UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Friday April 29 2005 2.30 to 4.00

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 2

(TURN OVER

1 (a) A petroleum company is attempting to decide whether to drill for oil at a particular site in the Pacific Ocean. The cost to drill at this site is $300,000. With no other information, the company believes that there is a 50% chance that the selected site actually contains oil. The estimated value will be $1,800,000 if oil is found. Before drilling, the company can conduct a geological survey for $30,000. The survey may provide strong evidence that there is oil or strong evidence that the site does not contain oil. Past history indicates that when there really is oil, the survey is correct 90% of the time; when there is no oil, the survey is correct 80% of the time. Assume that this oil company wishes to maximize its expected net profit.

(i) Determine the optimal strategy through the use of a decision tree when the company does not use the survey information. [20%]

(ii) Compute and interpret the expected value of perfect information (EVPI) in this decision problem. [15%]

(iii) Draw a decision tree including probabilities on chance nodes and payoff information on leaf nodes when the company uses the survey information. [40%]

(iv) Compute and interpret the expected value of sample information (EVSI) in this decision problem. [10%]

(b) Explain the Central Limit Theorem and its application using a relatively simple real-world example. [15%]

2 (a) A company has recorded the following list of service rates (customers/hour) for one of its servers. What are the mean service time and the service rate for this server? [15%]

Customers per hour	4	4	5	6	5	4
Frequency	3	4	3	5	5	6

(b) State and intuitively derive Little's formula in a steady state queueing system. [15%]

(c) Consider a single-server queueing system. The number of customers requiring service appears to follow a Poisson distribution, with a mean arrival rate of 24 per hour. Customers are served on a first-come, first-served basis, and they are always willing to wait for service. The time it takes to serve a customer appears to be exponentially distributed, with a mean of 2 minutes.

(i) What is the average number of customers in the system? [10%]

(ii) How much time should a customer expect to spend in the system? [10%]

(iii) What percentage of the time is the system idle? [10%]

(d) Consider the birth-and-death stochastic process with the following mean rates. The birth rate and the death rate are λ_n and μ_n, respectively, when the population size of the process is n.

(i) Develop the balance equations. [20%]

(ii) Solve these balance equations to find the steady-state probability distribution $p_0, p_1, \ldots$, as functions of birth rates and death rates. [20%]

3 (a) Consider the Markov chain that has the following (one-step) transition matrix.

State	**0**	**1**	**2**	**3**	**4**
0	0	0.2	0.5	0.3	0
1	0	0	0	1	0
2	0	0.2	0	0.1	0.7
3	0	1	0	0	0
4	0.8	0.1	0	0.1	0

(i) Draw the transition network. Determine the classes of this Markov chain. [15%]

(ii) For each of the classes identified in part (i), determine the period of the states in that class. [10%]

(iii) Is this Markov chain irreducible or aperiodic or regular? Explain your answers. [10%]

(b) A production process contains a machine that deteriorates rapidly in both quality and output under heavy usage, so much that it is inspected at the end of each day. Immediately after inspection, the condition of the machine is noted and classified into one of four possible states: New, OK, Worn or Fail. The process can be modeled as a Markov chain, which is regular, with its one-step transition matrix P given by

State	**New**	**OK**	**Worn**	**Fail**
New	0	0.9	0.1	0
OK	0	0.6	0.3	0.1
Worn	0	0	0.6	0.4
Fail	1	0	0	0

(i) Find the steady-state probabilities. [20%]

(ii) If the costs of being in states New, OK, Worn and Fail are 0, $1,000, $3,000, and $6,000, respectively, what is the long run expected average cost per day? [10%]

(iii) Derive the expected first passage time formula. Find the expected first passage time from states New to New, that is, the expected length of time a machine can be used before it must be replaced. [35%]

4 (a) The demand for a product in each of the last five weeks is shown below.

Week	1	2	3	4	5
Demand (in thousand)	13	17	19	23	24

(i) Use a two-week moving average to generate a forecast for demand in week 6. [10%]

(ii) Apply exponential smoothing with a smoothing constant of 0.8 to generate a forecast for demand in week 6. [10%]

(iii) What criteria do you use to judge which forecasting method is preferred to other forecasting methods? [5%]

(b) The Independent Estate has recently done a study of homes that have sold in the Liverpool area within the past 18 months. Data were recorded for the asking price and the number of weeks the home was on the market before it sold. A random sample of 17 houses, which were sold at the asking price with prices ranging between £50,000 and £150,000, has been used to develop the Excel regression results as shown in the table below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.705948
R Square	0.498363
Adjusted R Square	0.464921
Standard Error	11.964179
Observations	17

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2133.111647	2133.11164	14.902111	0.001541
Residual	15	2147.123648	143.14157		
Total	16	4280.235294			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-16.225062	12.202527	-1.329648	0.203502	-42.234148	9.784024
Asking Price	0.000528	0.000137	3.860325	0.001541	0.000237	0.000820

(i) What is the regression model estimated? Explain what the variables and constants mean, if appropriate. [10%]

(ii) Define and discuss the correlation coefficient. What is the correlation coefficient of this example? [5%]

(iii) Provide a 95% confidence interval for the slope of the regression equation. [5%]

(iv) Use the regression equation to predict the number of weeks a house for which the asking price is £100,000 will be on the market before it is sold. Give a rough 95% confidence interval for the predicted number of weeks on the market before the house is sold. Interpret your result intuitively. [10%]

(v) Can you use the above regression equation to predict the number of weeks on the market before the house is sold at a price of about £250,000? Explain your answer. [5%]

(vi) Why is the slope in the regression equation a random variable? What type of probability distribution does this random variable have? Derive the mathematical formulae for the mean and the standard deviation of the slope random variable assuming that the standard error S_e is given. [25%]

(c) Discuss the underlying assumptions of the simple linear regression. [15%]

END OF PAPER