{1}{2}\mathrm{A}x^2 + \frac{1}{2}\mathrm{B}y^2$, and $0.2 < \mathrm{A/B} < 5$

Ratio of semi-axes $b/a \cong (\mathrm{A/B})^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with $R = (\mathrm{AB})^{-1/2}$ or better $R_e = [\mathrm{AB}(\mathrm{A}+\mathrm{B})/2]^{-1/3}$.

For **approach** use circular contact equation with $R = (\mathrm{AB})^{-1/2}$ (**not** R_e)

Hertzian contact frictional losses



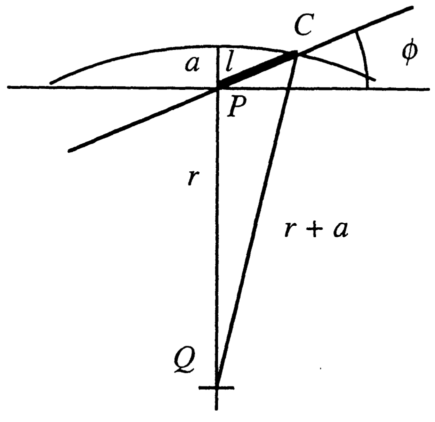
INVOLUTE GEARING

Spur gears

pitch cylinder radii	r	with suffix 1 or 2
base cylinder radii	r_b	
addendum cylinder radii	r_a	
number of teeth	N	
addendum	$a = r_a - r$	
pressure angle	ϕ	

circumferential pitch	$p = 2\pi r/N$
base pitch	$p_b = p\cos\phi$
module	$m = p/\pi = 2r/N$
ratio of contact	r_c
radius of curvature at pitch point	$\rho = r\sin\phi$

Path of contact



$$l = \left\{r^2\sin^2\phi + a(2r+a)\right\}^{1/2} - r\sin\phi$$

For a standard 20° spur wheel with N teeth of module m this becomes

$$\frac{l}{m} = \left(0.02924N^2 + N + 1\right)^{1/2} - 0.1710N$$

Standard tooth forms

Addendum $a = m$, Dedendum $= \frac{7}{6}m$, pressure angle = 20°.

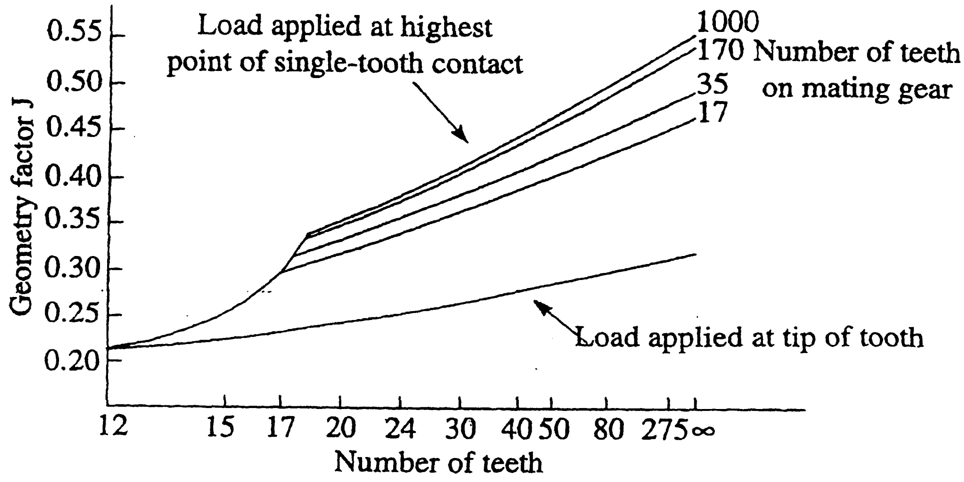
Modules: 0.3 – 1.0 mm in 0.1 mm steps
1.0 – 4.0 mm in 0.25 mm steps
4.0 – 7.0 mm in 0.5 mm steps
7.0 – 16.0 mm in 1.0 mm steps
16.0 – 24.0 mm in 2.0 mm steps
24.0 – 45.0 mm in 3.0 mm steps
45.0 – 75.0 mm in 5.0 mm steps

Friction in spur gears

$$\frac{\text{average friction loss}}{\text{power transmitted}} \approx \mu\pi\left\{\frac{1}{N_1} + \frac{1}{N_2}\right\}$$

Tooth failure

Allowable bending stress σ_b according to AGMA guidelines given by $\sigma_b = \frac{P'_T}{Jm}$ where P'_T is force per unit face-width acting tangentially to pitch circle and J given in the figure below for 20° spur gears. Typical values of σ_b shown in table.



Typical allowable tooth stresses (AGMA)

Material	Condition	Bending fatigue strength σ_b (MPa)	Surface fatigue strength σ_s (MPa)
Steel	Through hardened and tempered	170-390	590-1200
	Carburised and case hardened	380-480	1250-1550
Cast iron	As cast	69-90	450-590
Nodular iron	Quenched, annealed and tempered	150-300	500-800
Malleable iron	Pearlitic	70-145	500-650

EPICYCLIC SPEED RULE

$$\omega_s = (1+R)\omega_c - R\omega_a \quad \text{where } R = \frac{A}{S}$$

ROLLING ELEMENT BEARINGS

Fatigue life

$L = a_1 a_{23}(C/P)^p$ p = 3 for ball and 10/3 for roller bearings

Fatigue probability %	10	5	4	3	2	1
Life adjust factor a_1	1	0.62	0.53	0.44	0.33	0.21

Minimum radial load F_{rm}

For a ball bearing $F_{rm} = k_r\left(\frac{\nu n}{1000}\right)^{2/3}\left(\frac{d_m}{100}\right)^2$

For a roller bearing $F_{rm} = k_r\left(6+\frac{4n}{n_r}\right)\left(\frac{d_m}{100}\right)^2$

F_{rm} is the minimum radial load in N, d_m is the mean bearing diameter in mm, ν is the kinematic viscosity in mm^2s^{-1}, n the speed in rpm and n_r the limiting speed for oil lubrication. k_r is typically 25 for ball bearings and 150 for roller bearings.

Bearing choice

The information on the following pages concerning minimum loads, viscosities and standard bearing sizes and ratings is extracted from the SKF General Bearing Catalogue and is copied with permission. It is SKF copyright and is not to be further reproduced.

MPFS, DJC, JAW
January 03

Required viscosities and the effect of viscosity ratio on a_{23}

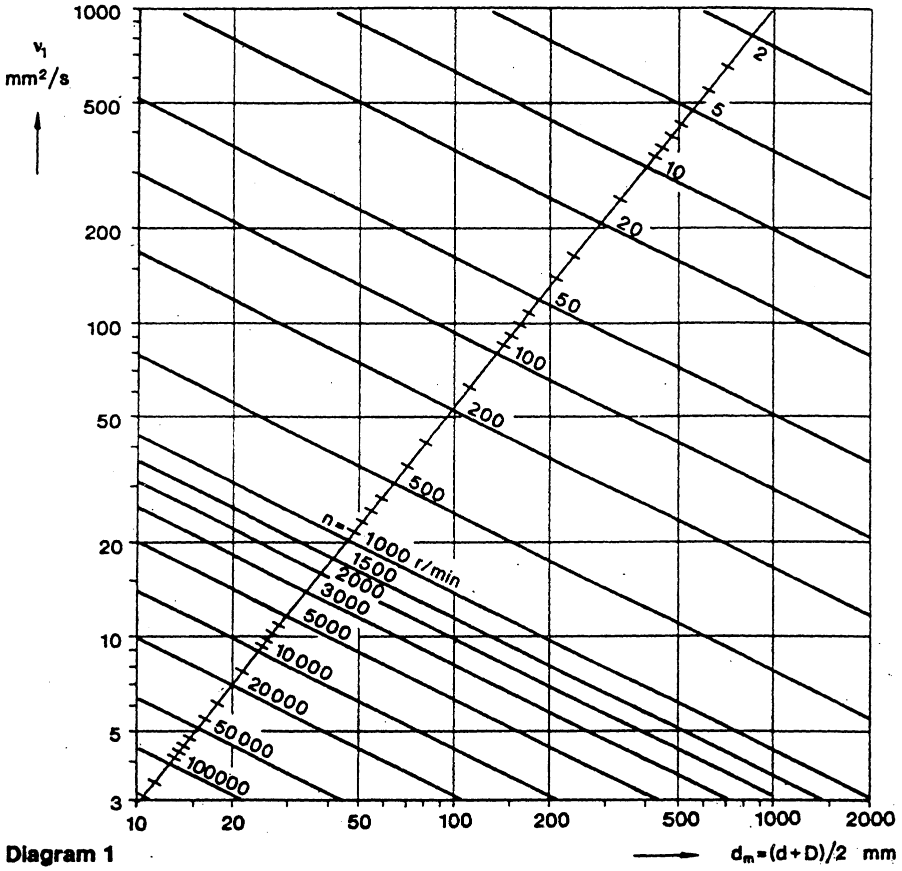


Diagram 1

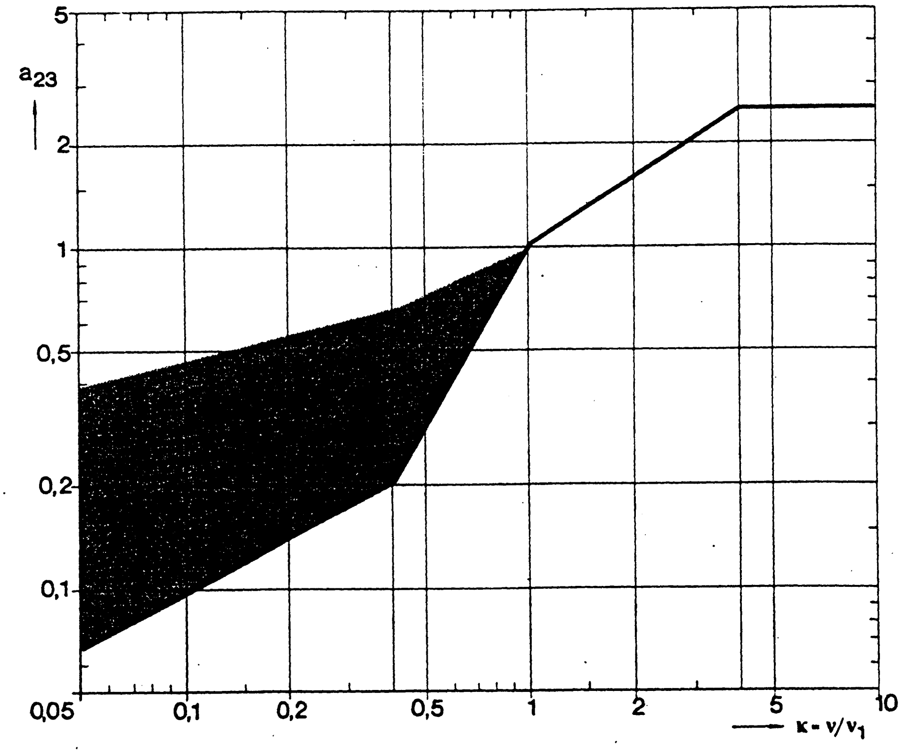
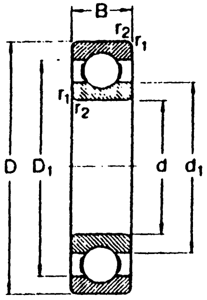
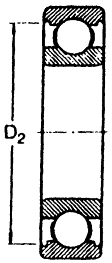


Diagram 3

Deep groove ball bearings
single row
d 35–55 mm



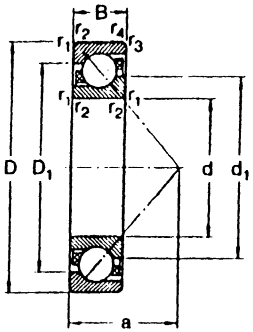
With full outer ring shoulders



With recessed outer ring shoulders

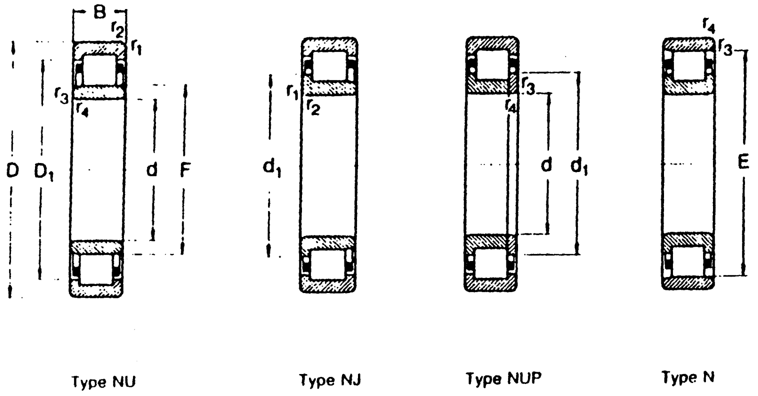
Principal dimensions			Basic load ratings dynamic	static	Fatigue load limit	Speed ratings Lubrication grease	oil	Mass	Designation
d	D	B	C	C_0	P_u				
mm			N		N	r/min		kg	–
35	47	7	4 750	3 200	166	13 000	16 000	0,030	**61807**
	55	10	9 560	6 200	290	11 000	14 000	0,080	**61907**
	62	9	12 400	8 150	375	10 000	13 000	0,11	**16007**
	62	14	15 900	10 200	440	10 000	13 000	0,16	**6007**
	72	17	25 500	15 300	655	9 000	11 000	0,29	**6207**
	80	21	33 200	19 000	815	8 500	10 000	0,46	**6307**
	100	25	55 300	31 000	1 290	7 000	8 500	0,95	**6407**
40	52	7	4 940	3 450	186	11 000	14 000	0,034	**61808**
	62	12	13 800	9 300	425	10 000	13 000	0,12	**61908**
	68	9	13 300	9 150	440	9 500	12 000	0,13	**16008**
	68	15	16 800	11 600	490	9 500	12 000	0,19	**6008**
	80	18	30 700	19 000	800	8 500	10 000	0,37	**6208**
	90	23	41 000	24 000	1 020	7 500	9 000	0,63	**6308**
	110	27	63 700	36 500	1 530	6 700	8 000	1,25	**6408**
45	58	7	6 050	4 300	228	9 500	12 000	0,040	**61809**
	68	12	10 100	6 700	285	9 000	11 000	0,14	**61909**
	75	10	15 600	10 800	520	9 000	11 000	0,17	**16009**
	75	16	20 800	14 600	640	9 000	11 000	0,25	**6009**
	85	19	33 200	21 600	915	7 500	9 000	0,41	**6209**
	100	25	52 700	31 500	1 340	6 700	8 000	0,83	**6309**
	120	29	76 100	45 000	1 900	6 000	7 000	1,55	**6409**
50	65	7	6 240	4 750	250	9 000	11 000	0,052	**61810**
	72	12	14 600	10 400	500	8 500	10 000	0,14	**61910**
	80	10	16 300	11 400	560	8 500	10 000	0,18	**16010**
	80	16	21 600	16 000	710	8 500	10 000	0,26	**6010**
	90	20	35 100	23 200	980	7 000	8 500	0,46	**6210**
	110	27	61 800	38 000	1 600	6 300	7 500	1,05	**6310**
	130	31	87 100	52 000	2 200	5 300	6 300	1,90	**6410**
55	72	9	8 320	6 200	325	8 500	10 000	0,083	**61811**
	80	13	15 900	11 400	560	8 000	9 500	0,19	**61911**
	90	11	19 500	14 000	695	7 500	9 000	0,26	**16011**
	90	18	28 100	21 200	900	7 500	9 000	0,39	**6011**
	100	21	43 600	29 000	1 250	6 300	7 500	0,61	**6211**
	120	29	71 500	45 000	1 900	5 600	6 700	1,35	**6311**
	140	33	99 500	62 000	2 600	5 000	6 000	2,30	**6411**

Angular contact ball bearings
single row
d 10–65 mm



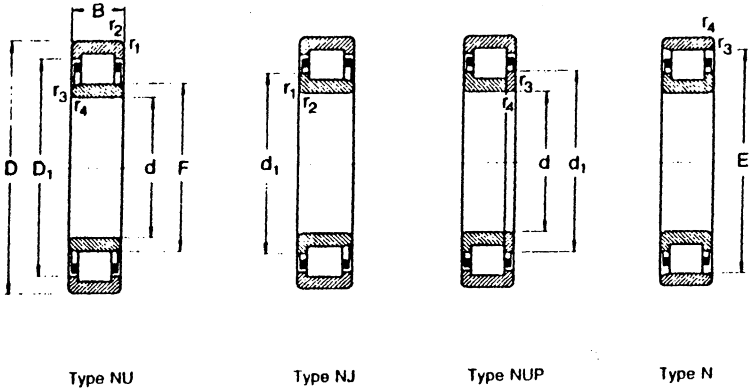
Principal dimensions			Basic load ratings dynamic	static	Fatigue load limit	Speed ratings Lubrication grease	oil	Mass	Designation
d	D	B	C	C_0	P_u				
mm			N		N	r/min		kg	–
10	30	9	7 020	3 350	140	19 000	28 000	0,030	**7200 BE**
12	32	10	7 610	3 800	160	18 000	26 000	0,036	**7201 BE**
	37	12	10 600	5 000	208	17 000	24 000	0,060	**7301 BE**
15	35	11	8 840	4 800	204	17 000	24 000	0,045	**7202 BE**
	42	13	13 000	6 700	280	15 000	20 000	0,080	**7302 BE**
17	40	12	11 100	6 100	260	15 000	20 000	0,065	**7203 BE**
	47	14	15 900	8 300	355	13 000	18 000	0,11	**7303 BE**
20	47	14	14 000	8 300	355	12 000	17 000	0,11	**7204 BE**
	52	15	19 000	10 400	440	11 000	16 000	0,14	**7304 BE**
25	52	15	15 600	10 200	430	10 000	15 000	0,13	**7205 BE**
	62	17	26 000	15 600	655	9 000	13 000	0,23	**7305 BE**
30	62	16	23 800	15 600	655	8 500	12 000	0,20	**7206 BE**
	72	19	34 500	21 200	900	8 000	11 000	0,34	**7306 BE**
35	72	17	30 700	20 800	880	8 000	11 000	0,28	**7207 BE**
	80	21	39 000	24 500	1 040	7 500	10 000	0,45	**7307 BE**
40	80	18	36 400	26 000	1 100	7 000	9 500	0,37	**7208 BE**
	90	23	49 400	33 500	1 400	6 700	9 000	0,63	**7308 BE**
45	85	19	37 700	28 000	1 200	6 700	9 000	0,42	**7209 BE**
	100	25	60 500	41 500	1 730	6 000	8 000	0,85	**7309 BE**
50	90	20	39 000	30 500	1 290	6 000	8 000	0,47	**7210 BE**
	110	27	74 100	51 000	2 200	5 300	7 000	1,10	**7310 BE**
55	100	21	48 800	38 000	1 630	5 600	7 500	0,62	**7211 BE**
	120	29	85 200	60 000	2 550	4 800	6 300	1,40	**7311 BE**
60	110	22	57 200	45 500	1 930	5 000	6 700	0,80	**7212 BE**
	130	31	95 600	69 500	3 000	4 500	6 000	1,75	**7312 BE**
65	120	23	66 300	54 000	2 280	4 500	6 000	1,00	**7213 BE**
	140	33	108 000	80 000	3 350	4 300	5 600	2,15	**7313 BE**

Cylindrical roller bearings
single row
d 40–45 mm



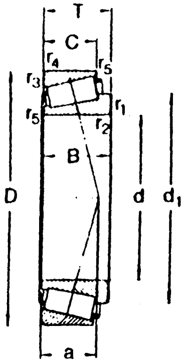
Principal dimensions d	 D	 B	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation
mm			N		N	r/min		kg	–
40 (cont.)	90	23	80 900	78 000	10 200	6 700	8 000	0,65	**NU 308 EC**
	90	23	80 900	78 000	10 200	6 700	8 000	0,67	**NJ 308 EC**
	90	23	80 900	78 000	10 200	6 700	8 000	0,68	**NUP 308 EC**
	90	23	80 900	78 000	10 200	6 700	8 000	0,64	**N 308 EC**
	90	33	112 000	120 000	15 300	6 300	7 500	0,94	**NU 2308 EC**
	90	33	112 000	120 000	15 300	6 300	7 500	0,96	**NJ 2308 EC**
	90	33	112 000	120 000	15 300	6 300	7 500	0,98	**NUP 2308 EC**
	110	27	96 800	90 000	11 600	6 000	7 000	1,30	**NU 408**
	110	27	96 800	90 000	11 600	6 000	7 000	1,30	**NJ 408**
	110	27	96 800	90 000	11 600	6 000	7 000	1,35	**NUP 408**
45	75	16	44 600	52 000	6 300	9 000	11 000	0,26	**NU 1009 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,43	**NU 209 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,44	**NJ 209 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,45	**NUP 209 EC**
	85	19	60 500	64 000	8 150	6 700	8 000	0,43	**N 209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,52	**NU 2209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,54	**NJ 2209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,55	**NUP 2209 EC**
	85	23	73 700	81 500	10 600	6 700	8 000	0,52	**N 2209 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,90	**NU 309 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,92	**NJ 309 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,95	**NUP 309 EC**
	100	25	99 000	100 000	12 900	6 300	7 500	0,88	**N 309 EC**
	100	36	138 000	153 000	20 000	5 600	6 700	1,30	**NU 2309 EC**
	100	36	138 000	153 000	20 000	5 600	6 700	1,30	**NJ 2309 EC**
	100	36	138 000	153 000	20 000	5 600	6 700	1,35	**NUP 2309 EC**
	120	29	106 000	102 000	13 400	5 600	6 700	1,65	**NU 409**
	120	29	106 000	102 000	13 400	5 600	6 700	1,65	**NJ 409**
	120	29	106 000	102 000	13 400	5 600	6 700	1,70	**NUP 409**

Cylindrical roller bearings
single row
d 50–55 mm



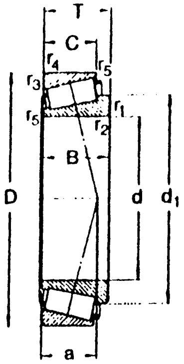
Principal dimensions d	 D	 B	Basic load ratings dynamic C	static C_0	Fatigue load limit P_u	Speed ratings Lubrication grease	oil	Mass	Designation
mm			N		N	r/min		kg	–
50	80	16	30 800	34 500	4 000	8 500	10 000	0,31	**NU 1010**
	90	20	64 400	69 500	8 800	6 300	7 500	0,48	**NU 210 EC**
	90	20	64 400	69 500	8 800	6 300	7 500	0,49	**NJ 210 EC**
	90	20	64 400	69 500	8 800	6 300	7 500	0,51	**NUP 210 EC**
	90	20	64 400	69 500	8 800	6 300	7 500	0,48	**N 210 EC**
	90	23	78 100	88 000	11 400	6 300	7 500	0,56	**NU 2210 EC**
	90	23	78 100	88 000	11 400	6 300	7 500	0,58	**NJ 2210 EC**
	90	23	78 100	88 000	11 400	6 300	7 500	0,59	**NUP 2210 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,15	**NU 310 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,15	**NJ 310 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,20	**NUP 310 EC**
	110	27	110 000	112 000	15 000	5 000	6 000	1,15	**N 310 EC**
	110	40	161 000	186 000	24 500	5 000	6 000	1,70	**NU 2310 EC**
	110	40	161 000	186 000	24 500	5 000	6 000	1,75	**NJ 2310 EC**
	110	40	161 000	186 000	24 500	5 000	6 000	1,80	**NUP 2310 EC**
	130	31	130 000	127 000	16 600	5 000	6 000	2,00	**NU 410**
	130	31	130 000	127 000	16 600	5 000	6 000	2,05	**NJ 410**
55	90	18	57 200	69 500	8 300	7 000	8 500	0,40	**NU 1011 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,66	**NU 211 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,67	**NJ 211 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,69	**NUP 211 EC**
	100	21	84 200	95 000	12 200	6 000	7 000	0,66	**N 211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,79	**NU 2211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,81	**NJ 2211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,82	**NUP 2211 EC**
	100	25	99 000	118 000	15 300	6 000	7 000	0,79	**N 2211 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,45	**NU 311 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,50	**NJ 311 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	1,55	**NUP 311 EC**
	120	29	138 000	143 000	18 600	4 800	5 600	[illegible]	[illegible]

Taper roller bearings
single row
d 35–50 mm



Principal dimensions			Basic load ratings dynamic	static	Fatigue load limit	Speed ratings Lubrication grease	oil	Mass	Designation	Dimension Series to ISO 355
d	D	T	C	C_0	P_u					
mm			N		N	r/min		kg	–	–
35	80	22,75	72 100	73 500	8 500	5 000	6 700	0,52	**30307**	2FB
(cont.)	80	22,75	61 600	67 000	7 800	4 500	6 000	0,52	**31307**	7FB
	80	32,75	95 200	106 000	12 200	4 800	6 300	0,73	**32307**	2FE
	80	32,75	93 500	114 000	13 200	4 500	6 000	0,80	**32307 B**	5FE
40	68	19	52 800	71 000	7 800	5 300	7 000	0,27	**32008 X**	3CD
	75	26	79 200	104 000	11 600	5 000	6 700	0,51	**33108**	2CE
	80	19,75	61 600	68 000	7 650	4 800	6 300	0,42	**30208**	3DB
	80	24,75	74 800	86 500	9 800	4 800	6 300	0,53	**32208**	3DC
	80	32	105 000	132 000	15 300	4 300	5 600	0,77	**33208**	2DE
	85	33	121 000	150 000	17 300	4 500	6 000	0,90	**T2EE 040**	2EE
	90	25,25	85 800	95 000	11 000	4 500	6 000	0,72	**30308**	2FB
	90	25,25	73 700	81 500	9 650	4 000	5 300	0,72	**31308**	7FB
	90	35,25	117 000	140 000	16 300	4 000	5 300	1,00	**32308**	2FD
	90	35,25	108 000	140 000	16 300	4 000	5 300	1,10	**32308 B**	5FD
45	75	20	58 300	80 000	8 800	4 800	6 300	0,34	**32009 X**	3CC
	80	26	84 200	114 000	12 900	4 500	6 000	0,56	**33109**	3CE
	85	20,75	66 000	76 500	8 650	4 500	6 000	0,48	**30209**	3DB
	85	24,75	80 900	98 000	11 200	4 500	6 000	0,58	**32209**	3DC
	85	24,75	73 700	93 000	11 000	4 300	5 600	0,60	**32209 B**	5DC
	85	32	108 000	143 000	16 300	4 000	5 300	0,82	**33209**	3DE
	95	29	89 700	112 000	12 900	3 600	4 800	0,92	**T7FC 045**	7FC
	95	36	147 000	186 000	21 200	4 000	5 300	1,20	**T2ED 045**	2ED
	100	27,25	108 000	120 000	14 600	4 000	5 300	0,97	**30309**	2FB
	100	27,25	91 300	102 000	12 500	3 400	4 500	0,95	**31309**	7FB
	100	38,25	140 000	170 000	20 400	3 600	4 800	1,35	**32309**	2FD
	100	38,25	134 000	176 000	20 000	3 600	4 800	1,45	**32309 B**	5FD
50	80	20	60 500	88 000	9 650	4 500	6 000	0,37	**32010 X**	3CC
	80	24	69 300	102 000	11 400	4 500	6 000	0,45	**33010**	2CE
	82	21,5	72 100	100 000	11 000	4 500	6 000	0,43	**K-JLM 104948/K-JLM 104910**	–
	85	26	85 800	122 000	13 700	4 300	5 600	0,59	**33110**	3CE
	90	21,75	76 500	91 500	10 400	4 300	5 600	0,54	**30210**	3DB
	90	24,75	82 500	100 000	11 600	4 300	5 600	0,61	**32210**	3DC
	90	24,75	82 500	104 000	12 500	4 000	5 300	0,65	**32210 B**	5DC
	90	28	106 000	140 000	16 300	4 000	5 300	0,75	**K-JM 205149/K-JM 205110**	–
	90	28	106 000	140 000	16 300	4 000	5 300	0,75	**K-JM 205149/K-JM 205110 A**	–
	90	32	114 000	160 000	18 300	3 800	5 000	0,90	**33210**	3DE
	100	36	154 000	200 000	22 800	3 800	5 000	1,30	**T2ED 050**	2ED
	105	32	108 000	137 000	16 000	3 200	4 300	1,20	**T7FC 050**	7FC

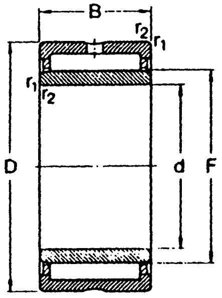
Taper roller bearings
single row
d 50–65 mm



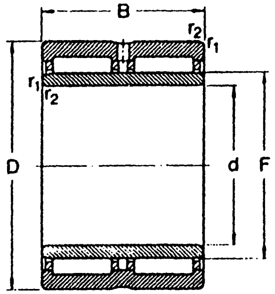
Principal dimensions			Basic load ratings dynamic	static	Fatigue load limit	Speed ratings Lubrication grease	oil	Mass	Designation	Dimension Series to ISO 355
d	D	T	C	C_0	P_u					
mm			N		N	r/min		kg	–	–
50	110	29,25	125 000	140 000	17 000	3 600	4 800	1,25	**30310**	2FB
(cont.)	110	29,25	106 000	120 000	14 300	3 200	4 300	1,20	**31310**	7FB
	110	42,25	172 000	212 000	24 500	3 200	4 300	1,80	**32310**	2FD
	110	42,25	161 000	216 000	25 000	3 200	4 300	1,85	**32310 B**	5FD
55	90	23	78 100	112 000	12 500	4 000	5 300	0,56	**K-JLM 506849/K-JLM 506810**	–
	90	23	80 900	116 000	13 200	4 000	5 300	0,55	**32011 X**	3CC
	90	27	89 700	137 000	15 300	4 000	5 300	0,67	**33011**	2CE
	95	30	110 000	156 000	18 000	3 800	5 000	0,86	**33111**	3CE
	100	22,75	89 700	106 000	12 200	3 800	5 000	0,70	**30211**	3DB
	100	26,75	106 000	129 000	15 000	3 800	5 000	0,83	**32211**	3DC
	100	26,75	101 000	127 000	15 300	3 600	4 800	0,87	**32211 B**	–
	100	35	138 000	190 000	22 000	3 400	4 500	1,20	**33211**	3DE
	110	39	179 000	232 000	26 500	3 400	4 500	1,70	**T2ED 055**	2ED
	115	34	125 000	163 000	19 600	3 000	4 000	1,60	**T7FC 055**	7FC
	120	31,5	142 000	163 000	19 600	3 200	4 300	1,55	**30311**	2FB
	120	31,5	121 000	137 000	17 000	2 800	3 800	1,55	**31311**	7FB
	120	45,5	198 000	250 000	29 000	3 000	4 000	2,30	**32311**	2FD
	120	45,5	190 000	260 000	30 000	2 800	3 800	2,50	**32311 B**	5FD
60	95	23	82 500	122 000	13 700	3 800	5 000	0,59	**32012 X**	4CC
	95	24	84 200	132 000	15 000	3 600	4 800	0,62	**K-JLM 508748/K-JLM 508710**	–
	95	27	91 300	143 000	16 000	3 800	5 000	0,71	**33012**	2CE
	100	30	117 000	170 000	19 600	3 600	4 800	0,92	**33112**	3CE
	110	23,75	99 000	114 000	13 400	3 400	4 500	0,88	**30212**	3EB
	110	29,75	125 000	160 000	19 000	3 400	4 500	1,15	**32212**	3EC
	110	38	168 000	236 000	27 000	3 000	4 000	1,60	**33212**	3EE
	115	39	168 000	250 000	27 500	3 000	4 000	1,85	**T5ED 060**	5ED
	115	40	194 000	260 000	30 000	3 200	4 300	1,85	**T2EE 060**	2EE
	125	37	154 000	204 000	24 500	2 600	3 600	2,05	**T7FC 060**	7FC
	130	33,5	168 000	196 000	23 600	3 000	4 000	1,95	**30312**	2FB
	130	33,5	145 000	166 000	20 400	2 600	3 600	1,90	**31312**	7FB
	130	48,5	229 000	290 000	34 000	2 600	3 600	2,85	**32312**	2FD
	130	48,5	220 000	305 000	35 500	2 600	3 600	2,80	**32312 B**	5FD
65	100	23	84 200	127 000	14 300	3 400	4 500	0,63	**32013 X**	4CC
	100	27	96 800	156 000	17 600	3 400	4 500	0,78	**33013**	2CE
	110	28	123 000	183 000	21 200	3 200	4 300	1,05	**K-JM 511946/K-JM 511910**	–
	110	34	142 000	208 000	24 500	3 200	4 300	1,30	**33113**	3DE
	120	24,75	114 000	134 000	16 300	3 000	4 000	1,15	**30213**	3EB
	120	32,75	151 000	193 000	23 200	3 000	4 000	1,50	**32213**	3EC
	120	39	161 000	240 000	27 500	3 000	4 000	1,95	**T5ED 065**	5ED

Needle roller bearings with flanges with inner ring

d 40–65 mm



Series NKI(S), NA 49



Series NA 69

Principal dimensions			Basic load ratings dynamic	static	Fatigue load limit	Speed ratings Lubrication grease	oil	Mass	Designation
d	D	B	C	C_0	P_u				
mm			N		N	r/min		kg	–
40	55	20	27 500	57 000	7 200	6 300	9 000	0,14	**NKI 40/20**
	55	30	40 200	93 000	12 000	6 300	9 000	0,22	**NKI 40/30**
	62	22	42 900	71 000	9 150	5 600	8 000	0,23	**NA 4908**
	62	40	67 100	125 000	16 000	5 600	8 000	0,43	**NA 6908**
	65	22	42 900	72 000	9 150	5 600	8 000	0,28	**NKIS 40**
42	57	20	29 200	61 000	7 650	6 000	8 500	0,15	**NKI 42/20**
	57	30	41 800	98 000	12 900	6 000	8 500	0,22	**NKI 42/30**
45	62	25	38 000	78 000	10 000	5 600	8 000	0,23	**NKI 45/25**
	62	35	49 500	110 000	14 300	5 600	8 000	0,32	**NKI 45/35**
	68	22	45 700	78 000	10 000	5 300	7 500	0,27	**NA 4909**
	68	40	70 400	137 000	17 300	5 300	7 500	0,50	**NA 6909**
	72	22	44 600	78 000	10 000	5 000	7 000	0,34	**NKIS 45**
50	68	25	40 200	88 000	11 200	5 300	7 500	0,27	**NKI 50/25**
	68	35	52 300	122 000	16 000	5 300	7 500	0,38	**NKI 50/35**
	72	22	47 300	85 000	11 000	5 000	7 000	0,27	**NA 4910**
	72	40	73 700	150 000	19 000	5 000	7 000	0,52	**NA 6910**
	80	28	62 700	104 000	13 700	4 500	6 300	0,52	**NKIS 50**
55	72	25	41 800	96 500	12 200	4 800	6 700	0,27	**NKI 55/25**
	72	35	55 000	134 000	17 600	4 800	6 700	0,38	**NKI 55/35**
	80	25	57 200	106 000	13 700	4 500	6 300	0,40	**NA 4911**
	80	45	89 700	190 000	24 000	4 500	6 300	0,78	**NA 6911**
	85	28	66 000	114 000	15 000	4 300	6 000	0,56	**NKIS 55**
60	82	25	44 000	95 000	12 000	4 300	6 000	0,40	**NKI 60/25**
	82	35	60 500	146 000	19 000	4 300	6 000	0,55	**NKI 60/35**
	85	25	60 500	114 000	14 600	4 300	6 000	0,43	**NA 4912**
	85	45	93 500	204 000	26 000	4 300	6 000	0,81	**NA 6912**
	90	28	68 200	120 000	15 600	4 000	5 600	0,56	**NKIS 60**
65	90	25	61 600	120 000	15 300	4 000	5 600	0,46	**NA 4913**
	90	25	52 800	106 000	13 700	4 000	5 600	0,47	**NKI 65/25**
	90	35	73 700	163 000	21 600	4 000	5 600	0,66	**NKI 65/35**
	90	45	95 200	212 000	27 000	4 000	5 600	0,83	**NA 6913**
	95	28	70 400	132 000	17 000	3 800	5 300	0,64	**NKIS 65**

PART IIA 2005

3C4 Machine design – transmissions

Engineering Tripos Part IIA 2005

Principal Assessor: **Dr D J Cole**

Module 3C4

Machine Design – Transmissions

1. (b) (ii) $\left(\dfrac{36}{5}\right)^2 \dfrac{L\omega^2}{\pi^2}$

2. (a) (i) $\dfrac{T_A}{T_S} = \dfrac{A}{S} = R$

(iii) $\dfrac{T_A \omega_A}{T_S \omega_S} = \dfrac{2R}{1-R}$

3. (a) fraction lost = 1/2

(b) (ii) $J\omega_1^2 \left(\dfrac{1}{G_1^2} - \dfrac{1}{G_1 G_2} + \dfrac{1}{2G_2^2} \right)$

(iii) fraction lost = 1/3

4. (b) peak power $= \Omega^2 c$

(c) (i) max power in $= \dfrac{\Omega^2 c}{3}$, max power out $= \dfrac{2\Omega^2 c}{3}$

(ii) initial energy required $= \dfrac{2}{9\sqrt{3}} \Omega^2 c \tau$

(iii) maximum energy stored $= \dfrac{4}{9\sqrt{3}} \Omega^2 c \tau$

(d) increase in mean power $= \dfrac{4}{9\sqrt{3}} \Omega^2 c \tau (1-\eta)$, assuming small $(1-\eta)$