

ENGINEERING TRIPOS PART IIB

Module 3C6 Examination, 2006

Answers

$$1 \quad (\text{b}) \quad x=0, \theta=0; \quad x=L, d\theta/dx=0. \quad \omega_n = \sqrt{\frac{G}{\rho}} \left(n + \frac{1}{2} \right) \frac{\pi}{L}; \quad u_n(x) = \sin \left(n + \frac{1}{2} \right) \frac{\pi x}{L}$$

$$(\text{c}) \quad K \frac{\partial^2 \theta}{\partial t^2} = -GJ \frac{\partial \theta}{\partial x} \quad \text{at } x=L; \quad \tan kL = \frac{J\rho}{Kk}, \quad \text{with } k^2 = \frac{\rho\omega^2}{G}$$

$$2 \quad (\text{b}) \quad \omega_n^2 \approx \left(n + \frac{1}{2} \right)^4 \pi^4 \frac{EI}{mL^4}; \quad (\text{c}) \quad \omega^2 = 504 \frac{EI}{mL^4}$$

$$3 \quad (\text{a}) \quad T = \frac{1}{2} \left(\frac{3}{2} m \dot{y}_1^2 + \frac{3}{2} m \dot{y}_2^2 \right); \quad V = \frac{1}{2} k y_1^2 + \frac{1}{2} k (y_2 - y_1)^2; \quad \frac{3m}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad k \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$(\text{b}) \quad \omega^2 = \frac{k}{m} \left(1 \pm \frac{\sqrt{5}}{3} \right); \quad [0.618 \quad 1]^T; \quad [-1.618 \quad 1]^T$$

$$(\text{c}) \quad 0.33 \text{ rads}; \quad (\text{d}) \quad 0.28 \text{ rads}$$

$$4 \quad (\text{a}) \quad T = \frac{1}{2} J (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 + \dot{\theta}_4^2 + \dot{\theta}_5^2)$$

$$V = \frac{1}{2} k [(\theta_2 - \theta_1)^2 + (\theta_2 - \theta_1)^2 + (\theta_3 - \theta_2)^2 + (\theta_4 - \theta_3)^2 + (\theta_5 - \theta_4)^2]$$

$$(\text{c}) \quad \omega_1^2 \approx 0.4 \frac{k}{J}$$