UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Tuesday 10 May 2005 2.30 to 4.00

Module 3D1

SOIL MECHANICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (19 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) A sample of very soft clay was placed in an oedometer fitted with porous platens top and bottom. It was brought to a vertical effective stress of 20 kPa, at which its thickness was recorded to be 19.0 mm. The vertical stress was then doubled each hour for 2 hours, and then similarly halved each hour for two hours, until the sample had been brought back into equilibrium under 20 kPa. During this process, the thickness *h* was monitored. The sample thicknesses after 1 hour's equilibration are given below.

σ' kPa	20	40	80	40	20
h mm after 1 hour	19.00	17.90	16.81	17.08	17.35

Finally, the sample was stripped out and found to be saturated with a water content of 20.8%. The specific gravity of its grains was found to be 2.70.

Find appropriate values for the parameters Γ, λ and κ that describe the elasto-plastic compression and swelling of clays. [30%]

(b) In the case of the compression from 40 kPa to 80 kPa, sample thicknesses are available for the first 16 minutes, as recorded below.

t min	0	1	2	4	8	16
h mm	17.91	17.62	17.51	17.34	17.12	16.93

Estimate the coefficient of consolidation C_v of the clay for this increment of loading. [30%]

(c) (i) Explaining your reasoning, make an estimate of C_v for the subsequent unloading from 80 kPa to 40 kPa. [20%]

(ii) An engineer with access to the intermediate consolidation times in all the test stages notices that C_v remains roughly constant while the clay remains on its normal compression line. What does this imply about the permeability of the clay, and about the micromechanics of normal compression? [20%]

2 A long flood embankment was constructed entirely with silt from the flood-plain. It stands 4 m high, with side slopes of 1V:2H. The responsible agency has received reports of poor performance, with the embankment slumping and cracking. The agency has asked that an undisturbed core sample be taken from the centre of the embankment immediately prior to the flooding season, and that piezometers be sealed in place to record any changes in pore water pressure at that location.

(a) A core of 100 mm diameter and 300 mm length was extracted from 2 m depth in the embankment. It was found to weigh 4845 g. A water content sample having an initial weight of 20.32 g was dried in the standard fashion to 16.93 g. Finally, a density bottle was used to discover that a 15.41 g sample of the dried, powdered, soil displaced 5.749 g of distilled water when all air had been eliminated. Calculate the bulk density (ρ) and water content (w) of the core, and the specific gravity (G_s) of the grains. Deduce the voids ratio (e) and saturation ratio (S_r) of the core, and comment on whether these values could be consistent with the embankment having been compacted at its optimum water content. [30%]

(b) Piezometers located at depths of 1 m and 3 m beneath the crest of the embankment displayed gauge pore pressures of –40 kPa and –20 kPa at time t_1 just before the wet season's first flood. Three days later at time t_2, after continuous flooding at the embankment crest, the readings had changed to 0 kPa and 10 kPa respectively. It is also envisaged that there may be a time t_3 at which the whole embankment comes into pore pressure equilibrium with a flood that submerges the embankment. Sketch three profiles of vertical effective stress on the embankment centreline for times t_1, t_2 and t_3. List any significant assumptions or simplifications. [30%]

(c) Define and sketch isochrones of excess pore pressure appropriate to the three times t_1, t_2 and t_3 so as to estimate the coefficient of consolidation in swelling, and the duration of a flood that could result in 90% of the available swelling on the centreline. You will have to make significant assumptions, which should be recorded. [30%]

(d) Discuss possible explanations for the poor performance of the embankment. [10%]

3 (a) Using the SSA Cam Clay model of soil behaviour distinguish carefully between those cases in which soil tends to contract on yielding, and those in which it tends to dilate. Make a sketch on (τ, σ'), (v, σ') diagrams to illustrate your answer, and show how excess pore pressures in undrained tests replace volume changes in drained tests. [30%]

(b) Derive various relationships between the normalised shear strength ($\tau_{strength}/\sigma'_o$) and the over-consolidation ratio (σ'_c/σ'_o) of a soil pre-consolidated to a maximum historic normal effective stress σ'_c, and allowed to recover to an initial effective stress σ'_o prior to shearing at constant σ. Take each of the following strength criteria for $\tau_{strength}$:

(i) ultimate shear stress ($\tau_{u,ult}$) recorded during an undrained shear test;

(ii) maximum shear stress ($\tau_{u,max}$) recorded during an undrained shear test;

(iii) ultimate shear stress ($\tau_{d,ult}$) recorded during a drained shear test;

(iv) maximum shear stress ($\tau_{d,max}$) recorded during a drained shear test.

Evaluate the normalised shear strength in each case, for over-consolidation ratios of 1 and 10, using data for London Clay where necessary. Do not try to account for residual friction on slickensides, but mention where this phenomenon might occur. [50%]

(c) For both low over-consolidation ratio (e.g. 1) and high over-consolidation ratio (e.g. 10), discuss the significance of the relative values calculated for each of the cases (i) to (iv). Refer to contrasting slope stability problems when making cuttings in soft muds and stiff clays. [20%]

4 (a) A triaxial test apparatus is arranged so that the top platen can either be pushed down or pulled up while the cell pressure, which acts both on the top platen and on the cylindrical membrane retaining the soil, is kept constant. Sketch these alternate total stress paths on a (q, p) diagram, labelling them "compression" and "extension" as appropriate. [20%]

(b) TA-AS Cam Clay recognises that two different critical state stress ratios $(q/p')_{crit}$ need to be respected in triaxial compression and extension tests respectively if, as observed, there is to be a unique value of the Mohr-Coulomb angle of friction in a critical state. Derive expressions for M_{comp} and M_{extn} as functions of ϕ_{crit}. [20%]

(c) Ham River Sand is brought to its minimum achievable voids ratio, and confined in the triaxial apparatus under a cell pressure of 1MPa, with zero initial deviatoric stress. It is to be subjected to drained tests. One sample is to be compressed, and another is to be extended, until they reach their maximum deviatoric stresses. By making use of correlations based on the relative dilatancy index, estimate q_{comp} and q_{extn} in these two cases. [30%]

(d) Estimate the mean effective stress p'_{crit} that would have had to be applied in (c) so as to bring the soil into a critical state, and deduce $q_{comp,crit}$ and $q_{extn,crit}$. [20%]

(e) An engineer has been taught to connect strength data for soils with a given initial voids ratio, such as those calculated in (c) and (d), with straight line strength envelopes on a (q, p') diagram. Sketch this construction, and explain its shortcomings. [10%]

END OF PAPER

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Engineering Tripos Part IIA

3D1 & 3D2
Soil Mechanics Data Book

Data Book 2003/2004

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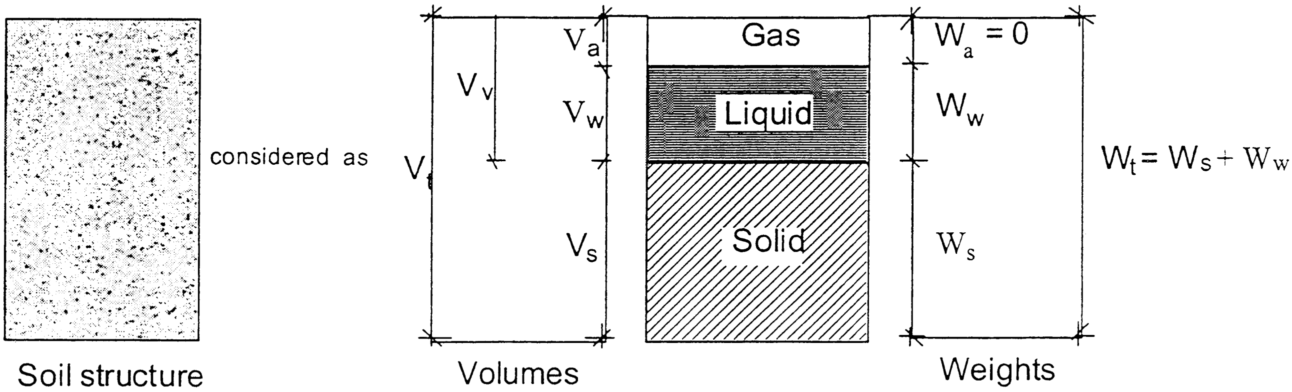
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\frac{e(1 - S_r)}{1+e}\right)$

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Soil classification (BS1377)

Liquid limit	w_L
Plastic Limit	w_P
Plasticity Index	$I_P = w_L - w_P$
Liquidity Index	$I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$

(at the same water content)

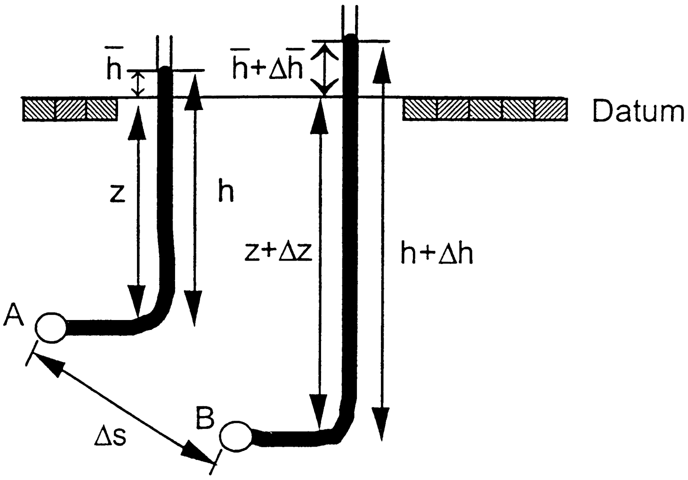
Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc.	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.
C_U	uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} > 1\mu$m	:	$k \cong 0.01$ (D_{10} in mm)^2 m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

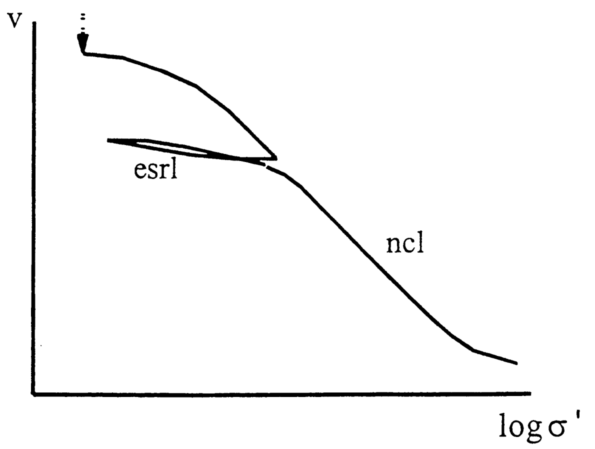
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

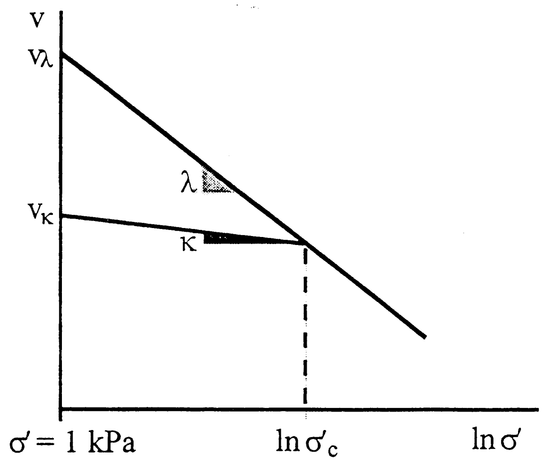
One-Dimensional Compression

✣ Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

✣ Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

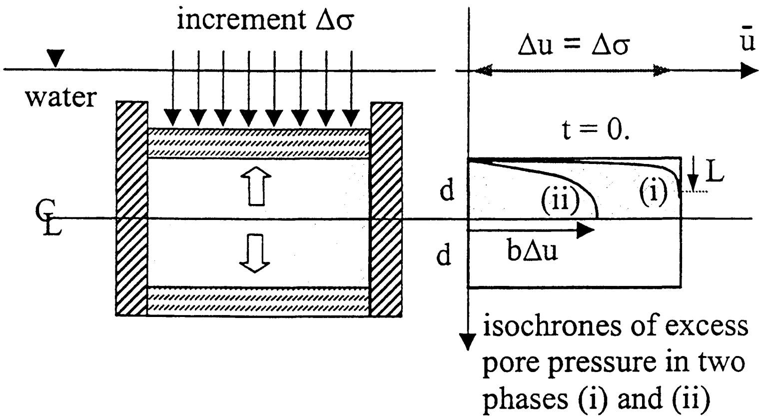
Tangent 1D plastic compression modulus $E_o = v\,\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\,\sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

✣ **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $\quad L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $\quad b = \exp(1/4 - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(1/4 - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

✣ Principle of effective stress (saturated soil)

total stress σ = effective stress σ' + pore water pressure u

✣ Principal components of stress and strain

sign convention	compression positive
total stress	σ_1, σ_2, σ_3
effective stress	σ_1', σ_2', σ_3'
strain	ε_1, ε_2, ε_3

✣ Simple Shear Apparatus (SSA) ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

✣ Biaxial Apparatus - Plane Strain (BA-PS) ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

✣ Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	$= \sigma'_a + u$
total radial stress	σ_r	$= \sigma'_r + u$
total mean normal stress	p	$= (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	$= (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	$= \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	$= q/p'$
axial strain	ε_a	
radial strain	ε_r	
volumetric strain	ε_v	$= \varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	$= \frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	$= \sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	$= p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

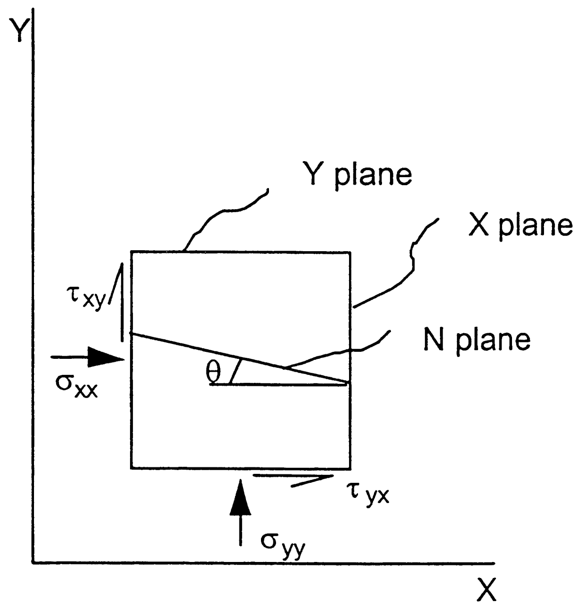
isotropic compression in which p' increases at zero q

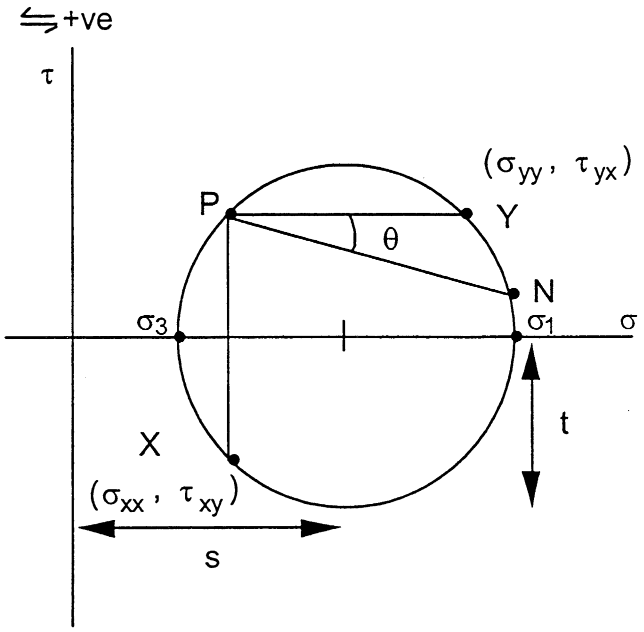
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

✣ Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

✤ Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

✤ General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

✤ General yield surface

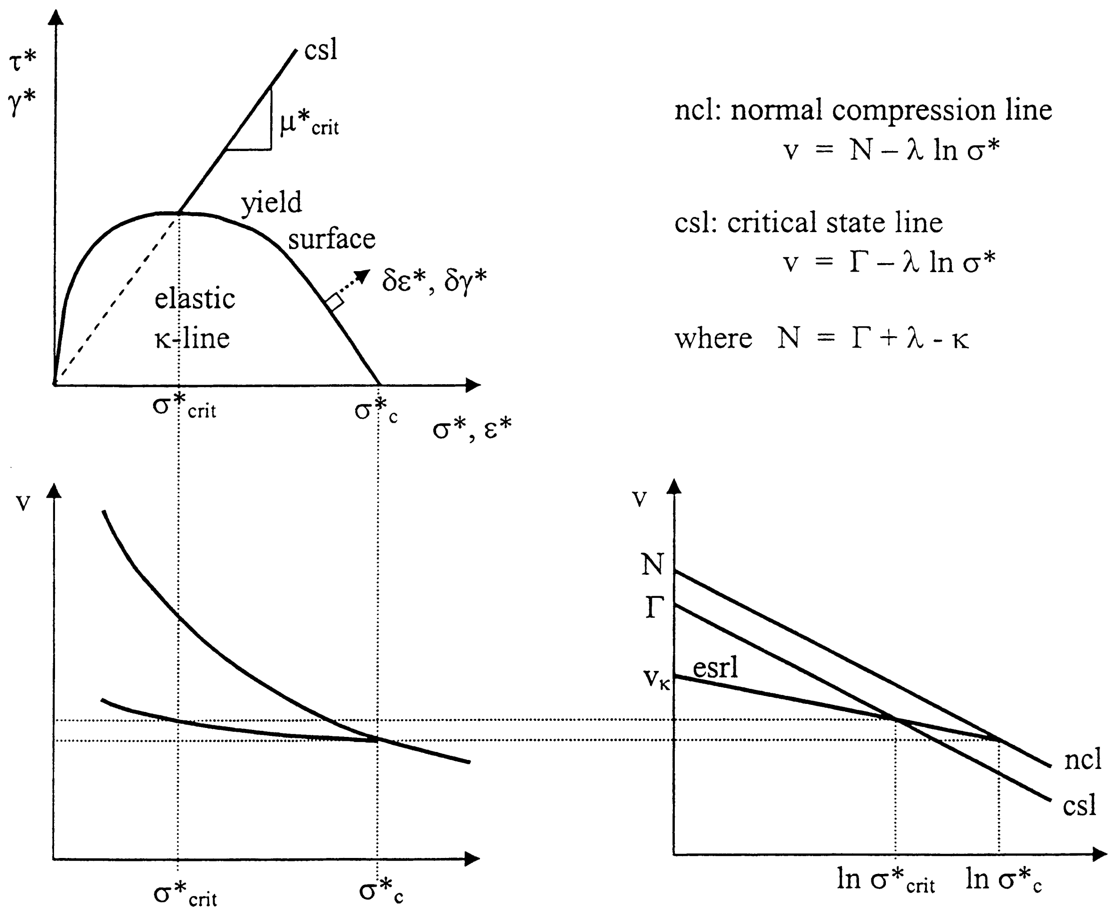
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

✤ Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

✣ The yield surface in (σ^*, τ^*, v) space



ncl: normal compression line

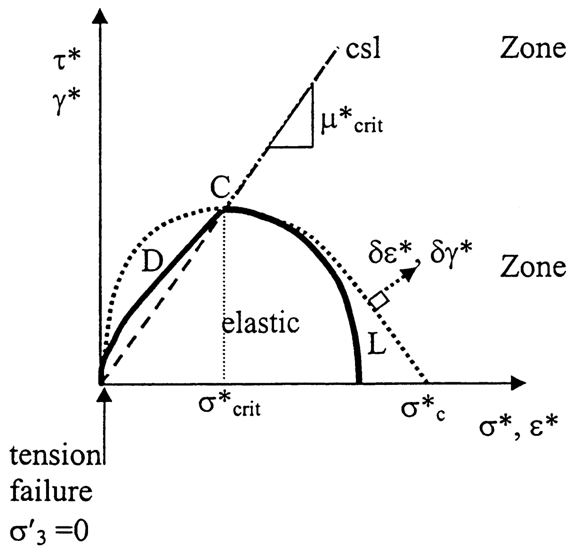
$$v = N - \lambda \ln \sigma^*$$

csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$

✣ Regions of limiting soil behaviour



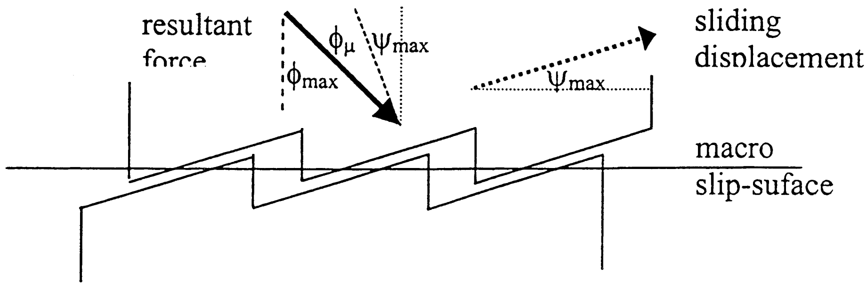
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

✣ Friction and dilatancy: the saw-blade model of direct shear

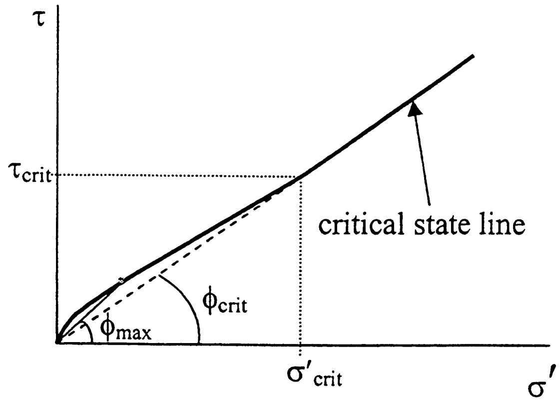


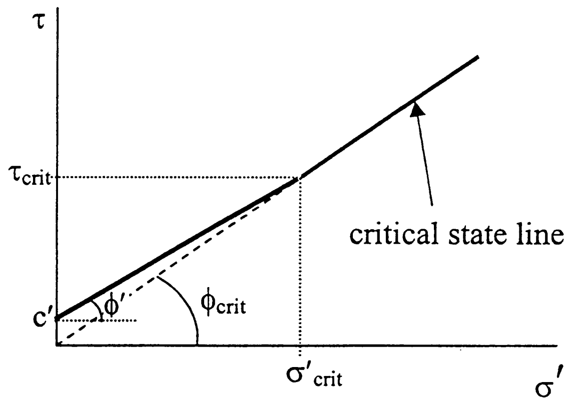
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

✣ Friction and dilatancy: secant and tangent strength parameters





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\,\tau_{crit}$

✚ Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ $\quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p')$ $\quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

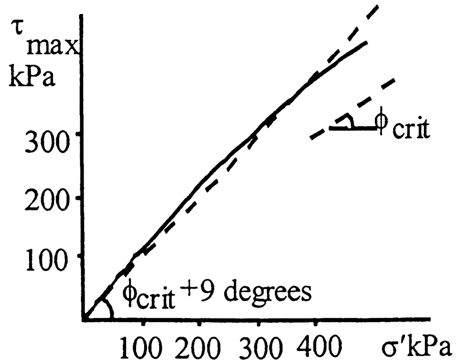
Relative dilatancy index $I_R = I_D I_C - 1$ $\quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

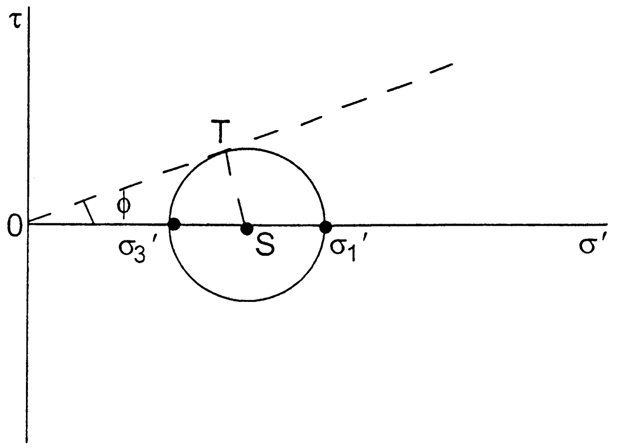
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' < 400$ kPa

✣ Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

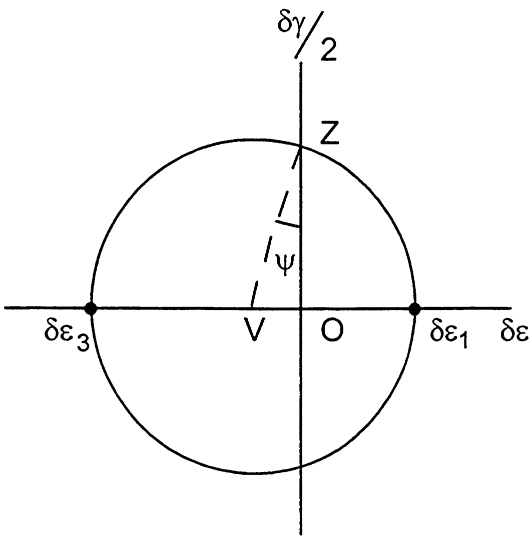
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ'_{crit} after large shear strains

✣ Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$

✣ Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

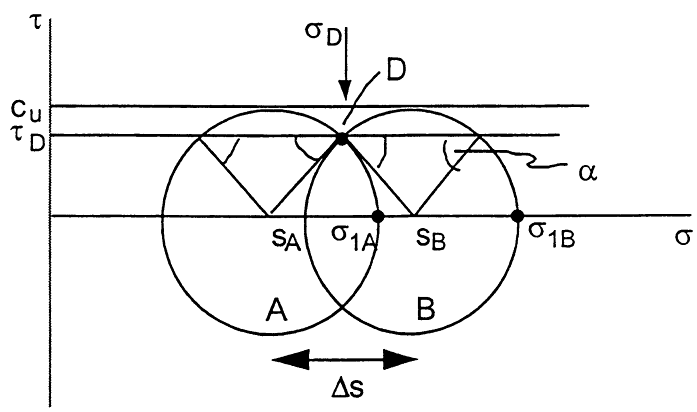
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

✣ Stress conditions across a discontinuity

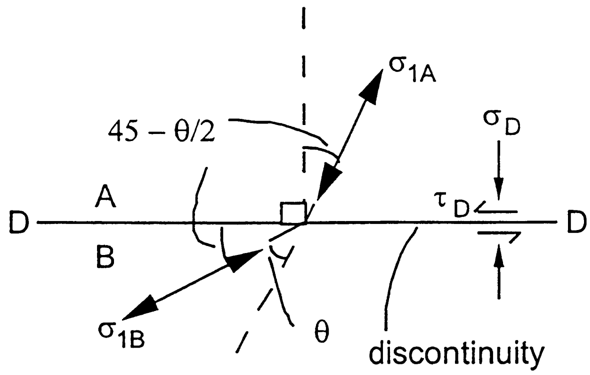


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1(B)} - \sigma_{1(A)} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = c_u$$

$$\tau_D / c_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi'$

✣ Limiting stresses

$$\sin\phi' = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principle effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Earth pressure coefficient K: $\sigma'_h = K\sigma'_v$

Earth pressure at rest for normally consolidated soils $K_0 = 1 - \sin\phi'$

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure:

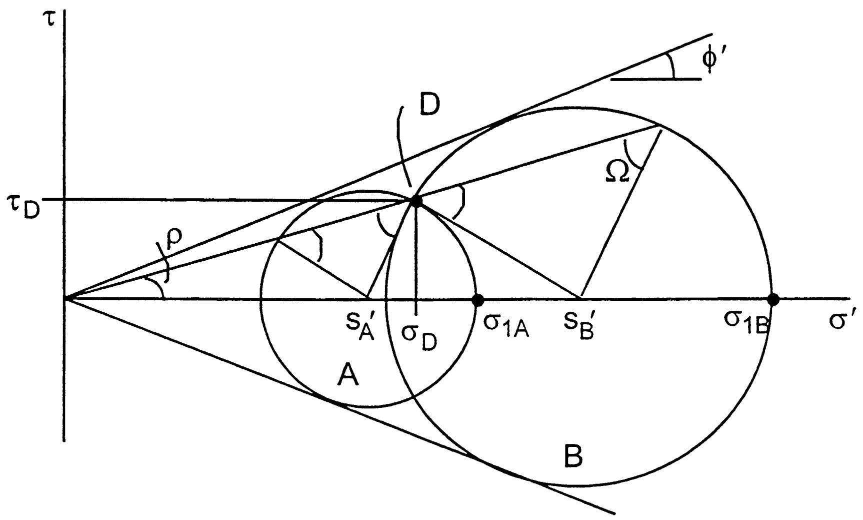
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

✣ Stress conditions across a discontinuity

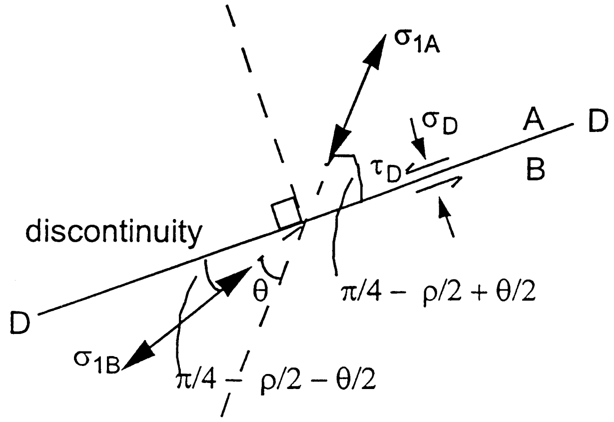


Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B



$\sin\rho = \cos\theta \sin\phi'$

$s'_B/s'_A = \cos(\theta-\rho)/\cos(\theta+\rho)$

In limit with $\delta\theta \rightarrow 0$

$\rho \rightarrow \phi'$,

$ds' = 2s' . \delta\theta \tan\phi'$.

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Formula for shallow foundation design

For clay in undrained conditions (ϕ=0 and q_s calculated based on total stress)

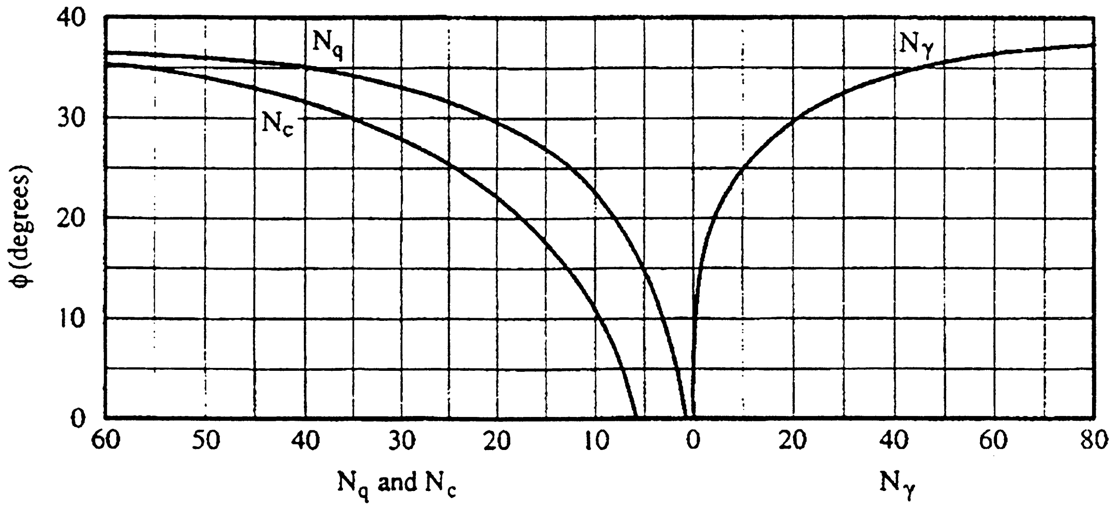
$$q_f = cN_c\zeta_c + q_s\zeta_s$$

For sand and clay in drained conditions (q_s calculated based on effective stress)

$$q_f = 0.5\gamma' BN_\gamma\zeta_\gamma + q_sN_q\zeta_s$$

$$\zeta_c = \zeta_{cd} \times \zeta_{cs} \times \zeta_{ci} \times \zeta_{c\beta} \times \zeta_{c\delta}$$
$$\zeta_r = \zeta_{\gamma d} \times \zeta_{\gamma s} \times \zeta_{\gamma i} \times \zeta_{\gamma\beta} \times \zeta_{\gamma\delta}$$
$$\zeta_s = \zeta_{sd} \times \zeta_{ss} \times \zeta_{si} \times \zeta_{s\beta} \times \zeta_{s\delta}$$

Correction factors		
	- Foundation depth	ζ_{cd} $\zeta_{\gamma d}$ ζ_{sd}
	- Foundation shape	ζ_{cs} $\zeta_{\gamma s}$ ζ_{ss}
	- Inclined loading	ζ_{ci} $\zeta_{\gamma i}$ ζ_{si}
	- Surface slope	$\zeta_{c\beta}$ $\zeta_{\gamma\beta}$ $\zeta_{s\beta}$
	- Base tilt	$\zeta_{c\delta}$ $\zeta_{\gamma\delta}$ $\zeta_{s\delta}$



[REMARKS]

(a) For clay in undrained conditions,

$N_c = 5.14$	for strip footing
$N_c = 5.69$	for circular footing (smooth)
$N_c = 6.05$	for circular footing (rough)
$N_c = 5(1+0.2\ B/L)$	for a rectangular footing of dimensions B x L (L>B)

(b) For sand and clay in drained conditions (use the above chart or the following equations)

$N_q = \tan^2(\pi/4 + \phi/2)e^{(\pi\tan\phi)}$

$N_\gamma = 2(N_q - 1)\tan\phi$

(c) For more complicated geometries, apply the correction factors using Table 1.

Table 1 Correction factors

	Cohesion	Self-weight	Surcharge
1. Foundation shapes	$\zeta_{CS} = 1 + \frac{B'}{L'}\frac{N_q}{N_c}$	$\zeta_{\gamma S} = 1 - 0.4\frac{B'}{L'}$	$\zeta_{qS} = 1 + \frac{B'}{L'} \tan\phi$
2. Inclined loading	$\zeta_{Ci} = (1 - 2i/\pi)^2$	$\zeta_{\gamma i} = (1 - i/\phi)^2$	$\zeta_{qi} = (1 - \frac{2i}{\pi})^2$
3 Foundation depth	$\zeta_{Cd} = 1 + 0.4\xi \ (\phi = 0)$ $= \zeta_{qd} - \frac{1 - \zeta_{qd}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma d} = 1.0$	$\zeta_{qd} = 1 + 2\tan\phi(1 - \sin\phi)^2 \xi$
4. Surface slope	$\zeta_{C\beta} = 1 - [2\beta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\beta} - \frac{1 - \zeta_{q\beta}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\beta} = (1 - \tan\beta)^2$ $Note: N_\gamma = -2\sin\beta (\phi = 0)$	$\zeta_{q\beta} = (1 - \tan\beta)^2$
5. Base tilt	$\zeta_{C\delta} = 1 - [2\delta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\delta} - \frac{1 - \zeta_{q\delta}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\delta} = (1 - \delta\tan\phi)^2$	$\zeta_{q\delta} = (1 - \delta\tan\phi)^2$

V : vertical load, H : horizontal load, B : foundation width, L (>B) : foundation length, e_B : eccentricity parallel to B, e_L : eccentricity parallel to L; B' = B – $2e_B$, L' = L – $2e_L$; $i = \tan^{-1}(H/V)$ where i is in radians; ξ = D/B if D/B < 1; $\xi = \tan^{-1}(D/B)$ if D/B > 1, $\beta < \pi/4$ where β is in radians, $\delta < \pi/4$ where δ is in radians.

ENGINEERING TRIPOS PART IIA 2005
NUMERICAL ANSWERS, MODULE 3D1: SOIL MECHANICS

1. (a) $\Gamma = 2.031$, $\lambda = 0.143$, $\kappa = 0.035$.
   (b) $C_v = 6.2 \times 10^{-8}$ m^2/s.
   (c) (i) $C_v = 25 \times 10^{-8}$ m^2/s

2. (a) $\rho = 2056$kg/m^3, w = 0.2, Gs = 2.68, e = 0.564, Sr = 0.95.
   (b) at z =1m, at t_1, $\sigma_v' = 60$kPa, t_2, $\sigma_v' = 20$kPa, at t_3, $\sigma_v' = 10$kPa
   at z = 3m, at t_1, $\sigma_v' = 80$kPa, t_2, $\sigma_v' = 50$kPa, at t_3, $\sigma_v' = 30$kPa.
   (c) $C_v = 2.4$ m^2/day, duration = 5.5 days.

3. (b) (i) $\tau_{u,ult}/\sigma_o' = \tan\phi_{crit}\,(\sigma_c'/E\sigma_o')^{1-\kappa/\lambda}$
   (ii) $\tau_{u,max}/\sigma_o' = \tan\phi_{crit}\ln(\sigma_c'/\sigma_o')$
   (iii) $\tau_{d,ult}/\sigma_o' = \tan\phi_{crit}$
   (iv) $\tau_{d,max}/\sigma_o' = \tan\phi_{crit}\ln(\sigma_c'/\sigma_o')$

   For OCR = 1, (i) = (ii) = 0.23, (iii) = (iv) = 0.42,
   For OCR = 10, (i) = 2.24, (ii) = 0.98, (iii) = 0.42, (iv) = 0.98.

4 (b) $M_{comp} = 6\sin\phi_{crit}/(3 - \sin\phi_{crit})$, (b) $M_{extn} = 6\sin\phi_{crit}/(3 + \sin\phi_{crit})$
   (c) $q_{comp} = 2.69$MPa, $q_{extn} = -0.76$ MPa
   (d) $q_{comp,crit} = 7.09$MPa, $q_{extn,crit} = -4.95$ MPa

PART IIA 2005

3D1 Soil mechanics

Principal Assessor: **Dr A Al-Tabbaa**