ENGINEERING TRIPOS PART IIA

Tuesday 3 May 2005 2.30 to 4

Module 3D5

ENVIRONMENTAL ENGINEERING I

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheet (6 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Rain falls uniformly over a catchment for three hours and the excess rainfall is constant throughout that time. The distribution percentages for the outflow hydrograph over six successive three hour periods starting at the beginning of the rainfall are 4, 15, 30, 25, 15, 11. Estimate the distribution percentages over successive one hour intervals from uniform excess rainfall of one hour duration, and plot the shape of this hydrograph. [40%]

(b) Explain what is meant by subcritical, critical and supercritical flow in an open channel, and how the terms relate to the velocities (relative to the bed and relative to the fluid) of small disturbances on the water surface. Sketch the water surface profiles for flow over a small bump in a channel bed for the four cases:

(i) upstream subcritical, downstream subcritical;
(ii) upstream subcritical, downstream supercritical;
(iii) upstream supercritical, downstream supercritical;
(iv) upstream supercritical, downstream subcritical. [30%]

(c) Explain why the fall velocity of particles of sand (of diameter D) in water (as given on the Data Sheet) is proportional to $D^{1/2}$ for large particles, but proportional to D^2 for small particles. [30%]

2 A wide, smooth, flat flood drainage channel with a uniform rectangular cross-section flows into a large tidal lagoon. At time $t = 0$ the mean velocity in the channel is 0.5 m s^{-1} towards the lagoon, and the depth is 1.2 m. The water level may be assumed flat over the lagoon and channel. The water level in the lagoon is rising at a steady rate of 0.2 m per hour. Let x be the distance along the channel from the point where it enters the lagoon, and let the time t be measured in seconds. Let u_0 be the initial velocity (at time $t = 0$) of water in the channel defined as positive in the increasing x direction, and let d_0 be the initial water depth in the channel.

(a) Show that a positive characteristic has a slope on the (x, t) −plane of

$$\left[u_0 - 2(gd_0)^{1/2} + 3\left(g\left[d_0 + \frac{t}{18\times10^3}\right]\right)^{1/2} \right]^{-1}$$ [40%]

(b) At what time t does the positive characteristic passing through the point $x = 0$ and $t = 1$ hour intersect the line separating disturbed flow from undisturbed flow in the channel? Explain the physical significance of the intersection point, and without further calculation, describe which further physical principles could be applied to estimate the subsequent evolution of the water surface profile near this intersection point. [60%]

3 (a) Show how the Colebrook-White equation can be modified to eliminate the friction factor, and explain how this modified equation forms the basis of "pipe full" design charts of the type shown in Fig. 1.

Colebrook-White equation: $\dfrac{1}{\sqrt{\lambda}} = -2\log\left[\dfrac{k}{3.7d} + \dfrac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right]$ [30%]

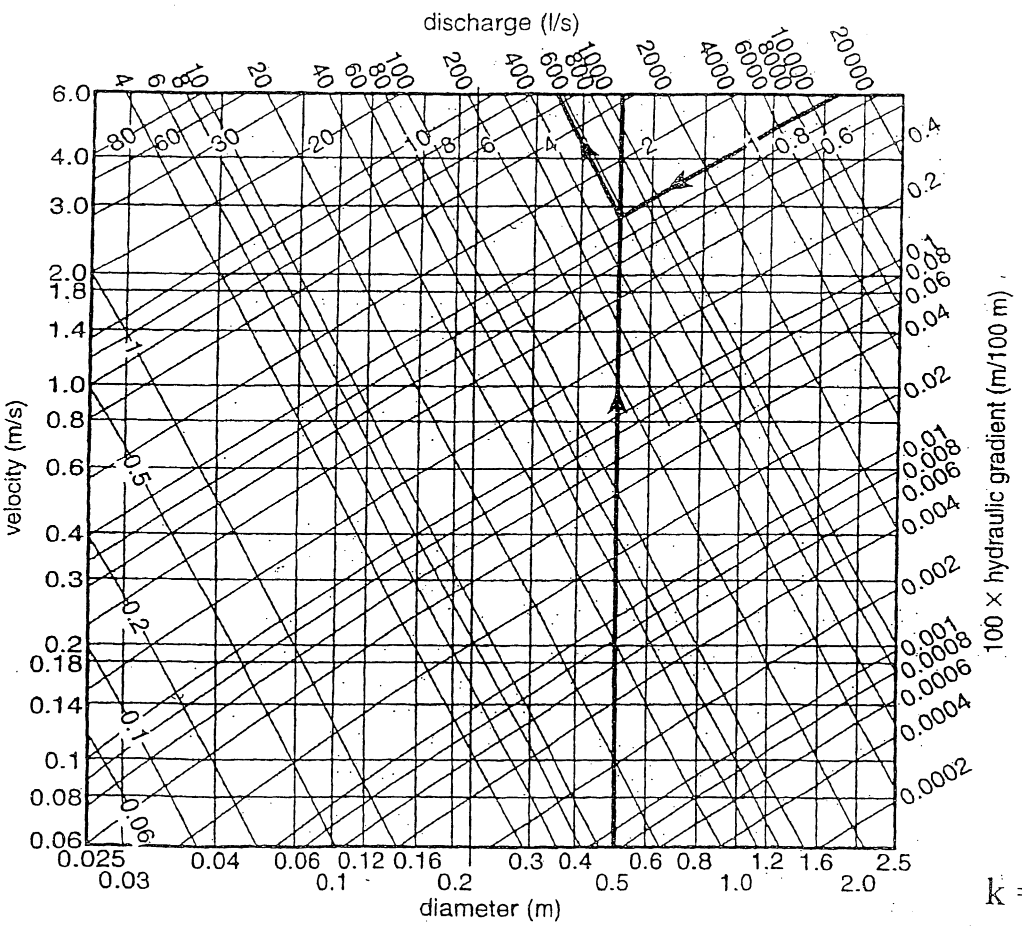

Fig. 1

(b) Water is to be transferred by pipeline from a water treatment works (water surface elevation 120 m) to a service reservoir on an adjacent hill one kilometre away (water surface elevation 150 m) across ground which has the profile shown in the following table:

Horizontal distance(m)	0	100	200	300	400	500	600	700	800	900	1000
Ground Elevation(m)	100	115	105	100	80	75	100	110	115	130	140

(The horizontal distances are measured from the treatment works. Note that the water surface level at the treatment works is 20 m above the local ground surface, and that of the reservoir is 10 m above its local ground surface).

A variable speed booster pump is available which showed the following characteristics when operated at 1000 r.p.m. :

Q (l/s)	0	100	200	300	400	500	600
H (m)	65	63	59	52	43.3	31	13

Cast iron pipe (with k = 0.03 mm) is available in three sizes, with internal diameters of 225 mm, 300 mm and 500 mm. The relevant Colebrook-White chart is given in Fig.1.

Design a pipe to transport 200 ls^{-1} from the water treatment works to the service reservoir using a combination of gravity and pumped flow. Select one pipe diameter suitable for the whole pipeline. Sketch the location of the pump and the hydraulic grade lines. Specify the speed at which the pump should be operated to deliver the required flow.

Notes:

Consider major losses due to pipe friction but ignore other minor losses.

The Darcy-Weisbach Equation: $h_f = \frac{\lambda L}{D} \frac{v^2}{2g}$

$Q_1/Q_2 = N_1/N_2$ and $H_1/H_2 = N_1^2/N_2^2$ [70%]

(TURN OVER

4 A wide channel of uniform cross-section has a bed slope of 0.001 and a depth of 1.0 m. Suspended sediment can be considered to be the sediment carried above the bed layer, which is 0.01 m thick. The concentration of suspended sediment at the top of the bed layer is 9.9 kg m^{-3}.

Assume the Karman constant is 0.4, the specific gravity of the sediment is 2.65, all grain diameters are 0.00021 m, the bed roughness height k_s is 0.01 m, the kinematic viscosity of water ν is 10^{-6} m^2s^{-1} and the water temperature is 20 °C.

(a) Show that the shear velocity u_* is 0.1 m s^{-1} and that the bed is hydraulically rough. [10%]

(b) Estimate the velocity at heights 0.01 m, 0.1 m, 0.2 m, 0.5 m and 1.0 m above the bed. [20%]

(c) Show that the fall velocity W of the particles in still water is such that $W / Ku_* \approx 1.0$. [10%]

(d) Estimate the sediment concentration at heights 0.01 m, 0.1 m, 0.2 m, 0.5 m and 1.0 m above the bed. [20%]

(e) Draw a graph showing the variation with height of the product (velocity × concentration of suspended sediment). [10%]

(f) Estimate the sediment transport rate in suspension per metre width of channel above the bed layer using:

(i) the trapezoidal rule to estimate the area under the graph in part (e); [10%]

(ii) the formulae and tables for sediment load in suspension given on the Data Sheet. [20%]

END OF PAPER

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Engineering Tripos Part IIA THIRD YEAR

Module 3D5: Environmental Engineering I

Data Sheet

(SI units throughout)

f-capacity equation

$$f = f_c + (f_0 - f_c)\, e^{-K_f t}$$

Spatially-varied flow (backwater curves)

$$\delta d = \ell\left(\frac{S - \overline{U}^2 C_f / 2gR}{1 - \overline{U}^2 / gd}\right)$$

Open-channel characteristics

$$\overline{U} + 2c - g(S - S_f)t = const \qquad \frac{dx}{dt} = \overline{U} + c$$

$$\overline{U} - 2c - g(S - S_f)t = const \qquad \frac{dx}{dt} = \overline{U} - c$$

Fall velocity of particles of sand in water (20°C)

For D < 0.0005 m $\quad W \doteq 56\times10^4 D^2(\rho_s - \rho)/\rho$

For D > 0.002 m (and shape factor=0.7) $\quad W \doteq 3.3\, D^{1/2}((\rho_s - \rho)/\rho)^{1/2}$

Initial motion of sediment on a flat bed

$$\frac{u_* k_s}{\nu} > 70 \qquad \frac{\tau_c}{(\rho_s - \rho)gD} = 0.05$$

Velocity in uniform flow in a channel

Chézy $\quad \overline{U} = C R^{\frac{1}{2}} S^{\frac{1}{2}}$

Manning $\quad \overline{U} = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$

Prandtl-Karman

Hydraulically smooth $\left(\frac{u_* k_s}{\nu} < 5\right)$

$$\frac{u}{u_*} = 2.5 \log_e\left(\frac{9.05 y u_*}{\nu}\right)$$

$$\frac{\overline{U}}{u_*} = 2.5 \log_e\left(\frac{3.66 R u_*}{\nu}\right)$$

Hydraulically rough $\left(\frac{u_* k_s}{\nu} > 70\right)$

$$\frac{u}{u_*} = 2.5 \log_e\left(\frac{30.2 y}{k_s}\right)$$

$$\frac{\overline{U}}{u_*} = 2.5 \log_e\left(\frac{12.1 R}{k_s}\right)$$

Variation of the concentration of sediment in suspension with distance from the bed

$$\frac{C}{C_a} = \left[\left(\frac{d-y}{y}\right)\left(\frac{a}{d-a}\right)\right]^{\frac{W}{Ku_*}}$$

Sediment load in suspension

$$\int_b^d C u dy = 11.6 u_* C_b b\left[I_1 \log_e(Ad) + I_2\right]$$

where $A = \frac{9.05 u_*}{\upsilon}$ for a hydraulically smooth bed

$= \frac{30.2}{k_s}$ for a hydraulically rough bed

b/d	W/Ku* = 0.2		W/Ku* = 0.6		W/Ku* = 1.0		W/Ku* = 1.5	
	I_1	I_2	I_1	I_2	I_1	I_2	I_1	I_2
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113

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Limiting shear stress for a particle on a slope

$$\frac{(\tau_c)_\theta}{\tau_c} = \cos\theta\left(1 - \frac{Tan^2\theta}{Tan^2\phi}\right)^{\frac{1}{2}}$$

"Regime" formulae

$$\overline{U} = 0.635 \quad f^{\frac{1}{2}}R^{\frac{1}{2}}$$

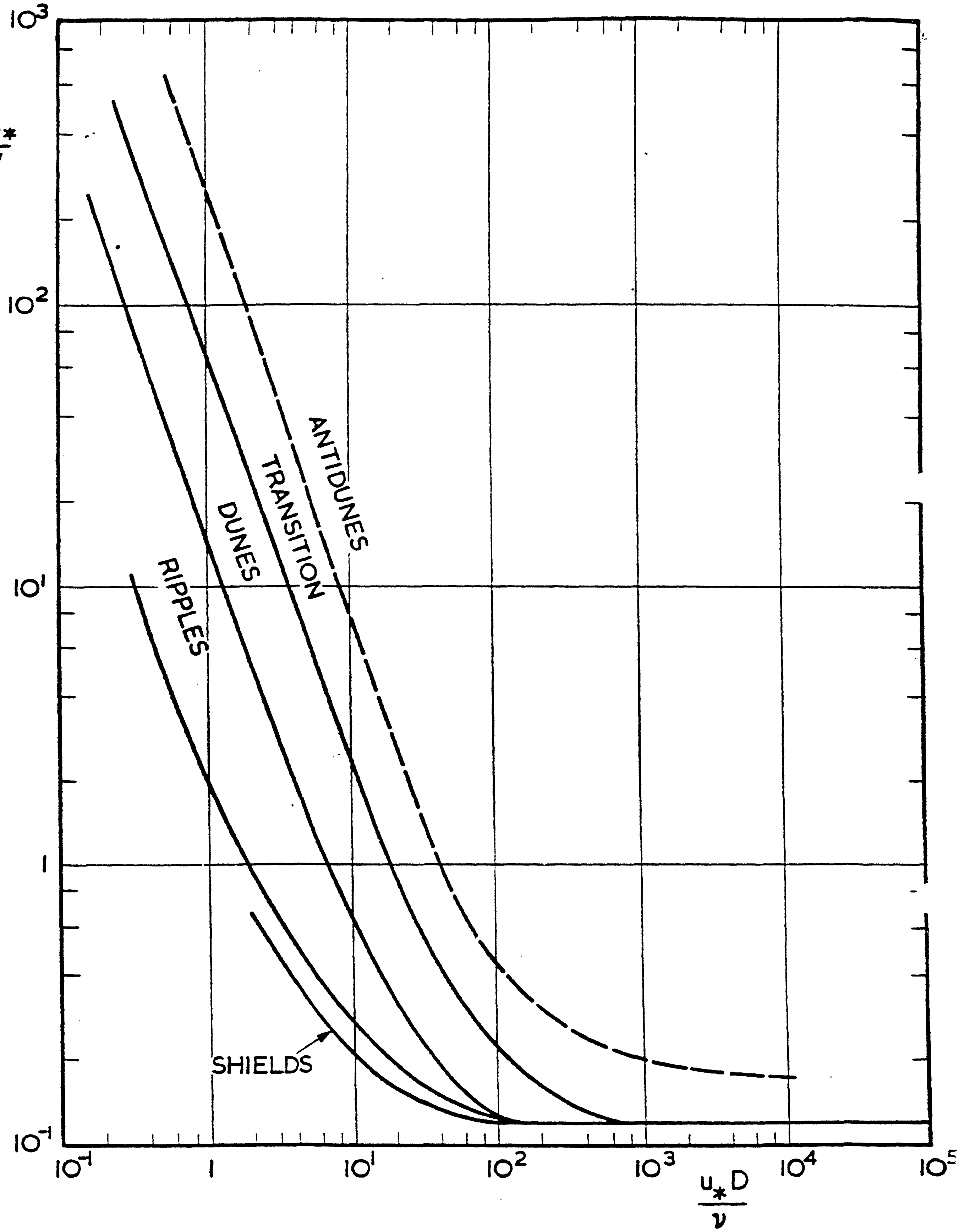
$$P = 4.83 \quad Q^{\frac{1}{2}}$$

$$R = 0.4725 \quad Q^{\frac{1}{3}}f^{-\frac{1}{3}}$$

$$S = 0.000303 \quad f^{\frac{5}{3}}Q^{-\frac{1}{6}}$$

SYMBOLS

C	Concentration of sediment. Chezy roughness coefficient $\left(= (2g/C_f)^{1/2}\right)$
C_f	friction coefficient
D	grain size or body diameter
D_{65}	grain size for which 65% by weight of grains have a smaller diameter
f	Lacey silt factor
f_0, f_c	coefficients in f-capacity equation
h_r	ripple height
K	Karman constant
K_f	coefficient in f-capacity equation
k_s	roughness height
P	wetted perimeter of a channel
Q	total flow rate of water
R	hydraulic radius $(= A/P)$
S	channel slope
t	time
u	horizontal component of fluid velocity
$\overline{U}$	mean velocity
u_*	$(\tau_0/\rho)^{1/2}$
v	vertical component of fluid velocity
W	fall velocity
x,y	co-ordinates
θ	angle of a slope to the horizontal
φ	angle of repose of sediment
ν	kinematic viscosity
ρ	density of fluid
ρ_s	density of sediment
τ_o	shear stress on the bed
τ_c	critical value of τ_o for sand movement



BED REGIMES IN OPEN CHANNEL FLOW (Albertson Simons & Richardson)

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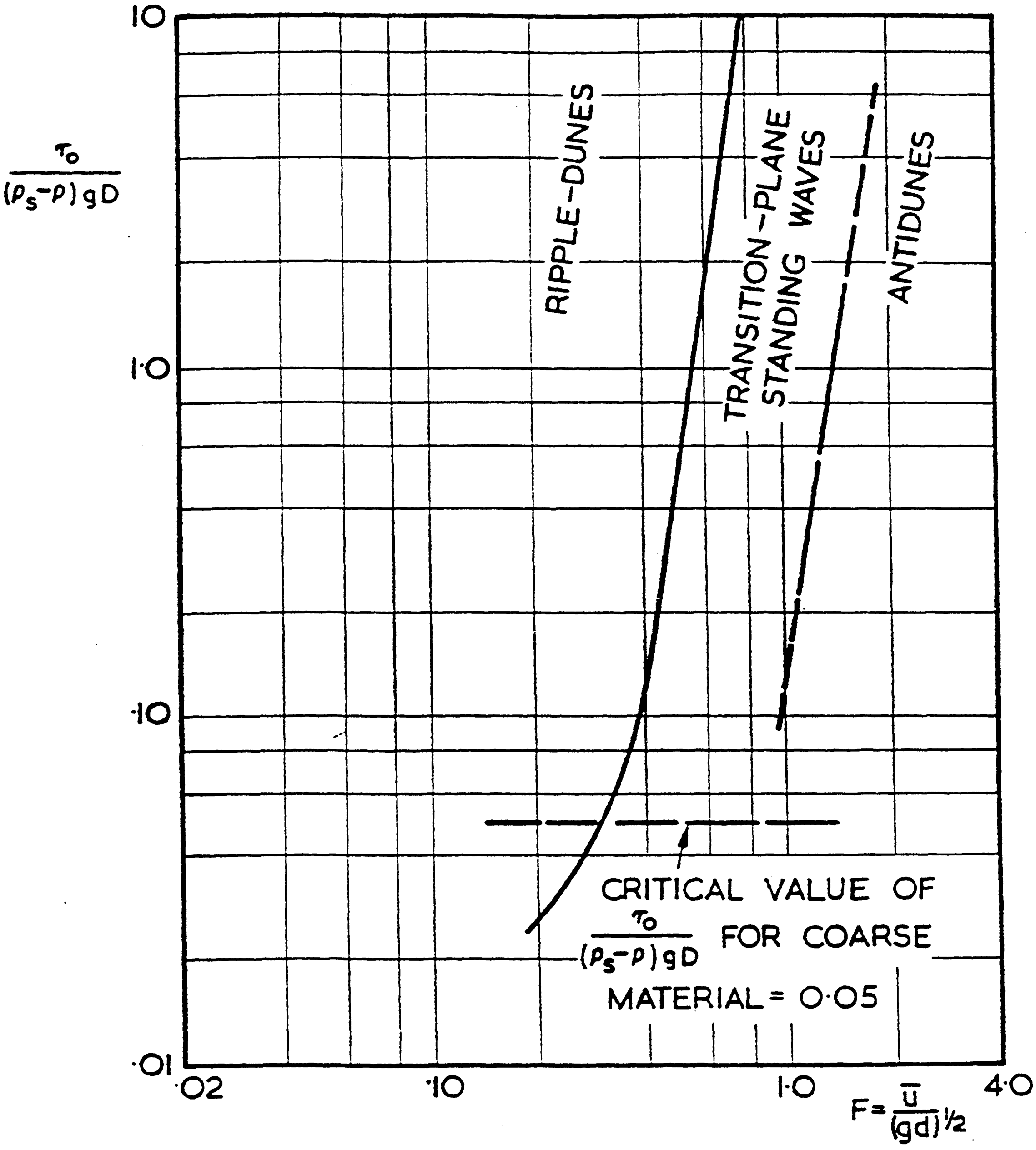

BED REGIMES IN OPEN-CHANNEL FLOW
(Garde & Albertson)

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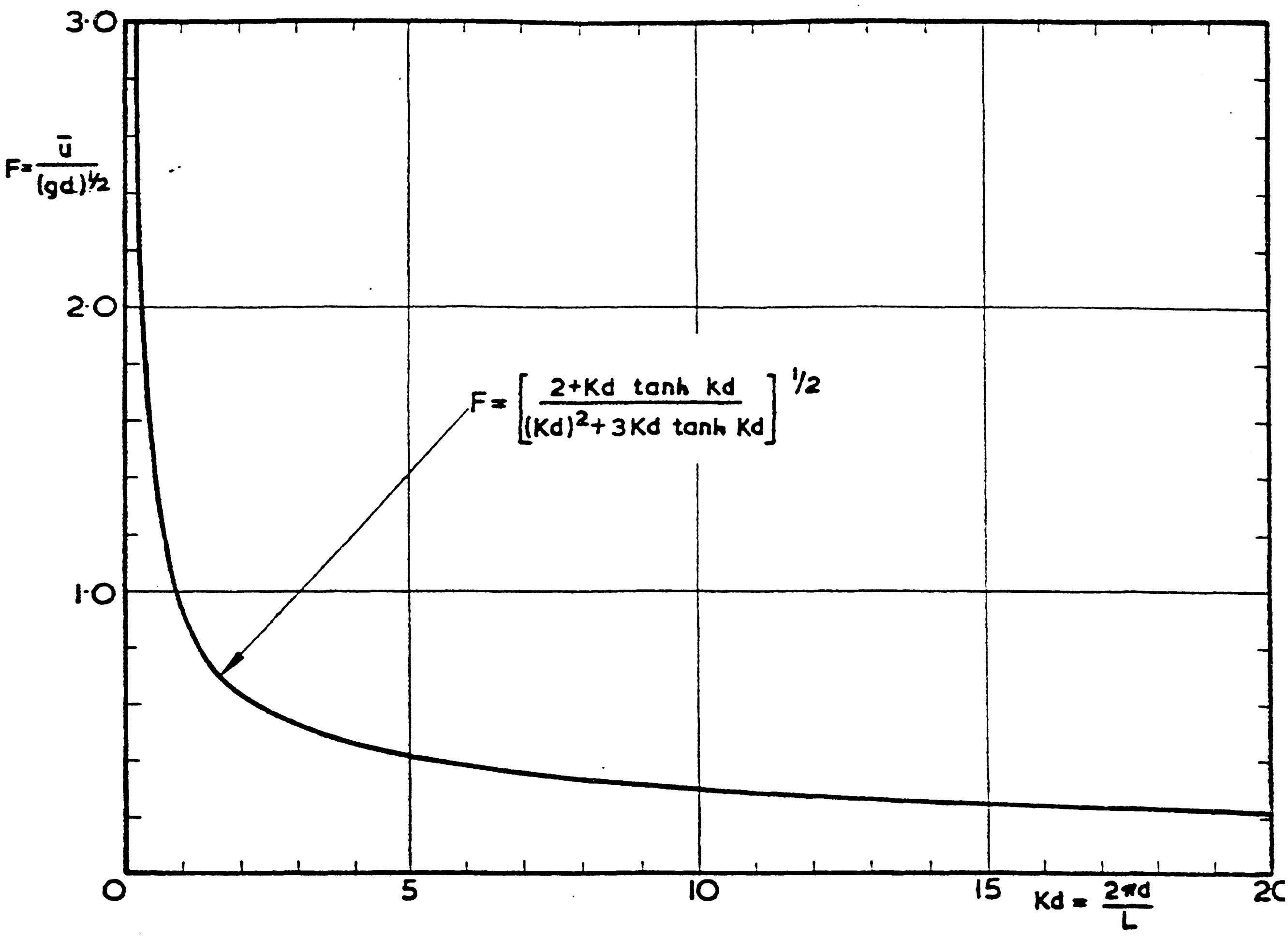
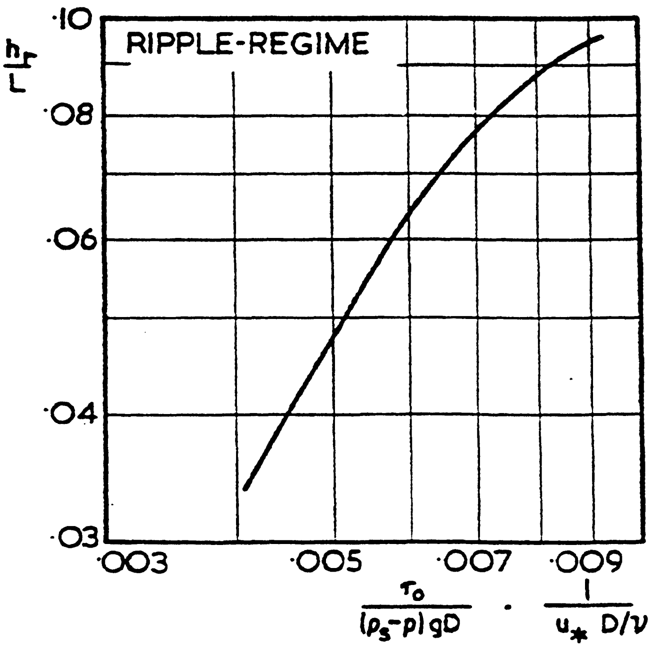
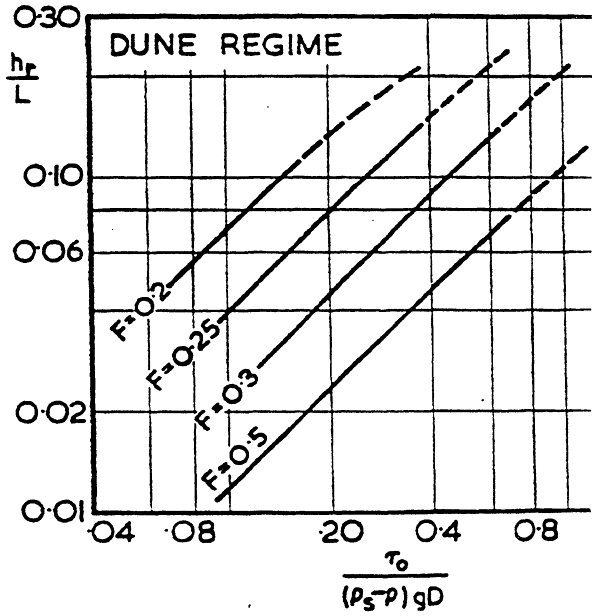


WAVELENGTH OF DUNES / ANTIDUNES (Kennedy)





RIPPLE/DUNE STEEPNESS (Garde & Albertson)

ENGINEERING TRIPOS PART IIA 2005
3D5: ENVIRONMENTAL ENGINEERING 1

Answers

1. a) 1,3,4,5,6,7,9,14,10,8,7,5,5,5,4,4,3

2 b) 4.55 hours

4 b) 0.85, 1.43, 1.6, 1.83, 2.0 m/s
d) 9.9, 0.9, 0.4, 0.1, 0 kg/m^3
f) i) 0.7 kg m^{-1} s^{-1}, ii) 0.97 kg m^{-1} s^{-1}

PART IIA 2005

3D5 Environment engineering I

Principal Assessor: **Mr F A McRobie**