UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Wednesday 11 May 2005   2.30 to 4

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Define "warping" in the context of beams loaded in torsion.

What is meant by the phrase "restrained warping torsion"? In what way does it differ from "St. Venant's torsion"?

For each of the following examples, write one sentence explaining whether restrained warping torsion is important. (Note: "varying torsion" refers to variation along the length of the beam of the internal torsional torque.)

(i) An I-beam, under uniform torsion, ends free to warp;

(ii) An I-beam, under uniform torsion, with warping prevented at the ends;

(iii) An I-beam, under varying torsion, with warping prevented at the ends;

(iv) A circular tube, under varying torsion, with warping prevented at the ends;

(v) A solid circular bar, under varying torsion, with warping prevented at the ends;

(vi) An angle section, under uniform torsion, ends free to warp;

(vii) An angle section, under varying torsion, with warping prevented at the ends;

(viii) An open box section, under uniform torsion, ends free to warp;

(ix) An open box section, under uniform torsion, with warping prevented at the ends;

(x) A closed box section, under uniform torsion, ends free to warp;

(xi) An closed box section, under varying torsion, with warping prevented at the ends;

(cont.

(xii) An I-beam, simply supported at both ends, subject to a load that causes lateral-torsional buckling [60%]

(b) A steel I-beam 12 m long has torsional constant $J = 3.6 \times 10^5$ mm^4 and restrained warping constant $\Gamma = 2.6 \times 10^{11}$ mm^6. It is built in at both ends and subject to a central point torque of 6 kNm aligned along the beam's longitudinal axis.

Calculate the rotation at the loading point ignoring the effects of warping restraint.

Calculate $\lambda = \sqrt{\dfrac{E\Gamma}{GJ}}$.

By carefully considering the symmetry of the warping displacement, use your calculated value of λ to estimate the rotation if restraint of warping is taken into account. [40%]

2 A curved beam with flexural stiffness EI and torsional stiffness GJ is mounted as a semi-circle in plan, built-in at both ends. It is subjected to a uniformly distributed load of intensity w per unit length of beam acting at right angles to the plane of the semi-circle.

(a) By considering the symmetry at the centre, what stress and strain resultants can be shown to be zero at that point? [20%]

(b) Use virtual work to calculate the bending moment at the centre of the beam due to applied load w. [60%]

(c) Without doing further calculations, explain how you would use virtual work to calculate the out-of-plane deflection at the centre of the beam. [20%]

3 (a) Briefly describe the difference in meaning of the eigenvalues in the classical and non-classical descriptions of elastic stability. [20%]

(b) Figure 1 shows three rigid bars of length L linked to form a pin-jointed structure ABCD. It is shown in a general deflected configuration with joints B and C having deflections u_1 and u_2 below the line AD. Point loads Q_1 and Q_2 are applied conjugate to u_1 and u_2. An axial load P is applied at D in the direction of A. The deflection y is conjugate to P. The linear spring at B has spring stiffness k. The rotational spring at C creates a torque equal to $G\theta$, where θ is the relative rotation (in radians) between bars BC and CD. Both springs are unstressed when ABCD is straight.

According to small deflection theory, the total potential energy $\pi\,(u_1, u_2)$ of the system can be written

$$\pi(u_1, u_2) \;=\; -\mathbf{Q}^T\mathbf{u} \;+\; \frac{1}{2}\mathbf{u}^T\mathbf{K}\,\mathbf{u}$$

where $\mathbf{Q} = (Q_1, Q_2)^T$, $\mathbf{u} = (u_1, u_2)^T$ and $\mathbf{K} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$.

(i) Determine the coefficients a, b and c in the tangent stiffness matrix $\mathbf{K}$ in terms of P, L, k and G. [40%]

(ii) When $G = kL^2$, show that buckling occurs when $P = 2kL/3$ and determine the corresponding buckling eigenvector. [20%]

(iii) With $G = kL^2$ still, loads P and $\mathbf{Q}$ are applied simultaneously, with $P = kL/4$ and $\mathbf{Q} = (3kL/400, \;\; -kL/400)^T$. Determine the corresponding equilibrium deflections $\mathbf{u} = (u_1, u_2)^T$. [20%]

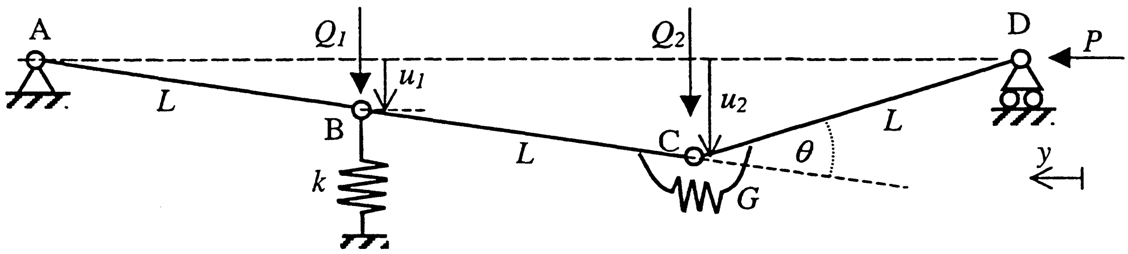

Fig. 1

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4 (a) Briefly explain the essential insight shown by Shanley which resolved "The Column Paradox". [10%]

(b) Figure 2(a) defines the usual notation for the elastic slope-deflection equations of a beam-column. The rigid-jointed frame shown in Fig. 2(b) has three beams and a column. The relevant factor $k = EI/L$ is the same for all four members. A point load P is applied at B as shown. Stating your assumptions, write down an appropriate set of slope-deflection equations in matrix form and show that the occurrence of instability is governed by an equation of the form

$$s^2\left(1-c^2\right)+\beta s+\gamma=0$$

where s and c are the stability functions for the column BC. Determine the numerical value of the coefficients β and γ, and sketch the shape of the buckled structure. [40%]

(c) Explain briefly how the Perry-Robertson approach to column buckling deals with inelastic behaviour. [10%]

(d) Determine the magnitude M_{cr} (in kNm) of the equal and opposite end-moments that would be sufficient to cause lateral torsional buckling of a steel $686 \times 254 \times 140$ Universal Beam of length 6 m, assuming that the ends are simply-supported for bending, fully restrained against torsion and free to warp.

Hint: with usual meanings, $M_{cr} = \dfrac{\pi}{L}\sqrt{GJEI_y}\left(1+\dfrac{\pi^2}{L^2}\dfrac{E\Gamma}{GJ}\right)^{1/2}$ with $\Gamma = \dfrac{I_y D^2}{4}$ [40%]

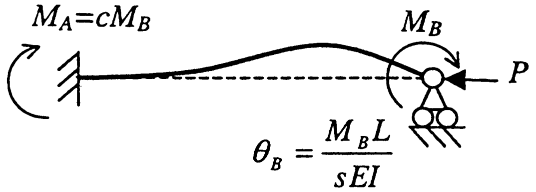

Fig. 2(a)

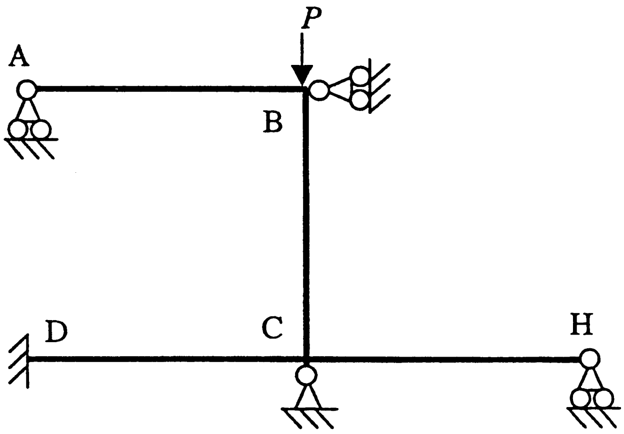

Fig. 2(b)

END OF PAPER

ENGINEERING TRIPOS PART IIA 2005
3D4: STRUCTURAL ANALYSIS AND STABILITY
Answers

1. a) Important for ii), iii) ix) and xii)
   b) 0.617 rads, 0.335 rads

2 $wR^2\left[\dfrac{4-\pi}{\pi}\right]\left\{\dfrac{GJ-EI}{GJ+EI}\right\}$

3. b) i) $a = k + \dfrac{G}{L^2} - \dfrac{2P}{L}$, $b = -\dfrac{2G}{L^2} + \dfrac{P}{L}$, $c = \dfrac{4G}{L^2} - \dfrac{2P}{L}$

   iii) $\dfrac{L}{700}\begin{bmatrix}7\\3\end{bmatrix}$

4 b) $\beta = 10$, $\gamma = 21$
   d) 1203 kNm

PART IIA 2005

3D4 Structural analysis and stability

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