UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Wednesday 11 May 2005 9 to 10.30

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (10 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) An engineer is interested in the maximum span over which a simply-supported beam bending about its major axis can carry its own weight plus a uniform live working load of 10 kNm^{-1} across the entire span. The extra deflection at midspan due to the working load is not to exceed 1/250th of the span, and the beam is to carry without collapse an ultimate load with factors γ_f of 1.4 on dead load and 1.6 on live load. Take γ_m = 1.0 throughout, and neglect any effects due to shear force.

Using material property information from the Ashby maps, estimate this maximum span for

(i) a 406 × 140 × 46 UB in ductile mild steel, and

(ii) a 400 mm deep by 150 mm wide rectangular section in softwood (pine) stressed along the grain. [40%]

(b) Figure 1 shows a schematic view of one bay of an extensive floor structure on many columns. The floor is to be designed at ULS for a uniformly distributed load, applied everywhere, of 18 kNm^{-2}, which includes an allowance for self-weight and appropriate load factors.

Assume for design purposes a load path with the slab spanning one-way simply-supported on to the secondary beams; the secondary beams fixed-ended at the primary beams; and the primary beams simply-supported at their connection to the columns.

(i) For what maximum shear force should the primary, and secondary beams be designed? [10%]

(ii) For what maximum bending moment should the primary, and secondary, beams be designed? [15%]

(iii) Briefly discuss what assumptions are being made in presuming that this load path can be used for design purposes. In what materials and circumstances are these assumptions likely to be valid? [15%]

(iv) Which of the shear forces in b(i) would be altered, and to what values, for another load path in which any load applied to a slab nearer to a primary beam than a secondary beam was carried by the slab direct to the primary beam? [20%]

(cont.

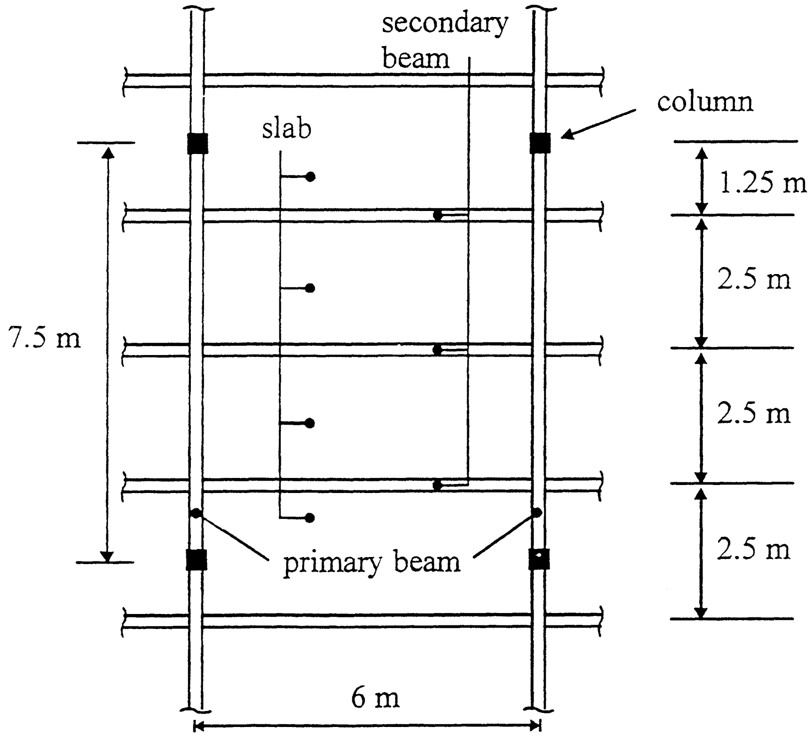
secondary beam
column
slab
1.25 m
2.5 m
7.5 m
2.5 m
2.5 m
primary beam
6 m

Fig. 1

2 (a) Using information from the Structures Data book, evaluate the shape-efficiency factor ϕ_f for strength in bending about the major axis, using linear-elastic theory, for:

(i) a 150 × 100 × 10 Rectangular Hollow Steel Section, and

(ii) a 152 × 89 × 16 UB.

What features give rise to the difference in efficiency, although both sections have similar depth? [15%]

(b) Comment briefly upon the relative advantages, efficiency and practicality of steel members with Circular Hollow Section, Rectangular Hollow Section and Universal Beam section, for use as nominally simply-supported beams with spans in the order of 5 to 10 m in the structural framework of ordinary buildings.

Discuss briefly whether the relative merits of the different shapes would be different in the case of a bridge beam with depth in the order of 2 m, to be built of stiffened steel plate welded together. [20%]

(c) The fully plastic moment M_p of a 305 × 165 × 54 UB in S355 steel with a partial safety factor γ_m of 1.1 bending about its major axis is 273 kNm.

(i) Such a beam is to be simply supported over a 5 m span with rotation about the beam axis and lateral deflection prevented only at the ends, leaving the flanges free to rotate in their own planes. Using the formula on the attached data sheet, and section properties from the Structures Data book, find the elastic critical value of an applied bending moment uniform across the span. Describe the associated buckling mode. [15%]

(ii) The actual bending moment diagram is symmetrical with maximum at midspan, and the equivalent uniform moment for lateral-torsional buckling is 80% of the maximum moment. Find the reduction factor to be applied to the fully plastic moment at midspan to allow for interaction with lateral-torsional buckling. [20%]

(cont.

(iii) Discuss briefly what steps might be taken to ensure that the fully plastic moment can be attained, and whether to use instead, if it were available, a rectangular box section with the same overall dimensions and thicknesses apart from having two webs each 4 mm thick (thereby increasing J by a factor of 150 and I_{yy} by a factor of 1.5 while leaving M_p unchanged). [30%]

3 (a) Briefly explain the difference between:

(i) a braced and an unbraced column;

(ii) a stocky and a slender column. [20%]

(b) A stocky reinforced concrete column with a square cross-section 300 mm × 300 mm contains equal areas of top and bottom steel with a yield stress of 460 MPa. The depth to the centroid of the top steel can be taken as 45 mm and the effective depth to the tension steel, 255 mm. The concrete has a compressive cube strength of 25 MPa. The partial material safety factors for concrete and steel are 1.5 and 1.15 respectively and ε_{cu} = 0.0035 and ε_y = 0.002. The Young's modulus of elasticity of the steel is 210 GPa.

(i) Find the area of reinforcing steel required to sustain a pure axial load that can range from 2000 kN in compression to 1000 kN in tension. [15%]

You are now advised that a total of 4 no. 32 mm bars will be used to reinforce the section. Calculations suggest that the column will fail under a pure axial load of 2200 kN. In addition, the applied axial load and moment corresponding to the yielding of the bottom reinforcement in tension and the simultaneous crushing of the top face of the concrete in compression are found to be 486 kN and 169 kNm respectively.

(ii) Find the applied axial load and moment corresponding to a strain in the bottom reinforcement of 0.0083 and simultaneous crushing of the top face of the concrete in compression. Comment on your result. [45%]

(iii) Using the information supplied and your answer to part (ii), sketch a column interaction diagram with a vertical axis of N/bh and a horizontal axis of M/bh^2. Describe qualitatively how the interaction diagram would change in size and shape if the column section dimensions were increased to 400 mm × 400 mm (and the concrete cover increased proportionally) whilst the concrete cube strength was increased to 30 MPa with the reinforcing steel remaining the same. [20%]

4 (a) Briefly compare and contrast the compressive and tensile behaviour of timber, advanced composites and toughened glass in structural design. [20%]

(b) A single lap timber joint is required to transfer a shear force of 10 kN which includes an allowance for self-weight and appropriate load factors. The joint will be used in a structure in service class 2 and subjected to permanent loading. The partial material safety factors for steel and timber are 1.15 and 1.3 respectively. Any holes required for the connection are pre-drilled. Relevant timber properties can be found on the 3D3 Timber Datasheet.

(i) Two 75×220 mm C24 softwood timbers are connected to form a tension joint as shown in Fig. 3(a). Find the number of 8 mm diameter grade 4.6 bolts ($f_{u,k}$ = 400 MPa) required to transfer the shear load. *Hint*: either mode *c* or mode *f* will control. [35%]

(ii) A second joint where a section of 75×220 mm C24 softwood timber is connected to a section of 75×220 mm C18 softwood timber at an angle of 90° as shown in Fig. 3(b) is considered. Calculate whether failure mode *a* or mode *b* will be more critical when the bolts described in part b(i) above are used to connect the timbers. If instead 6 mm diameter 125 mm long nails are used as shown in Fig. 3(c), will mode *a* or mode *b* be more critical? [20%]

(c) Briefly explain why the design process for a bolted timber joint differs from that of the design of a bolted structural steel joint. [25%]

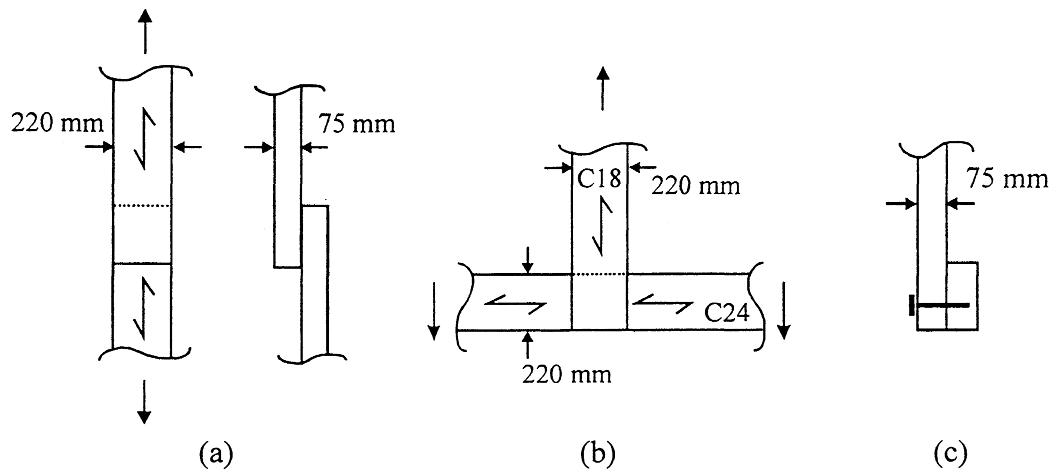

Fig. 3

END OF PAPER

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Module 3D3 Selection of material and shape – design for bending

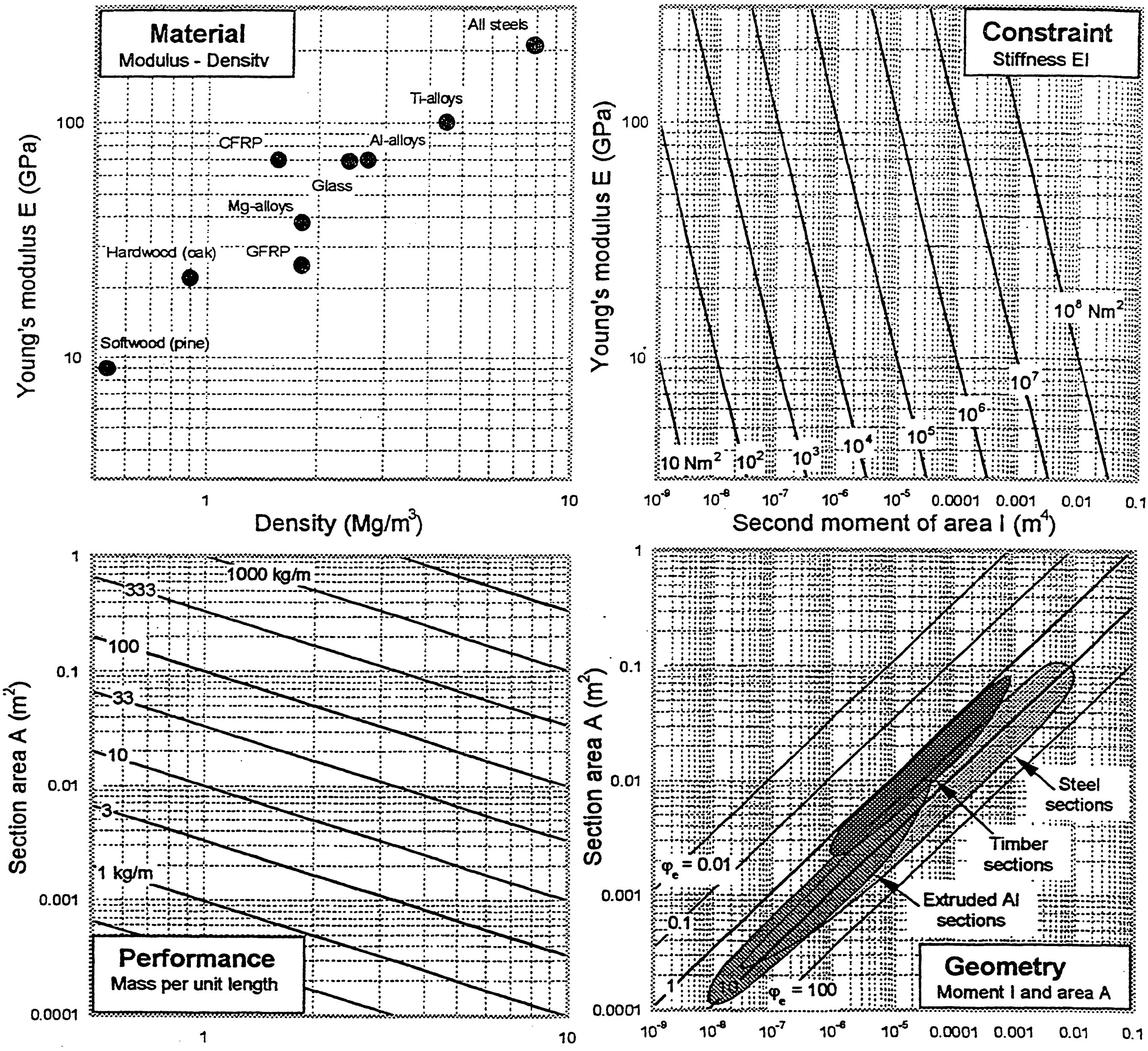


Figure 1. The chart-assembly for exploring structural sections for stiffness limited design. Each chart shares its axes with its neighbours.

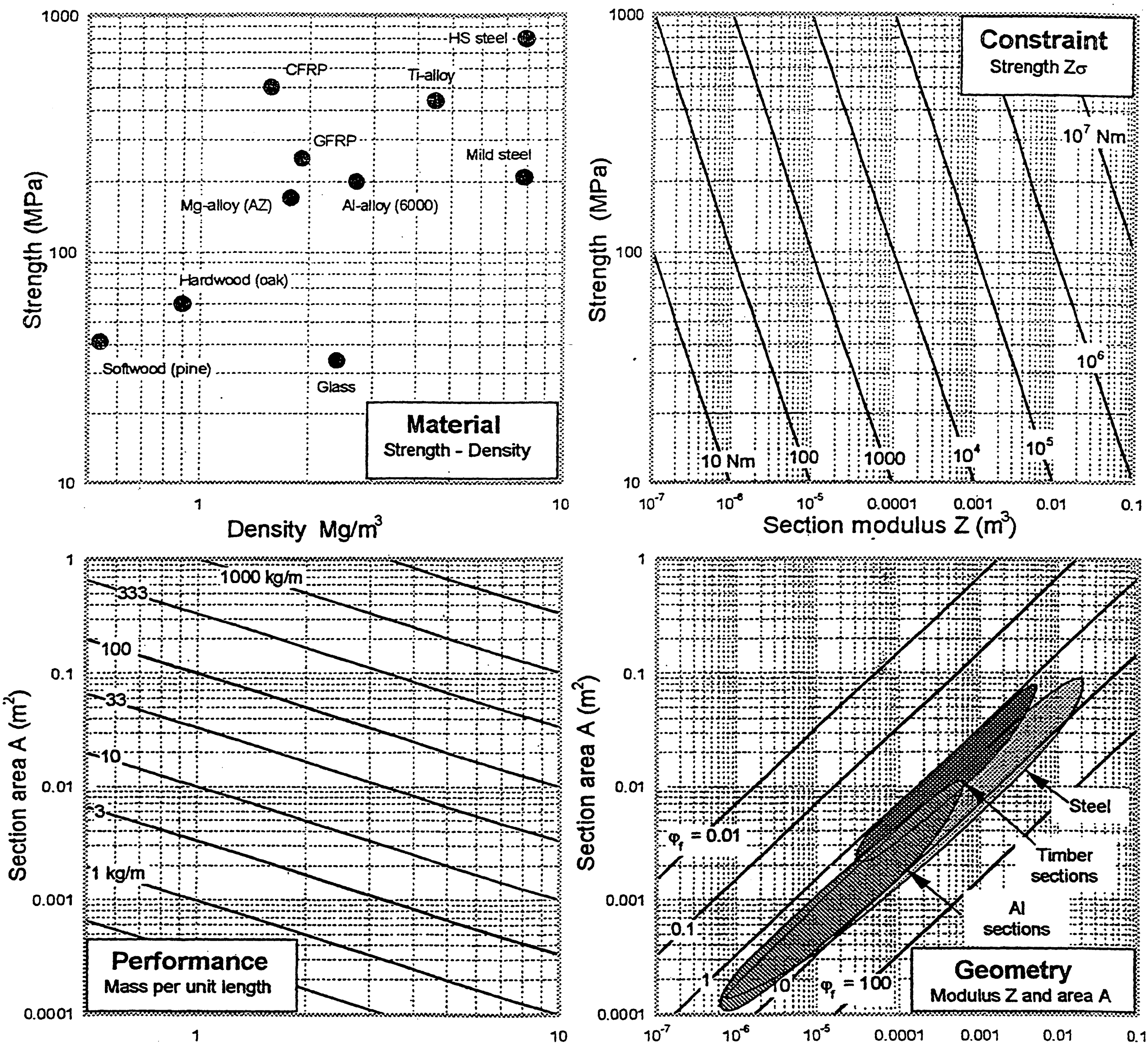


Figure 2 The chart-assembly for exploring structural sections for strength limited design. Like that for stiffness, each chart shares its axes with its neighbours.

Note: concept and charts are due to Prof. Michael Ashby, 2002
(see M.F. Ashby 'Materials selection in mechanical design' 2nd Edition, Butterworth 1999, JA163).

Module 3D3 – Structural Materials and Design   Steel Datasheet

Materials available (see Structures Data Book 1999 pp. 1 and 11)

The two common structural steels BS EN – S275 and S355 have characteristic yield strengths σ_y of 275 and 355 MPa respectively. Both satisfy the usual criteria for plastic design (adequate ratio of UTS to yield strength; adequate elongation to fracture). In design, the calculated strength (e.g. buckling resistance) for either material would be divided by a specified partial safety factor γ_m, often 1.1. Strength design is at ULS, with specified partial safety factors γ_f on loads, often 1.4 on dead load, 1.6 on live.

Tension members (axial force only)

Gross area A; net area A_n is A minus hole(s). Effective section A_e is KA_n but not greater than A, where factor K is 1.2 for S275, 1.1 for S355.

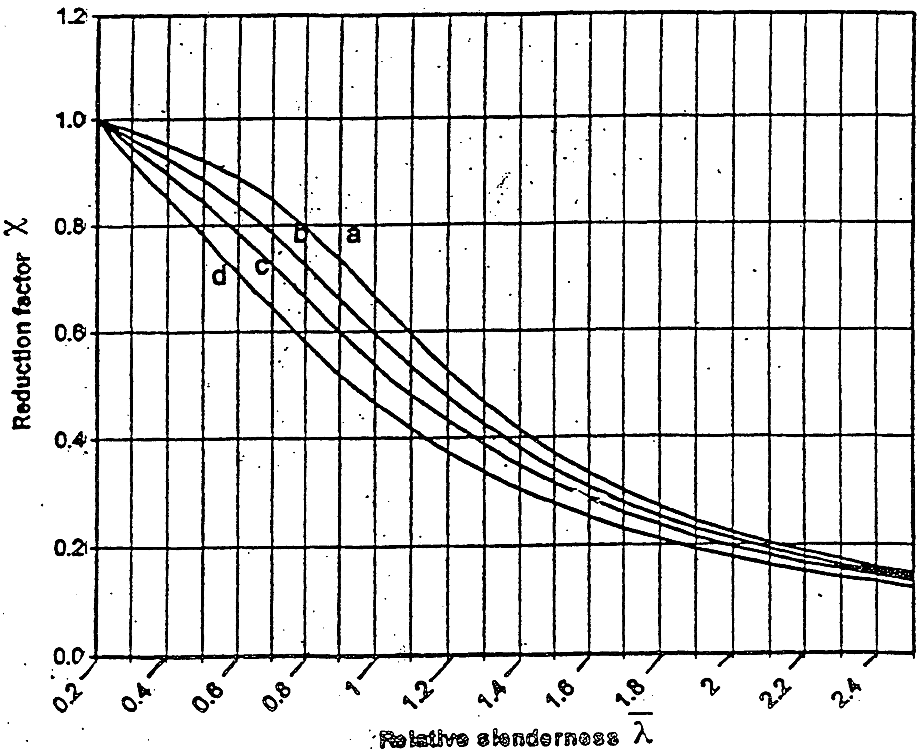
For eccentric connection, with area a_2 not connected at joint, effective area is often taken as $A_e - ca_2$, where factor c is 0.5 for bolted connections, 0.3 for welds.

Compression members (axial force only)

Joints do not normally control design, though criteria above should be checked. Base design against buckling on gross area A, yield strength σ_y, radius of gyration r, and column effective length L between points of contraflexure. Slenderness $\lambda = L/r$.

Define λ_0 as the slenderness at which the elastic critical stress for a perfect column equals the yield strength, so $\lambda_0 = \pi \{ E/\sigma_y \}^{1/2}$. Relative slenderness $\bar{\lambda} = \lambda/\lambda_0$

To allow for interaction between yield and buckling, use curves of reduction factor χ (on the full yield axial strength) plotted against the relative slenderness $\bar{\lambda}$. Typical curves are as shown below (these from the IStructE EC3 (Steel) Design Manual). Choice of empirical curve a to d depends on section type (extreme fibre distance y/r) and typical imperfection and residual stress magnitudes.



Beams (without axial force)

Moment - check maximum moment less than $\sigma_y Z_p$. Sections are Class 1 to 4, for ability to provide enough ductility for full plastic behaviour without local buckling.

Shear – yield strength q_w in shear often taken as $0.6\sigma_y$ for simplicity. But check for buckling in thin webs, for depth/thickness d/t and aspect ratio a/d, e.g. for $a/d > 1$, elastic critical average stress $q_{cr} = (0.75 + \{d/a\}^2)\,\{1000t/d\}^2$ in MPa.

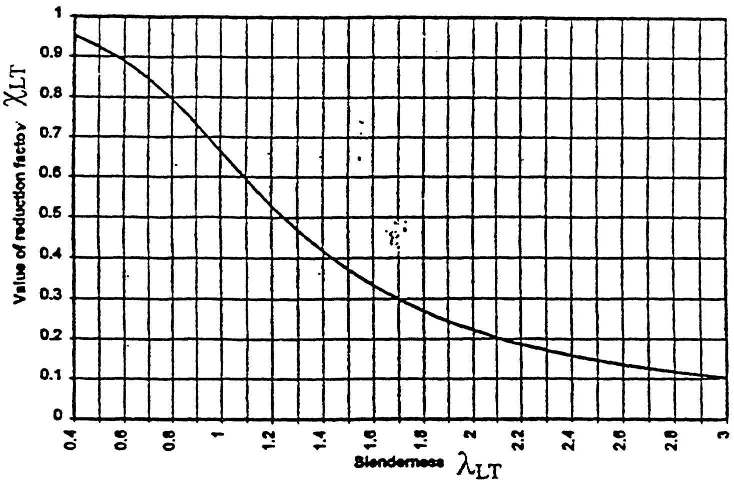
Bearing – check, using special formulae for the various failure modes, at supports and under high local loads, in case stiffener(s) are required.

Lateral-torsional buckling - for uniform bending moment M over a distance L between points where lateral displacement and rotation about beam axis are prevented, elastic critical value, with significant torsional stiffness due to restraint of warping,

$$M_c = \frac{\pi}{L}\sqrt{EI_{yy}\left\{GJ + \frac{\pi^2}{L^2}EC_w\right\}}.$$ For typical I-beam, basic torsion constant $J = \Sigma bt^3/3$

where b is the width of component plates of thickness t; and warping-restraint factor $C_w = D^2 I_{yy}/4$ where D is the distance between centres of flanges.

To allow for interaction between buckling and yield, again use empirical curves of the capacity reduction factor χ_{LT} against relative slenderness $\lambda_{LT} = \{M_p/M_{cr}\}^{1/2}$ (here M_{cr} is the hinge-position moment for elastic buckling).



Joints

Ductility of steel allows a reasonably simple equilibrium system to be envisaged for initial design – often with a transmitted force uniformly distributed across the various fasteners involved (particularly if they have similar properties). For a bolted joint in shear, a couple C about its centre can be taken simply by forces F_i on each bolt, perpendicular to the line to the centre and proportional to the distance d_i from the centre, so that $F_i = Cd_i/\Sigma d_i^2$, but other equilibrium systems can be envisaged.

Applied shear forces F on bolts are commonly checked against the shear strength (say $0.6\sigma_y$) of the bolt, depending on the number of shear planes activated; and against the bearing strength $\sigma_b dt$ in each plate (d is bolt diameter, t is plate thickness), despite the fact that bolts may not actually transmit force in this way. For M20 bolts, σ_y is typically 600 MPa, and in Codes nominal σ_b is typically of order 400 MPa in S275 steel.

(information mainly from Eurocode 3) CTM Jan. 2003

3D3 – Structural Materials and Design – Concrete Datasheet

Structural system	Span/effective depth ratio			
	IStructE*		EC2**	
	beam	slab	high	light
1. Simply supported beam One-way or two-way spanning simply supported slab	15	21	18	25
2. End span of continuous beam: one-way continuous slab; or two-way spanning slab continuous over one long side	20	27	23	32
3. Interior span of: beam; one-way or two way spanning slab	21	30	25	35
4. Slab supported on columns without beams (flat slab), based on longer span	n/a	26	21	30
5. Cantilever	6	8	7	10

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 4.2 [10.1], ** Table 4.14 [10.2]

Table 10.1 Span versus depth ratio

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width cover	240 70	200 50	200 45
Slabs with plain soffit	thickness cover	170 45	125 35	100 35

Extracts from Table 4.1 [10.1]

Table 10.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

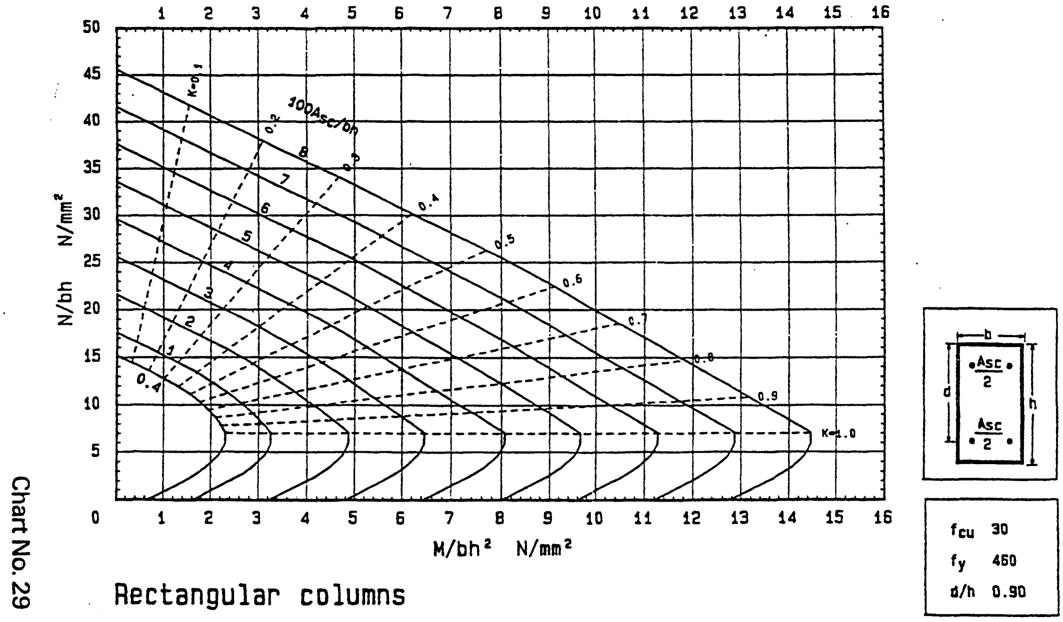


Fig 10.14 Interaction diagram from [10.3]

[10.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[10.2] Eurocode 2: Design of concrete structures DD ENV 1992-1-1:1992 – note** draft document
[10.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure

Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1 - 0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if x/d =0.5

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Bond anchorage

$$l_b = \frac{f_y r}{2\beta_a \sqrt{f_{cu}}}$$

Cracking

$$w_k = \beta s_{rm} \varepsilon_{sm}$$

$$s_{rm} = 50\text{ mm} + 0.25 k_1 k_2 \phi / \rho_r$$

where: w_k is the design crack width

β is a statistical variation coefficient, usually 1.7

s_{rm} is the average final spacing

ε_{sm} is the mean steel strain (cracked-elastic theory)

ϕ is the average bar size

ρ_r is the effective steel ratio A_s/A_{ceff}

Shear

Concrete contribution

$V_{Rd1}=\{\tau_{Rd} k\,(1.2 + 40\rho_l)\} b_w d$

where: τ_{Rd} is the basic shear strength of the concrete.

k is a factor that varies with effective depth, d (k = 1.6 - d but $k \geq 1$ and d in metres),

b_w is the minimum width

ρ_l is the reinforcement ratio = $A_s/b_w d$ but $\rho_l \leq 0.02$.

Shear stirrup contribution

$v_s = f_y A_v / \gamma_s b_w s$

Columns

Axial loading only

$$\sigma_u = 0.6\frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm)

6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 – Structural Materials and Design – Timber Datasheet

			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens $\parallel$	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens $\perp$	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp $\parallel$	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp $\perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod $\parallel$	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod $\perp$	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	2	3
Solid and glued laminated timber and plywood			
Permanent	0.60	0.60	0.50
Long-term	0.70	0.70	0.55
Medium-term	0.80	0.80	0.65
Short-term	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – Design shear stress

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – Design bearing stress

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$$\lambda_{rel,m} = \sqrt{f_{m,k} / \sigma_{m,crit}}$$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e)"

$$k_{crit} = \begin{cases} 1 & \text{for} \quad \lambda_{rel,m} \le 0.75 & (5.2.2c) \\ 1.56 - 0.75\lambda_{rel,m} & \text{for} \quad 0.75 < \lambda_{rel,m} \le 1.4 & (5.2.2d) \\ 1/\lambda_{rel,m}^2 & \text{for} \quad 1.4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm^2

For bolts up to 30 mm diameter at an angle α to the grain:

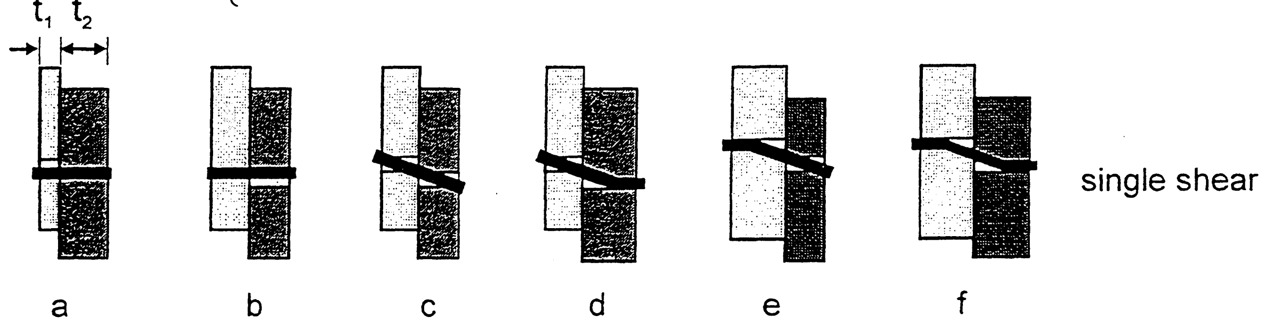
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear

$$R_d = \min. \begin{cases} f_{h,1,d}\, t_1\, d & (6.2.1a) \\ f_{h,1,d}\, t_2\, d\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}\, t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \dfrac{t_2}{t_1} + \left(\dfrac{t_2}{t_1}\right)^2\right] + \beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1 + \dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1.1\dfrac{f_{h,1,d}\, t_1\, d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}\, d\, t_1^2}} - \beta\right] & (6.2.1d) \\ 1.1\dfrac{f_{h,1,d}\, t_2\, d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}\, d\, t_2^2}} - \beta\right] & (6.2.1e) \\ 1.1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d} f_{h,1,d}\, d} & (6.2.1f) \end{cases}$$



Extract from [11.2] – Timber-to-timber and panel-to-timber joints

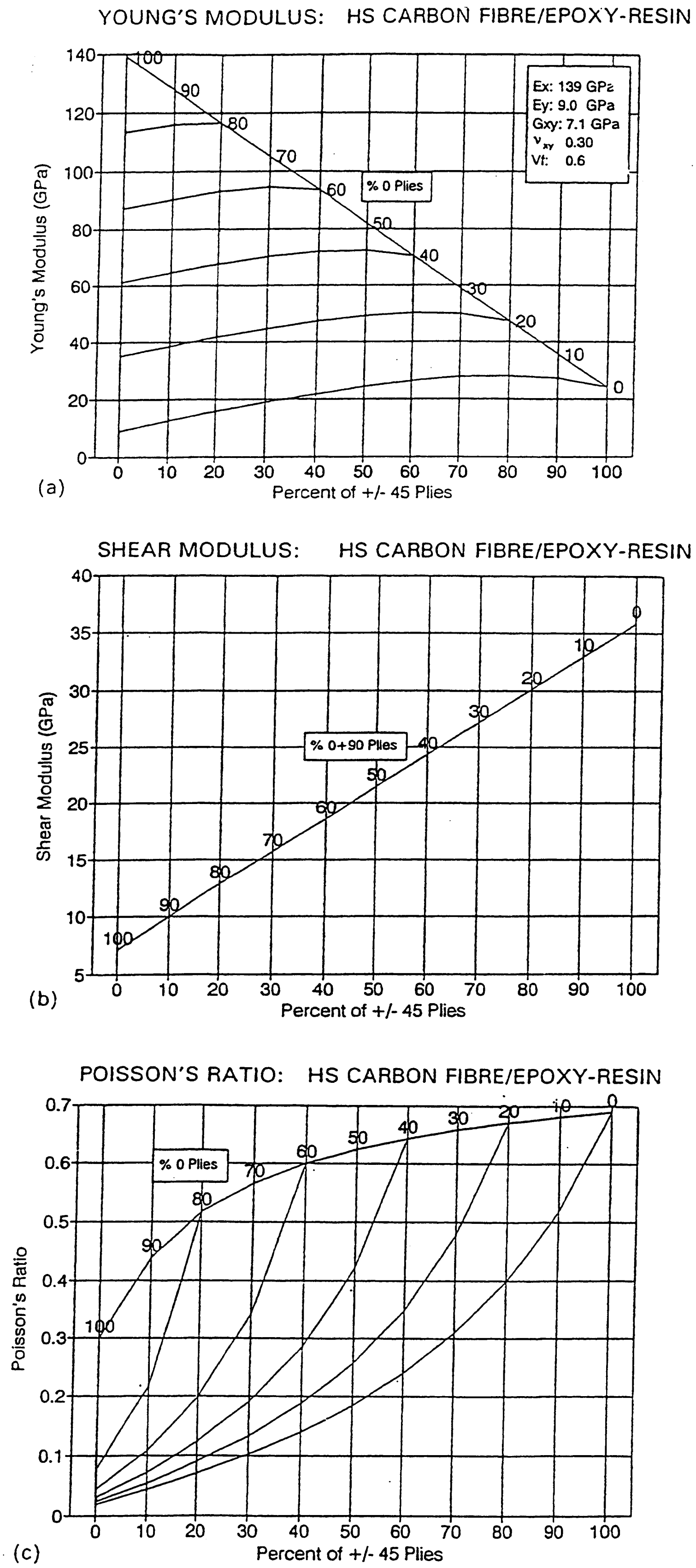


Figure 1. HS carbon/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

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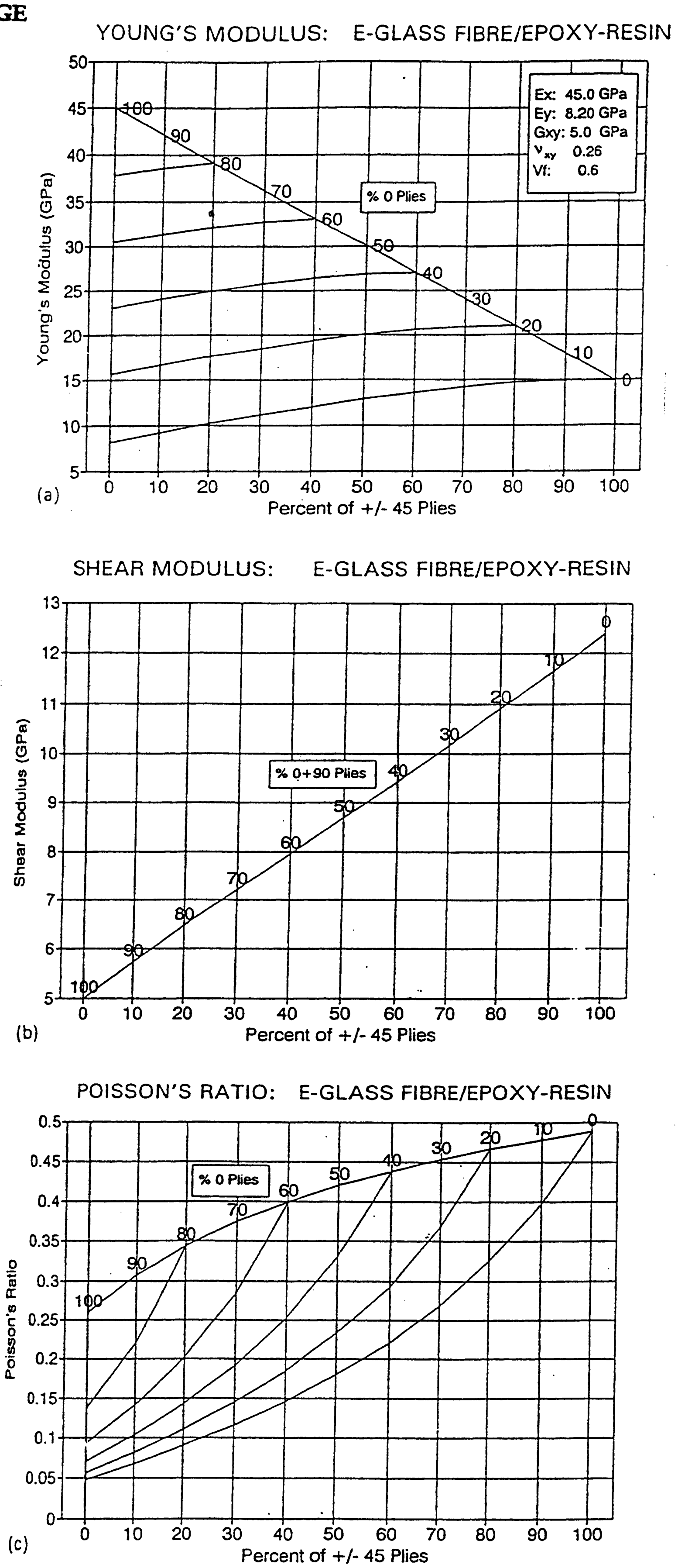

Figure 2. E-glass/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

[12.3] M.G. Bader, Materials selection, preliminary design and sizing for composite laminates. Journal Paper: Composites Part A, 27A, (1996), pp 65-70.

Engineering Tripos Part IIA, 2005

Paper 3D3 Structural Materials and Design

Answers

1. (a) (i) L_{max} (deflection) = 10.04 m
L_{max} (strength) = 9.69 m ** strength limit most critical
(ii) L_{max} (deflection) = 6.05 m ** deflection limit most critical
L_{max} (strength) = 9.04 m

(b) (i) V_{max} (primary) = 405 kN, V_{max} (secondary) = 135 kN
(ii) M_{max} (primary) = 844 kNm, M_{max} (secondary) = 101 kNm
(iv) the maximum shear force in the primary beam remains the same
the maximum shear force in the secondary beam reduces to 106.9kN

2. (a) (i) $\phi_f = 3.41$
(ii) $\phi_f = 7.15$

(c) (i) $M_c =$ 209 kNm, lateral torsional buckling
(ii) $\chi_{LT} = 0.66$

3. (b) (i) in compression ; need A_{stot} = 2750 mm^2
in tension; need A_{stot} = 2500 mm^2 , compression requirement controls
e.g. four 32 mm diameter bars
(ii) M_U = 136.8 kNm, $N_U \approx 0$ kN, almost case of pure bending

4. (b) (i) Mode c , R_d = 3032 kN
Mode f, R_d = 2368 kN **controls, therefore need 5 bolts
(ii) In both cases mode b is more critical

J.L. May, 2005

PART IIA 2005

3D3 Structural materials and design

Principal Assessor: **Dr J Lees**