ENGINEERING TRIPOS   PART IIA

Thursday 12 May 2005   2.30 to 4

Module 3D2

GEOTECHNICAL ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (19 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 At a site underlain by soft peaty clay, the water table is 2 m below the ground surface. High quality samples of clay have been taken from a depth of 7 m. Pressuremeter tests and consolidation tests have respectively established that at this depth the earth pressure at rest, $K_0 = 0.8$ and the overconsolidation ratio, $OCR = 1.5$. It has also been found that the unit weight of the clay, $\gamma = 17$ kN m^{-3} and the critical state parameter $M = 0.9$.

(a) Assuming that the coefficient of earth pressure at rest for the clay when it was originally in a normally consolidated state, $K_{0,nc} = 0.65$, calculate the present day and previous maximum effective vertical and horizontal stresses for the clay at a depth of 7 m. [20%]

(b) If in the future the ground water table is permanently lowered to 4 m below the ground surface, what would be the new OCR at a depth of 7 m? [10%]

(c) A large cylindrical oil tank is to be constructed on the site. Two triaxial tests are undertaken on the samples. The present day effective and total in-situ stresses were imposed on the samples before additional shearing was undertaken. Plot the effective and total stress paths in q-p' and q-p space for each test, details of which are as follows.

(i) Test A: an undrained test to simulate rapid filling of the tank. The axial stress was increased and the cell pressure was maintained constant until failure occurred. The undrained shear strength, c_u, was found to be 25 kN m^{-2}. What was the measured pore pressure? [30%]

(ii) Test B: a drained test was undertaken to approximately simulate slow filling of the tank, by increasing the axial stress while keeping the cell pressure constant. The pore pressure in the sample remained constant and equal to the in-situ value. What was the measured drained strength at failure? [30%]

(d) Give brief comments on the significance of Tests A and B in relation to the filling of the oil tank. [10%]

2 A self-boring cylindrical pressuremeter of diameter 80 mm is installed with no disturbance in a stiff clay to a depth of 15 m. The water table is 2 m below ground level. The unit weight of the clay, $\gamma = 20$ kN m^{-3}. At the depth of 15 m the coefficient of earth pressure at rest, $K_0 = 1.5$, the undrained shear strength $c_u = 150$ kN m^{-2} and the elastic shear modulus $G = 50$ MN m^{-2}.

The pressure is steadily increased under undrained conditions and the expansion of the flexible membrane is measured with strain-gauged feeler arms. By considering the pressuremeter expansion in terms of a cylindrical cavity, answer the following questions:

(a) At what pressure does 'lift-off' occur? [15%]

(b) At what pressure does yield of the clay first occur? What is the corresponding radial displacement of the membrane? [30%]

(c) What would be the theoretical maximum pressure that could be achieved, corresponding to very large expansion of the membrane? [15%]

(d) After yield has occurred, there is a plastic zone surrounded by an elastic zone. Within the elastic zone, at any radius r, the radial and circumferential stresses, σ_r and σ_θ respectively, are given by the following expressions:

$$\sigma_r = \sigma_{h0} + G\,\delta A/\pi r^2$$
$$\sigma_\theta = \sigma_{h0} - G\,\delta A/\pi r^2$$

where σ_{h0} is the insitu total horizontal stress in the ground, G is the elastic shear modulus and δA is the increase in cross-sectional area of the cavity. Show that the radius of the plastic zone, r_p, is given by the following expression:

$$r_p/r_c = [(G/c_u)(\delta A/A)]^{0.5}$$

where r_c is the radius of the cavity, A is the cross-sectional area of the cavity and the other symbols are as defined above. [20%]

(e) The pressure is increased to a value 50% higher than the value at which yield in the clay first occurs (as calculated in part (b) above). What is the radius of the corresponding plastic zone? [20%]

3 The stability of a sandy slope is a concern. The angle of the slope is 20°. Since the length of the slope is long, it is assumed that it is infinitely long as shown in Fig. 1. The dry unit weight of the sand is γ_d = 16 kN m^{-3}, whereas the saturated unit weight is γ_s = 18 kN m^{-3}.

A potential failure plane was found at 3 m depth as shown in Fig. 1. The friction angle of the sand at this location is 40 degrees. For the stability analysis of the infinite slope, forces acting on a soil block as shown in Fig. 1 are considered.

(a) When the slope is completely dry, calculate the normal stress σ and the shear stress τ acting at the base of the block. Is the slope likely to fail? [30%]

(b) Seepage is occurring parallel to the slope and the water table is at the slope surface as shown in Fig. 2. A standpipe piezometer is installed at Point A, which is located at a potential failure plane. Draw the equipotential lines and show that the pore pressure at Point A is 26.0 kPa. [15%]

(c) Using the pore pressure value obtained in part (b), calculate the effective normal and shear stresses acting at the base of the block. Is the slope likely to fail? [15%]

(d) Seepage is now occurring horizontally and the flow lines are shown in Fig. 3 and water is seeping out of the slope. Draw the equipotential lines and show that the pore pressure at Point A is 29.4 kPa. [15%]

(e) Using the pore pressure value obtained in part (d), calculate the effective normal and shear stresses acting at the base of the block. Is the slope likely to fail? [15%]

(f) If the slope is completely submerged into water and there is no seepage force, is the slope likely to fail? [10%]

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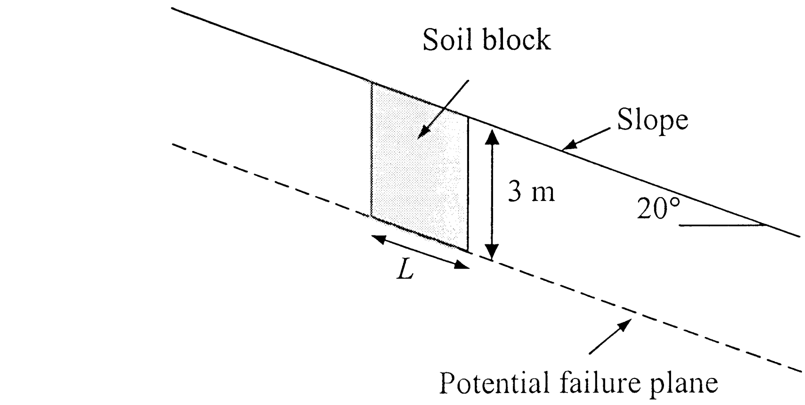

Fig. 1

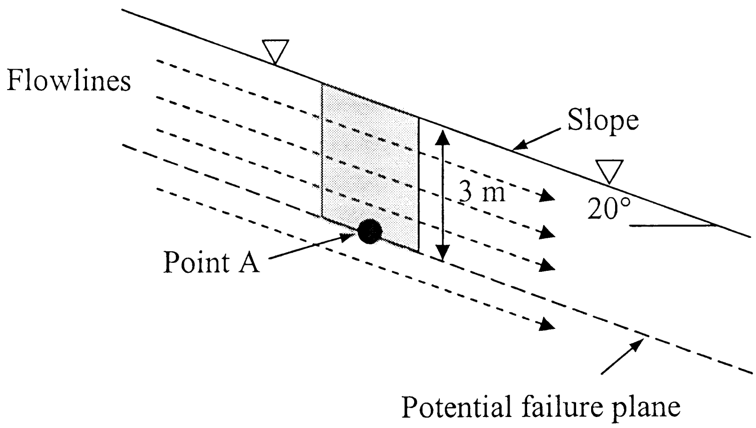

Fig. 2

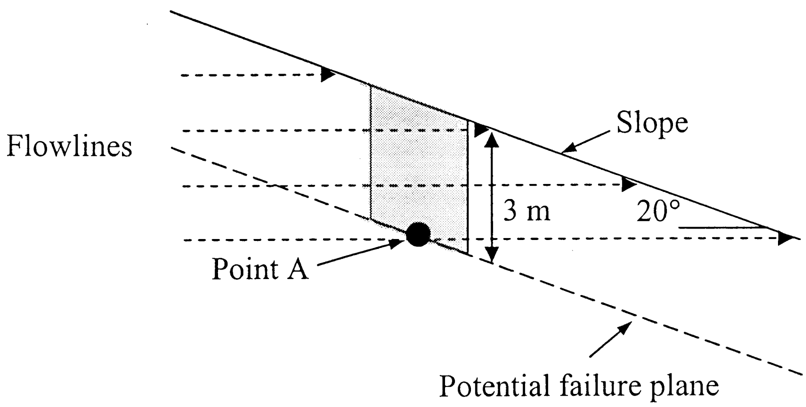

Fig. 3

(TURN OVER

4 A smooth, weightless spread foundation is placed on clay for a new building as shown in Fig. 4. The width of the foundation is 20 m and it is placed at 5 m depth. A vertical load Q is applied at the centre of the foundation. The side walls of the excavation are impermeable. Plane strain conditions are assumed. The water table is located at the surface. The saturated unit weight of the clay is 19 kN m^{-3}.

(a) An undisturbed clay sample was taken from the site and an undrained triaxial compression test was performed. The sample was initially consolidated isotropically to 300 kN m^{-2} with a back pressure of 200 kN m^{-2}. The sample failed when the axial stress was 400 kN m^{-2}. The pore pressure at failure was measured to be 230 kN m^{-2}. Draw total and effective stress Mohr circles at failure and determine the undrained shear strength. Show that the effective friction angle is 24.6°. [15%]

(b) An upper bound failure mechanism with four rigid soil blocks as shown in Fig. 5 is assumed to analyse the short-term stability of the foundation.

(i) The foundation is moving downwards at velocity v. Draw the velocity diagram of the sliding blocks. [10%]

(ii) Compute the rate of energy dissipation along the boundaries of the sliding blocks. [25%]

(iii) Determine the maximum value of Q that can be applied to the foundation. [15%]

(c) The long-term stability of the foundation can be evaluated using the bearing capacity formula available in the Data Book. For simplicity, ignore the shear resistance of the soil above the foundation level as shown in Fig. 6.

(i) Calculate the effective surcharge load q'_s. [15%]

(ii) Determine the maximum value of Q that can be applied to the foundation. [20%]

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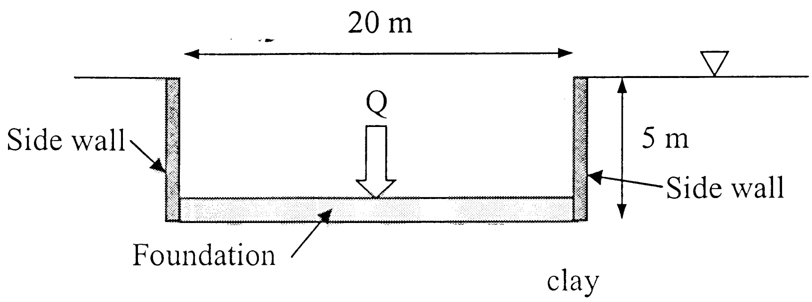

Fig. 4

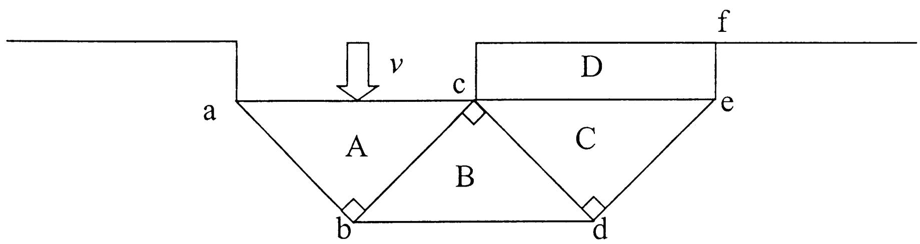

Fig. 5

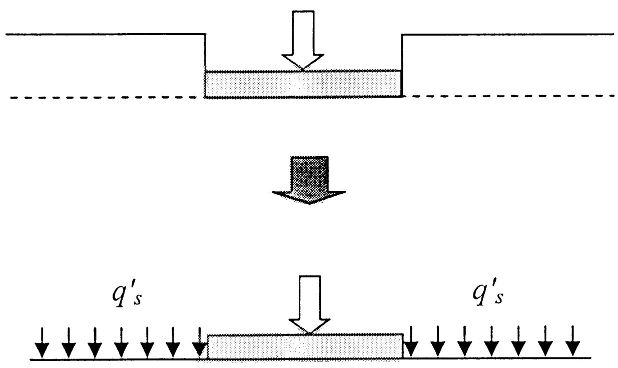

Fig. 6

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Soil Mechanics Data Book

Data Book 2003/2004

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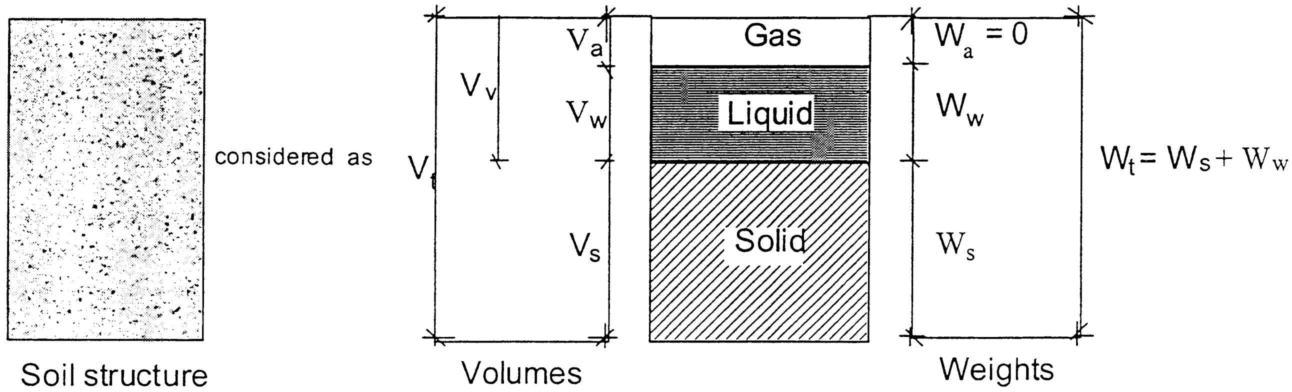
General definitions



Specific gravity of solid	G_s	
Voids ratio	e	$= V_v / V_s$
Specific volume	v	$= V_t / V_s = 1 + e$
Porosity	n	$= V_v / V_t = e/(1 + e)$
Water content	w	$= (W_w / W_s)$
Degree of saturation	S_r	$= V_w / V_v = (w\, G_s/e)$
Unit weight of water	γ_w	$= 9.81\ \text{kN/m}^3$
Unit weight of soil	γ	$= W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	γ'	$= \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	γ_d	$= W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	A	$= V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

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Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

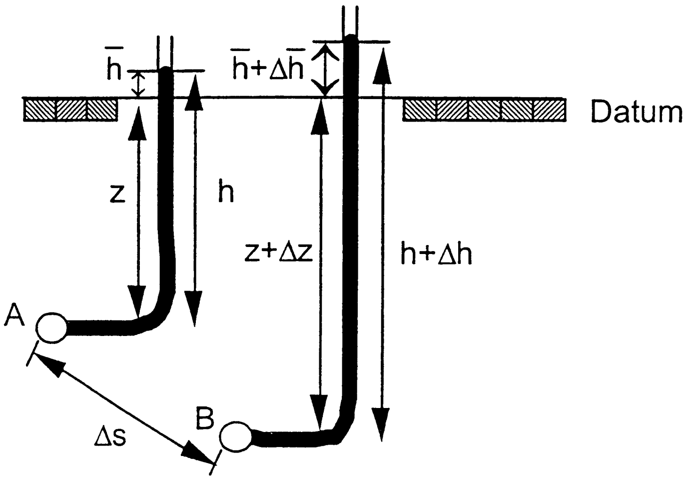
D — equivalent diameter of soil particle

D_{10}, D_{60} etc. — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U — uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
$10 \text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01 (D_{10} \text{ in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

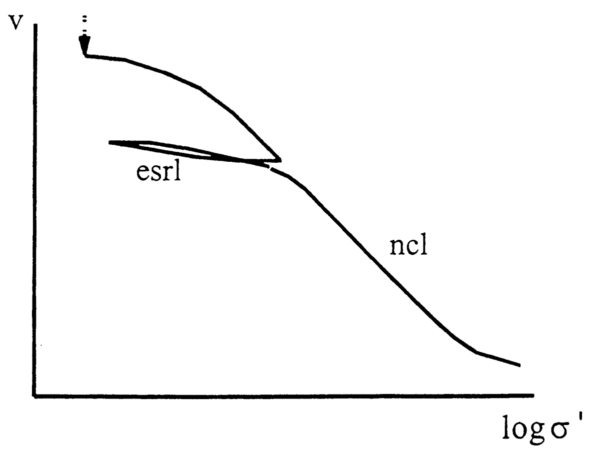
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

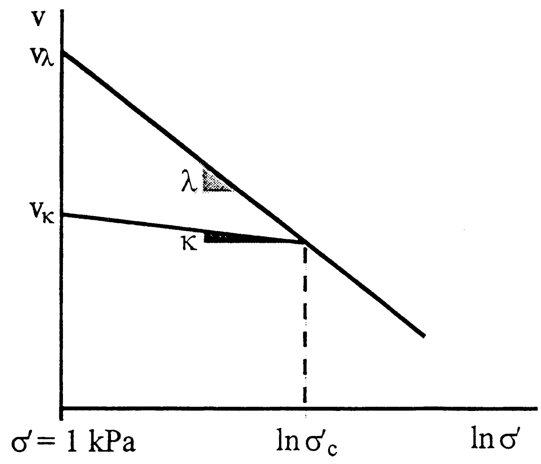
One-Dimensional Compression

✤ Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

✤ Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

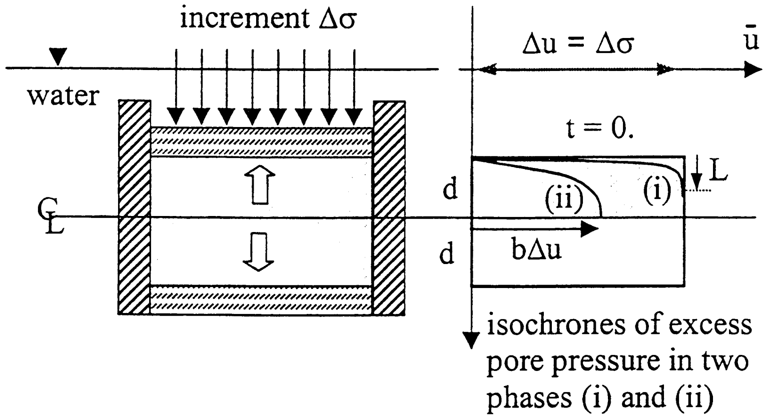
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

✣ Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

✣ Principle of effective stress (saturated soil)

total stress σ = effective stress σ' + pore water pressure u

✣ Principal components of stress and strain

sign convention	compression positive
total stress	$\sigma_1,\ \sigma_2, \sigma_3$
effective stress	$\sigma_1',\ \sigma_2',\ \sigma_3'$
strain	$\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3$

✣ Simple Shear Apparatus (SSA)

($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\quad \delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

✣ Biaxial Apparatus - Plane Strain (BA-PS)

($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

✣ Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

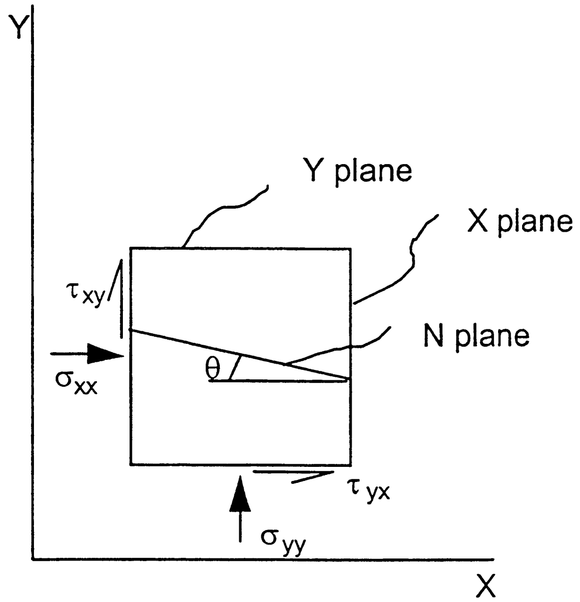
isotropic compression in which p' increases at zero q

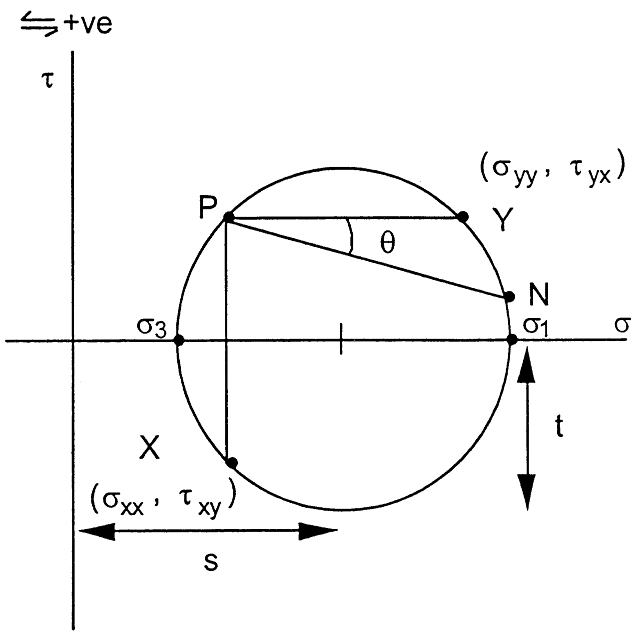
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

✣ Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

✤ Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

✤ General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

✤ General yield surface

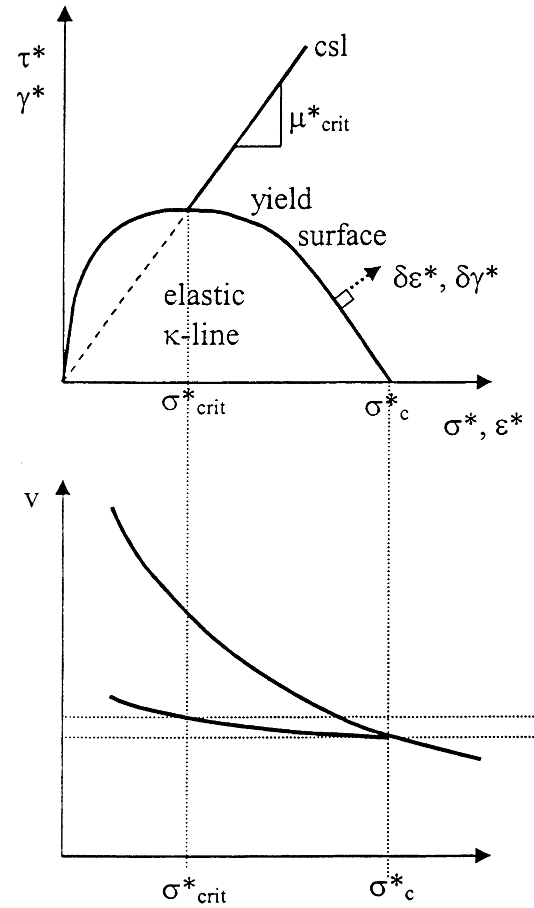
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

✤ Parameter values which fit soil data

	London Clay	**Weald Clay**	**Kaolin**	**Dog's Bay Sand**	**Ham River Sand**
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	-------------	-------------
w_P	0.26	0.18	0.42	-------------	-------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

✤ The yield surface in (σ^*, τ^*, v) space



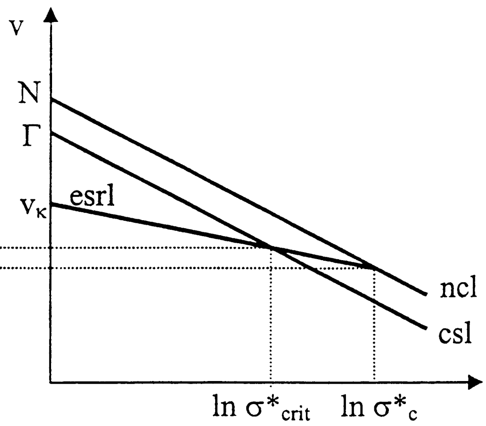
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

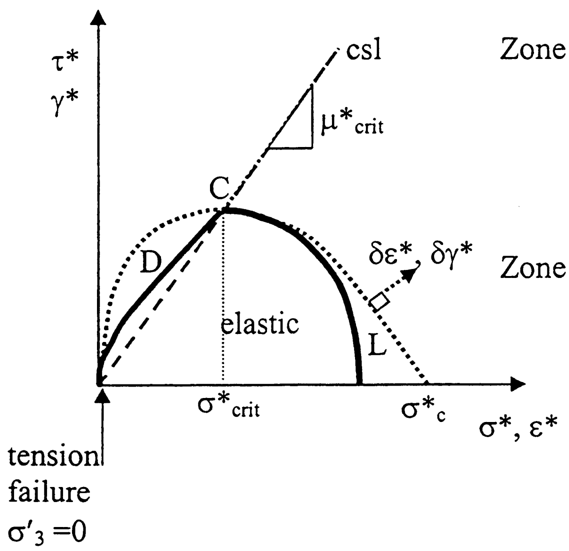
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



✤ Regions of limiting soil behaviour



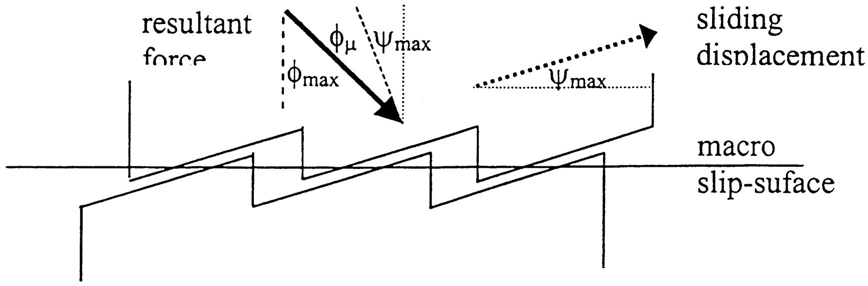
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

✣ Friction and dilatancy: the saw-blade model of direct shear

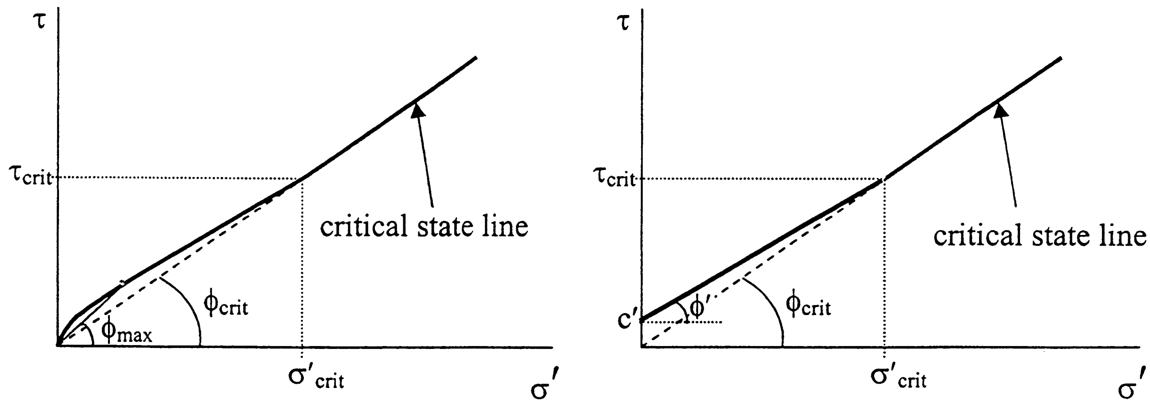


Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

✣ Friction and dilatancy: secant and tangent strength parameters



Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

✜ Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}\quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p')\quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

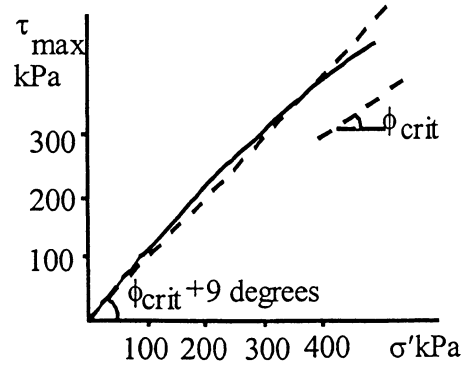
Relative dilatancy index $\quad I_R = I_D I_C - 1\quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

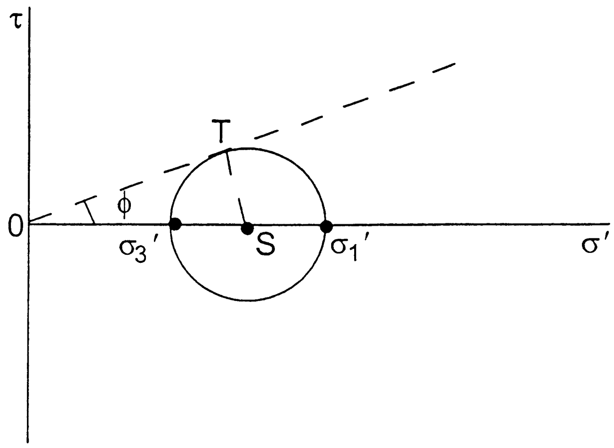
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9° \quad$ for $I_D = 1, \sigma' < 400$ kPa

✣ Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin \phi = TS/OS = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

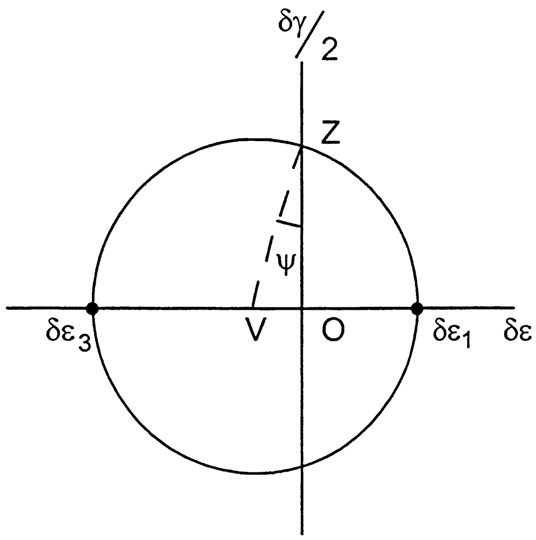
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ'_{crit} after large shear strains

✣ Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin \psi = VO/VZ = -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2} = -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$

✣ Limiting stresses

Tresca $$|\sigma_1 - \sigma_3| = q_u = 2c_u$$

von Mises $$(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

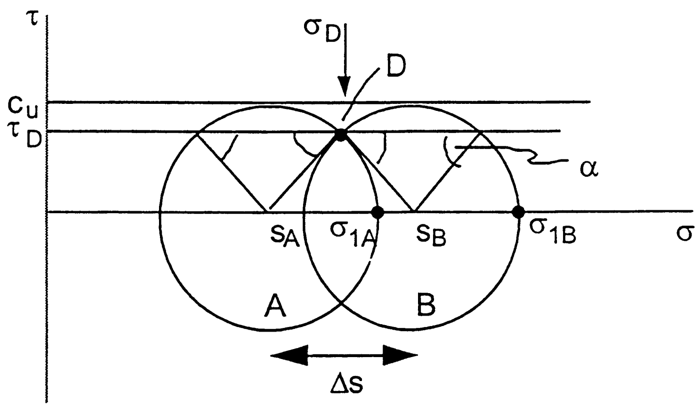
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

✣ Stress conditions across a discontinuity

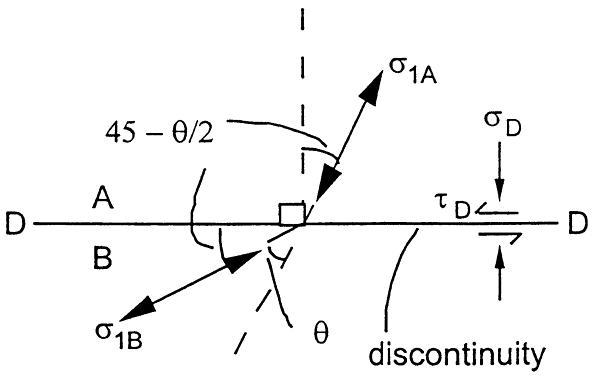


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1(B)} - \sigma_{1(A)} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = c_u$$

$$\tau_D / c_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi'$

✤ Limiting stresses

$$\sin\phi' = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principle effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Earth pressure coefficient K: $\sigma'_h = K\sigma'_v$

Earth pressure at rest for normally consolidated soils $K_0 = 1 - \sin\phi'$

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure:

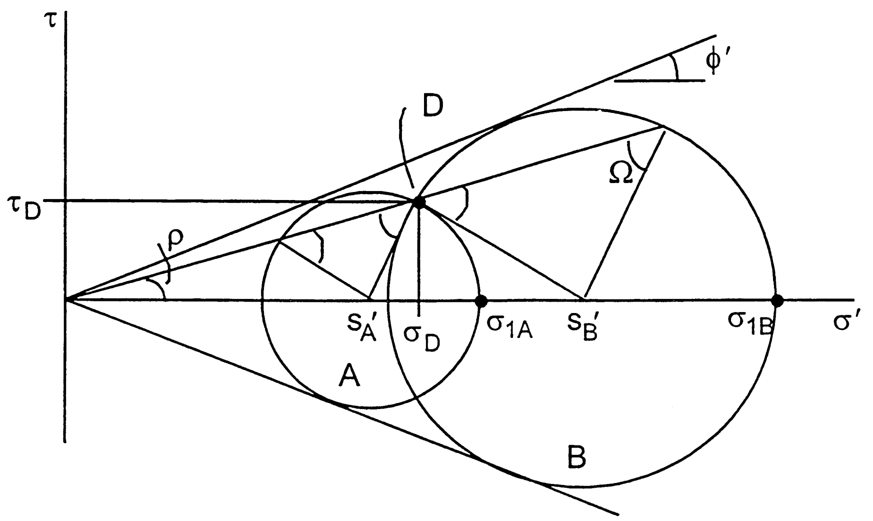
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

✤ Stress conditions across a discontinuity

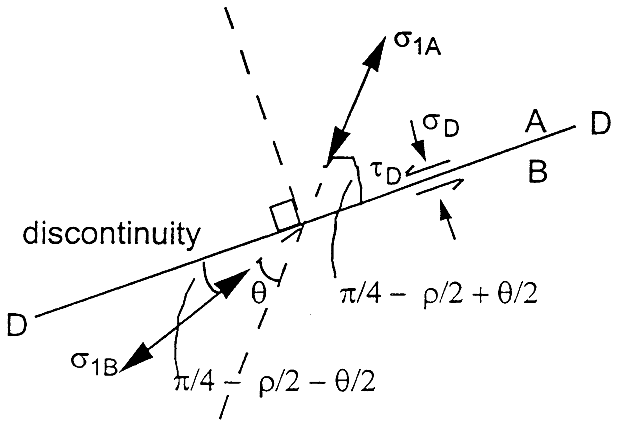


Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B



$\sin\rho = \cos\theta \sin\phi'$

$s'_B/s'_A = \cos(\theta-\rho)/\cos(\theta+\rho)$

In limit with $\delta\theta \rightarrow 0$

$\rho \rightarrow \phi'$,

$ds' = 2s' . \delta\theta \tan\phi'$.

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \frac{\delta A}{2\pi r}$

small shear strain $\gamma = \frac{2\rho}{r}$

Radial equilibrium: $r\frac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\frac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$

Formula for shallow foundation design

For clay in undrained conditions (ϕ=0 and q_s calculated based on total stress)

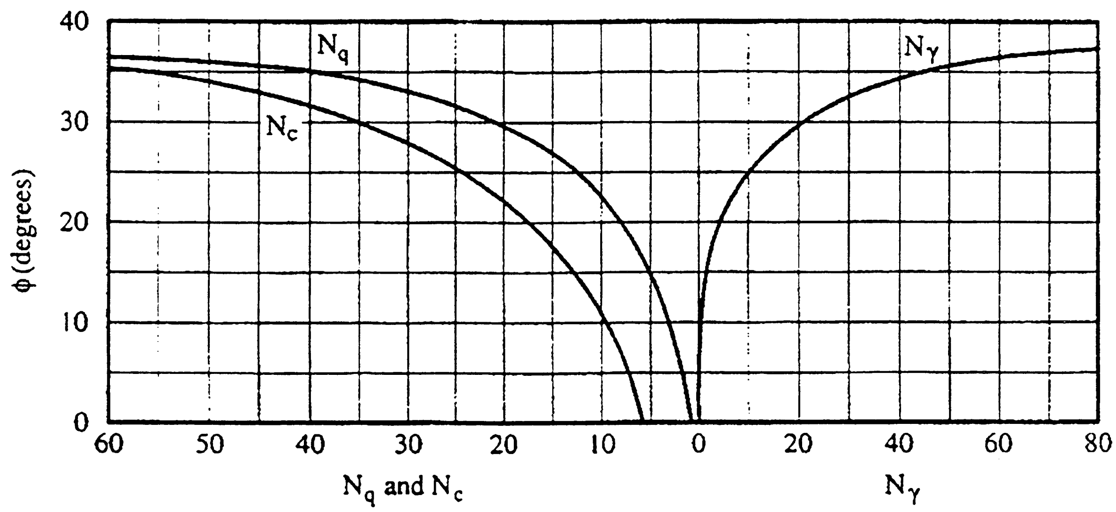
$$q_f = cN_c\zeta_c + q_s\zeta_s$$

For sand and clay in drained conditions (q_s calculated based on effective stress)

$$q_f = 0.5\gamma' B N_\gamma \zeta_\gamma + q_s N_q \zeta_s$$

$$\zeta_c = \zeta_{cd} \times \zeta_{cs} \times \zeta_{ci} \times \zeta_{c\beta} \times \zeta_{c\delta}$$
$$\zeta_r = \zeta_{\gamma d} \times \zeta_{\gamma s} \times \zeta_{\gamma i} \times \zeta_{\gamma\beta} \times \zeta_{\gamma\delta}$$
$$\zeta_s = \zeta_{sd} \times \zeta_{ss} \times \zeta_{si} \times \zeta_{s\beta} \times \zeta_{s\delta}$$

Correction factors		
	- Foundation depth	ζ_{cd} $\zeta_{\gamma d}$ ζ_{sd}
	- Foundation shape	ζ_{cs} $\zeta_{\gamma s}$ ζ_{ss}
	- Inclined loading	ζ_{ci} $\zeta_{\gamma i}$ ζ_{si}
	- Surface slope	$\zeta_{c\beta}$ $\zeta_{\gamma\beta}$ $\zeta_{s\beta}$
	- Base tilt	$\zeta_{c\delta}$ $\zeta_{\gamma\delta}$ $\zeta_{s\delta}$



[REMARKS]

(a) For clay in undrained conditions,

$N_c = 5.14$	for strip footing
$N_c = 5.69$	for circular footing (smooth)
$N_c = 6.05$	for circular footing (rough)
$N_c = 5(1+0.2\ B/L)$	for a rectangular footing of dimensions B x L (L>B)

(b) For sand and clay in drained conditions (use the above chart or the following equations)

$N_q = \tan^2(\pi/4 + \phi/2)e^{(\pi\tan\phi)}$

$N_\gamma = 2(N_q - 1)\tan\phi$

(c) For more complicated geometries, apply the correction factors using Table 1.

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Table 1 Correction factors

	Cohesion	Self-weight	Surcharge
1. Foundation shapes	$\zeta_{CS} = 1 + \frac{B'}{L'}\frac{N_q}{N_c}$	$\zeta_{\gamma S} = 1 - 0.4\frac{B'}{L'}$	$\zeta_{qS} = 1 + \frac{B'}{L'} \tan\phi$
2. Inclined loading	$\zeta_{Ci} = (1 - 2i/\pi)^2$	$\zeta_{\gamma i} = (1 - i/\phi)^2$	$\zeta_{qi} = (1 - \frac{2i}{\pi})^2$
3 Foundation depth	$\zeta_{Cd} = 1 + 0.4\xi \ (\phi = 0)$ $= \zeta_{qd} - \frac{1-\zeta_{qd}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma d} = 1.0$	$\zeta_{qd} = 1 + 2\tan\phi(1 - \sin\phi)^2 \xi$
4. Surface slope	$\zeta_{C\beta} = 1 - [2\beta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\beta} - \frac{1-\zeta_{q\beta}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\beta} = (1 - \tan\beta)^2$ $Note: N_\gamma = -2\sin\beta (\phi = 0)$	$\zeta_{q\beta} = (1 - \tan\beta)^2$
5. Base tilt	$\zeta_{C\delta} = 1 - [2\delta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\delta} - \frac{1-\zeta_{q\delta}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\delta} = (1 - \delta\tan\phi)^2$	$\zeta_{q\delta} = (1 - \delta\tan\phi)^2$

V : vertical load, H : horizontal load, B : foundation width, L (>B) : foundation length, e_B : eccentricity parallel to B, e_L : eccentricity parallel to L; B' = B – $2e_B$, L' = L – $2e_L$; i = $\tan^{-1}$(H/V) where i is in radians; ξ = D/B if D/B < 1; $\xi = \tan^{-1}$(D/B) if D/B > 1, $\beta < \pi/4$ where β is in radians, $\delta < \pi/4$ where δ is in radians.