PART IIA 2006
3A1: Fluid mechanics I (double module)
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Datasheet: Applications of External Flows; Viscous Flow & Boundary Layers Data Card; Incompressible Flow Data Card; Two-dimensional Flow

3A1 Fluid Mechanics 1 Exam Cribs 2006

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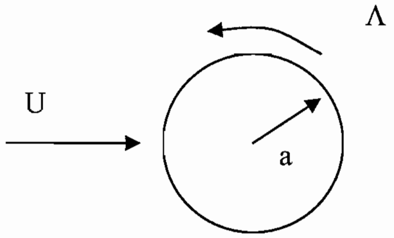


Doublet + vortex +free stream

(a) <u>doublet:</u> $F(z) = \frac{\mu}{2\pi z}$ at $z = -a$ $\frac{d}{\partial z}\left(\frac{\mu}{2\pi z}\right) = -U$ (stagnation)

$\therefore \frac{-\mu}{2\pi a^2} = -U \Rightarrow$ doublet strength, $\mu = 2\pi U a^2$.

<u>Vortex:</u> v_θ on surface of cylinder = $a\Omega$ $\therefore \oint \bar{u}.d\ell = 2\pi a^2 \Omega$

$\Rightarrow$ vortex circulation, $= 2\pi a^2 \Omega$.

(b) <u>Complex potential,</u>

$$F(z) = Uz + \frac{2\pi a^2 U}{2\pi z} + \frac{i\left(-2\pi a^2 \Omega\right)}{2\pi}\log z$$

$$\therefore F(z) = U\left(z + \frac{a^2}{z}\right) - ia^2\Omega \log z.$$

(c) for <u>stagnation points</u> $\frac{dF}{\partial z} = 0 = U\left(1 - \frac{a^2}{z^2}\right) - i\frac{a^2\Omega}{z}$

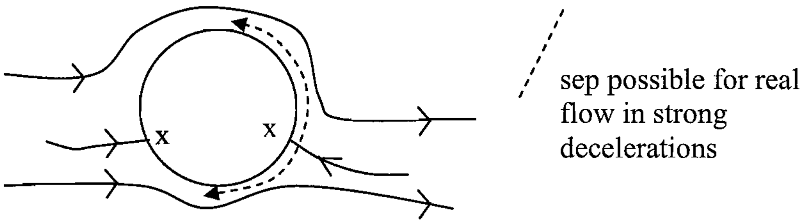
$$\therefore z - \frac{ia^2}{U}\Omega z - a^2 = 0$$

$$z = \left(\left(\frac{ia^2\Omega}{U}\right) = \sqrt{-\left(\frac{a^2\Omega}{U}\right)^2 + 4a^2}\right) \Big/ 2$$

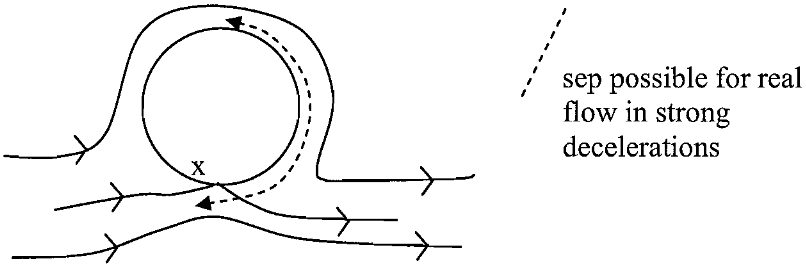
$$\therefore \frac{z}{a} = i\left(a\Omega/2U\right) \pm \sqrt{1 - \left(a\Omega/2U\right)}$$

Hence, depending on value of $\frac{a\Omega}{2U}$:

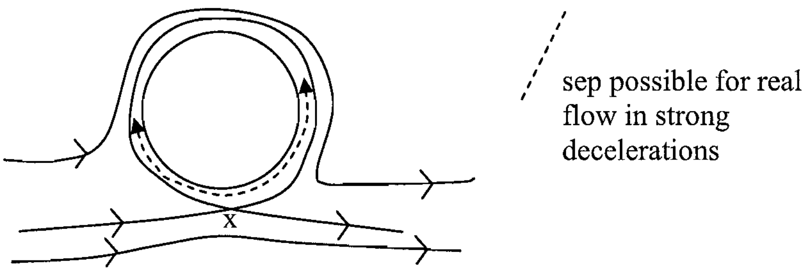
(i) $\frac{a\Omega}{2U} < 1$: **two** stagnation points on the surface of cylinder:



(ii) $\frac{a\Omega}{2U} = 1$: **one** stagnation point on the cylinder



(iii) $\frac{a\Omega}{2U} > 1$: **no** stagnation point on cylinder but in flow



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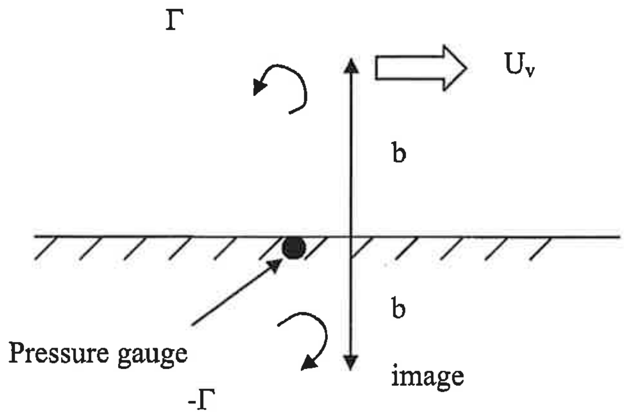


(a) from image, $U_v = \dfrac{\Gamma}{2\pi(2b)} = \dfrac{\Gamma}{4\pi b}$

(b) relative to moving vortex system

$$F(z) = \underline{(-U_v)}z - \frac{i\Gamma}{2\pi}\log\left(\frac{z-ib}{z+ib}\right)$$

Effective free stream

(c) hence, $\dfrac{dF}{\partial z} = -\dfrac{\Gamma}{4\pi b} - \dfrac{i\Gamma}{2\pi}\cdot\dfrac{1}{z-ib} + \dfrac{i\Gamma}{2\pi}\dfrac{1}{z+ib}$

Along the wall, z = x

$$\therefore \left.\frac{dF}{\partial z}\right|_{\substack{wall \\ y=0}} = -\frac{\Gamma}{2\pi}\left[\frac{1}{2b} + \frac{i(x+ib) - i(x-ib)}{(x+ib)(x-ib)}\right]$$

$$\therefore (u - iv)_{wall} = -\frac{\Gamma}{2\pi}\left[\frac{1}{2b} - \frac{2b}{x^2+b^2}\right]$$

NOTE: u=0, wall; as $x\to\infty$, $u\to\Gamma/4\pi b$

d) Stagnation pressure of the steady flow is $p_\infty + \frac{1}{2}\rho\left(\frac{\Gamma}{4\pi b}\right)^2$

So (Bernoulli): $p + \frac{1}{2}\rho\left(\frac{\Gamma}{4\pi b}\right)^2\left[\frac{4b^2}{x^2+b^2} - 1\right]^2 = p_\infty + \frac{1}{2}\rho\left(\frac{\Gamma}{4\pi b}\right)^2$

$$p - p_\infty = \frac{\rho\Gamma^2}{32\pi^2 b^2}\left[1 - \left(\frac{16b^4}{(x^2+b^2)^2} - \frac{8b^2}{x^2+b^2} + 1\right)\right]$$

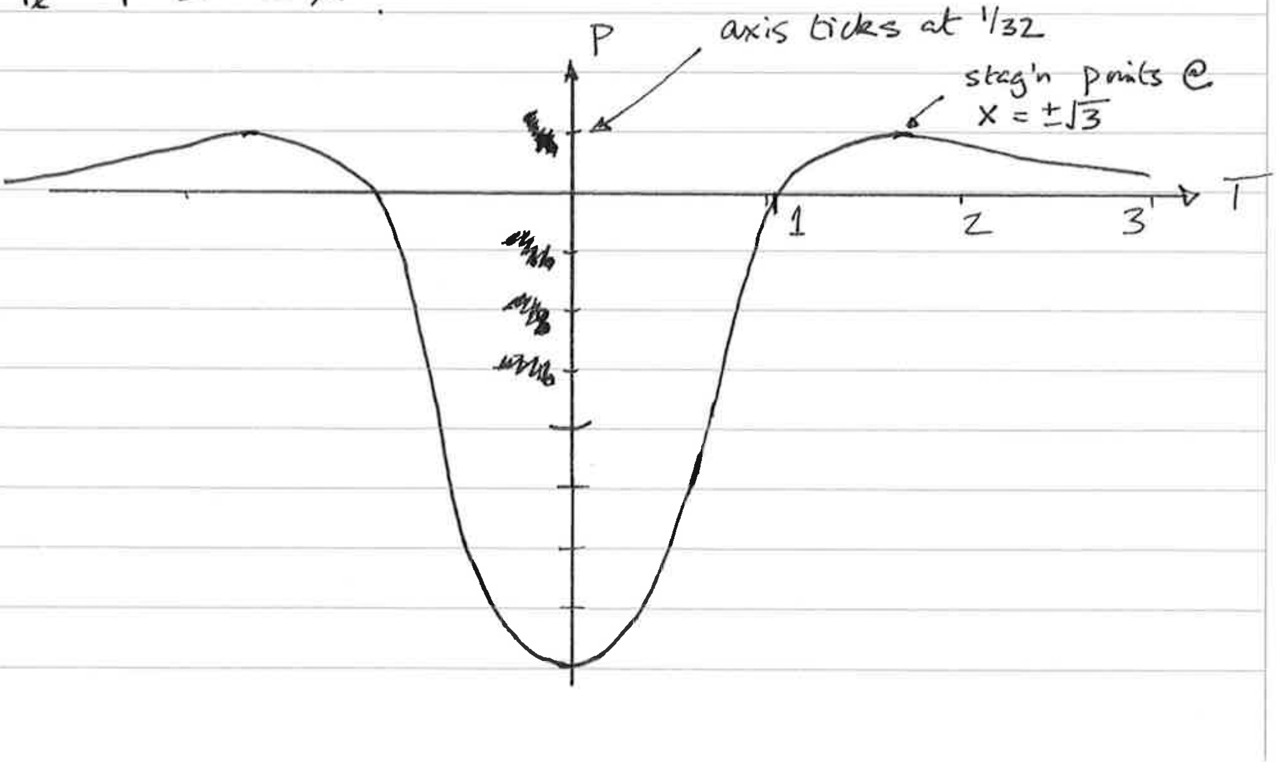
$$= \frac{\rho\Gamma^2}{4\pi^2 b^2}\,\frac{b^2}{x^2+b^2}\left[1 - \frac{2b^2}{x^2+b^2}\right]$$

$$P = \frac{p - p_\infty}{\rho\Gamma^2/\pi^2 b^2} = \frac{1}{4}\,\frac{1}{1+X^2}\left[1 - \frac{2}{1+X^2}\right] \qquad X = x/b$$

Now consider a stationary observer, with $X = 0$ @ $t = 0$

At time t, the observer sees the pressure at $x = -\frac{\Gamma}{4\pi b}t$

ie $T = -X$.



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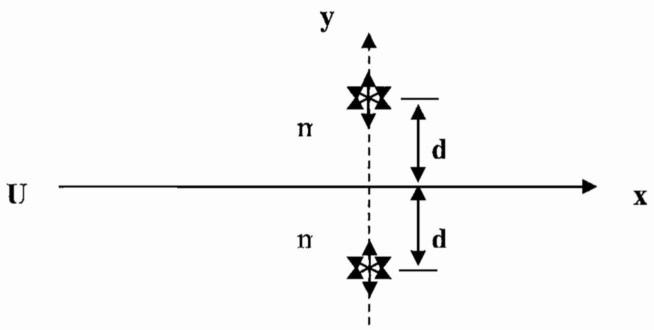


(a) $F(z) = Uz + \frac{m}{2\pi}\left[\log(z - id) + \log(z + ia)\right] = Uz + \frac{m}{2\pi}\log\left(z^2 + d^2\right)$

(b) stagnation points are where $\frac{dF}{\partial t} = 0$

$$\therefore \frac{d\mathrm{F}}{\partial \mathrm{t}} = U + \frac{m}{2\pi}\cdot\frac{1}{z - id} + \frac{m}{2\pi}\cdot\frac{1}{zid} = 0$$

$$\therefore \frac{U4\pi^2(z - id)(z + id) + 2\pi m(z + id) + 2\pi m(z - id)}{4\pi^2(z - id)(z + id)} = 0$$

$$\therefore 4\pi^2 U\left(z^2 + d^2\right) + 4\pi m z = 0$$

$$\therefore z^2 + \frac{m}{\pi U}z + d^2 = 0, \quad z = \frac{-\frac{m}{\pi U} \pm \sqrt{\left(\frac{m}{\pi U}\right)^2 - 4d^2}}{2}$$

$$\therefore z\Big|_{stag} = -\left(\frac{m}{2\pi U}\right) \pm \sqrt{\left(\frac{m}{2\pi U}\right)^2 - d^2}$$

(c) dye is released along the centre-line, y = 0

Dye from far upstream will pass between the source pair unless the stagnation point(s) are on the y = 0 axis

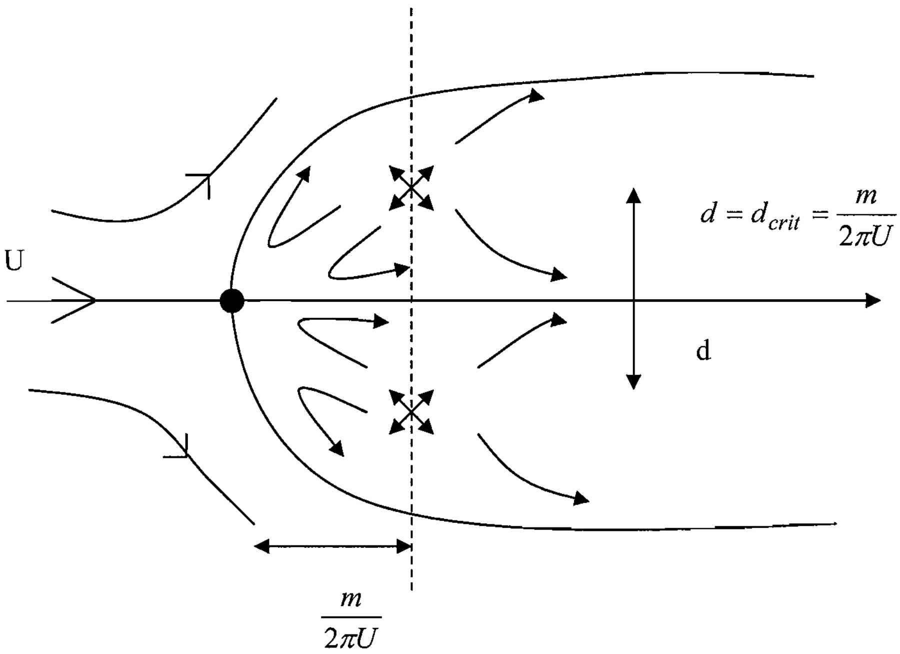
$\left(\frac{m}{2uU}\right)^2 < d^2$ (complex pair) off x-axis stag.points

$\therefore$ upstream flow passes between sources.

$d_{crit} = \dfrac{m}{2\pi U} \Leftarrow \left(\dfrac{m}{U}\right)^2 = d^2$ (coincident real) on x-axis stagnation point just blocks flow

$> d^2$ (real pair) on x-axis

(d) for the case $d = d_{crit}$, this is **also** the upstream, on-axis stagnation point:



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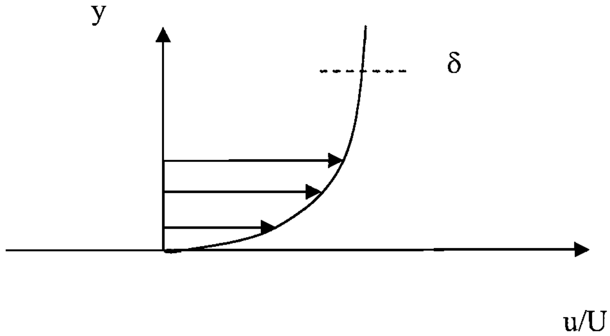
$$\frac{u}{U_\infty} = \left(\frac{Y}{\delta}\right)^{\frac{1}{7}}$$



(a)

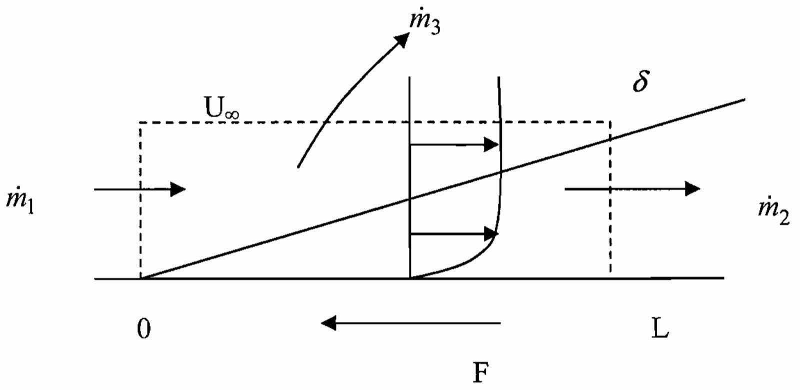
$$\frac{u}{U_\infty} = \left(\frac{Y}{\delta}\right)^{\frac{1}{7}} \quad @\ Y = 0, u = 0$$

$$@\ Y = 0, u = U_\infty$$

But $$@\ Y = 0, \left(\frac{\partial u}{\partial Y}\right) = \frac{U_\partial}{\delta^{\frac{1}{7}}} \cdot \frac{1}{7}(Y)^{-6/7} \rightarrow \infty!!!$$

This is NOT physical (in reality a laminar sublayer is in-between the turbulent layer and the wall)

(b)



for unit width

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = U_\infty \rho \int_o^\delta \left(\frac{Y}{\delta}\right)^{\frac{1}{7}} dY = U_\infty \rho \left(\frac{7}{8} \frac{1}{\delta^{\frac{1}{7}}} \left[Y^{\frac{8}{7}}\right]\right)_\partial^\delta$$

$$\therefore \dot{m}_3 = \frac{1}{8} \rho U_\infty \delta$$

Hence:

$$F = U_\infty^2 \rho \delta - \frac{1}{8} U_\infty^2 \rho \delta - \int_0^\delta U_\infty^2 \rho \frac{1}{\delta^{\frac{2}{7}}} Y^{\frac{2}{7}} dY$$

$$= \rho U_\infty^2 \delta \left[1 - \frac{1}{8} - \frac{1}{\delta^{9/7}} \cdot \frac{7}{9} \cdot \left[Y^{9/7}\right]\right]_o^\delta$$

$$\frac{7}{8} - \frac{7}{9} = \frac{\overset{7}{\cancel{63 - 56}}}{72}$$

$$\therefore F = \frac{7}{72}\rho U_\infty^2 \delta$$

Hence, $C_F = \dfrac{F}{\frac{1}{2}\rho U_\infty^2 L} = \dfrac{7}{36}\cdot\dfrac{\delta}{L}$

(c) Prandtl proposed $C_F \sim 0.05\,\mathrm{Re}_\delta^{-\frac{1}{4}}$

$$\therefore \frac{7}{36}\cdot\frac{\delta}{L} \sim 0.05\frac{\upsilon^{\frac{1}{4}}}{U_\infty^{\frac{1}{4}} 6^{\frac{1}{4}}}$$

$$\therefore 3.89\frac{\partial^{\frac{5}{4}}}{L^{\frac{5}{4}}} \sim \frac{\upsilon^{\frac{1}{4}}}{U_\infty^{\frac{1}{4}} L^{\frac{1}{4}}}$$

$$\therefore \frac{\delta}{L} \sim (3.89)^{-\frac{4}{5}}\cdot\left(\frac{1}{\mathrm{Re}_L}\right)^{\frac{1}{5}}$$

$$\therefore \frac{\delta}{L} \sim 0.34\,\mathrm{Re}_L^{-0.2}$$

5 (a) Results from no-slip condition/observation for real fluid with viscosity. Region over which fluid velocity goes from zero (at surface) to free-stream velocity
BUT
Under many conditions (high Reynolds number) this region is very thin compared with the streamwise length scale or physical size of surface.

Useful because a flow can often by divided into an inviscid flow and a boundary layer. Simplified equations are appropriate for these two regions that many be easily solved and solutions matched at the interface between the regions.

(b) <u>Momentum transfer</u>
– laminar flow; molocular viscosity
- turbulent flow; mass transfer by turbulent eddies that carry momentum.

At the surface
- laminar flow – velocity gradient x viscosity
- turbulent flow/smooth wall
    - o viscous sublayer – vel. grad x viscosity
- turb/rough wall
    - o form drag on roughness elements

Heat Transfer
As for momentum transfer except replace dynamic viscosity with thermal conductivity **except** for turbulent flow over rough wall. There is no thermal equivalent of "pressure differences" for form drag. Therefore surface heat transfer must be by molocular thermal conductivity alone.

Growth rates
Turbulent >> Laminar due to transfer by fluid elements rather than molocular effects.

Rough wall > Smooth wall

Roughness courses more turbulence and thus more transfer by fluid element.

Surface shear stress
Use of steady flow momentum equation stress that the larger the boundary layer growth rate the large the momentum deficit in the boundary layer and consequently the larger the surface shear stress.

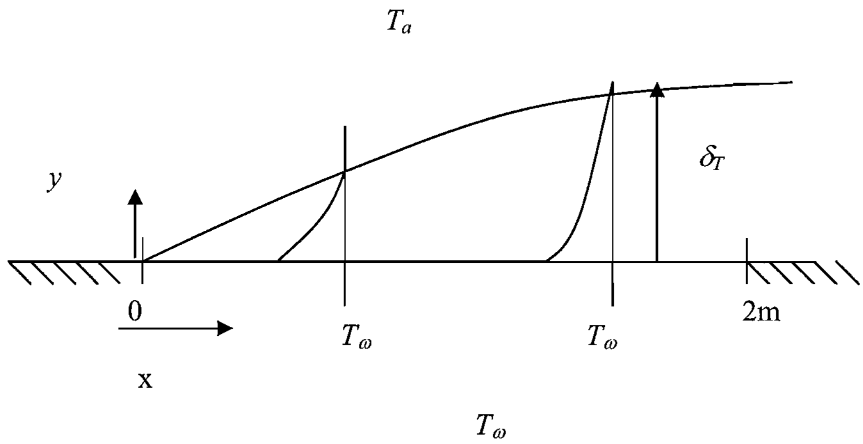
Heat Fluxes
As for surface shear stress **except** for rough wall where the lack of an equivalent to pressure may not make the rough wall case the best heat transfer – depends on roughness geometry – fins good; lateral bluff obstacles bad.

(c) Essentially it depends on the ration of the roughness dimension k to the thickness of the viscous sublayer ($\sim 10\, \nu/u_x$)

$k << 10\, \nu/u_x$ smooth eg $k < 5\, \nu/u_x$

$k >> 10\, \nu/u_x$ rough eg $k > 50\, \nu/u_x$

6 (a)



(b) $$\frac{T_a - T}{T_a - T_\omega} = f\left(\frac{y}{\delta_T}\right) = a + b\left(\frac{y}{\delta_T}\right) + c\left(\frac{y}{\delta_T}\right)^2$$

At $y = 0$ $T = T\omega$; At $y = \delta_T$ $T = Ta$

$$\left.\frac{\delta T}{\delta y}\right|_{y=0} = 0$$

Hence $a = 1$ $b = -2$ $c = 1$

$$\frac{T_a - T}{T_a - T_\omega} = 1 - 2\left(\frac{y}{\delta_T}\right) + \left(\frac{y}{\delta_T}\right)^2$$

(c) Steady flow energy equation gives

$$\frac{\partial}{\partial x}\int_0^{\delta_T} \rho C_p U(T - T_a)dy = -k\left.\frac{\partial(T - Ta)}{\partial y}\right|_{y=o}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{3}\delta_T\right) \bullet (T_\omega - T_a) = -\frac{\alpha}{U}\left(-\frac{2}{\delta_T} \bullet (T_\omega - T_a)\right)$$

$$\frac{\partial}{\partial x}(\delta_T) = \frac{6\alpha}{U}\left(1/\delta_T\right)$$

$$\delta_T \delta(\delta_T) = \frac{6\alpha}{U} \bullet \delta x$$

$$\frac{1}{2}\delta_T^2 = \frac{6\alpha x}{U} + C$$

$$\delta_T = \left(\frac{12\alpha x}{U}\right)^{\frac{1}{2}}$$

Surface heat flux density $= -k \left.\frac{\partial(T - Ta)}{\partial y}\right|_{y=0}$

$$= -k(T_\omega - T_a) \bullet \left(-\frac{2}{\delta_T}\right) = 2k\frac{T_\omega - Ta}{\delta T} = 2k(T_\omega - Ta)\left(\frac{12\alpha x}{U}\right)^{-\frac{1}{2}}$$

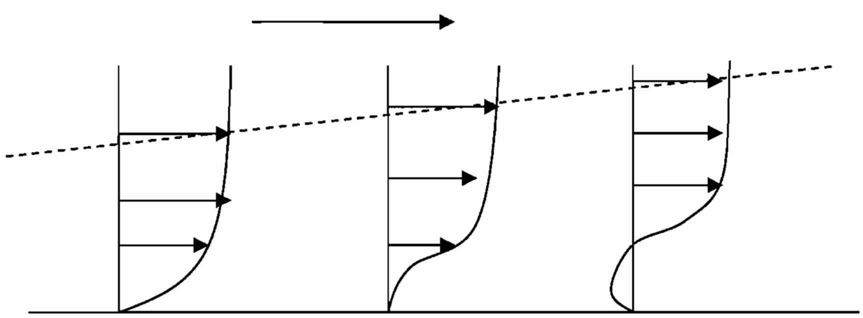
(d) Total heat flux = width x $x\int_0^L 2k(T_\omega - Ta)\left(\frac{12\alpha}{U}\right)^{-\frac{1}{2}} x^{-\frac{1}{2}} dx$

$$= \text{width } x2k(T_\omega - Ta) \bullet \left(\frac{12\alpha}{U}\right)^{-\frac{1}{2}} \quad 2L^{\frac{1}{2}} \text{ watts}$$

$$= 3\text{m x 2 x } 0.024\frac{\text{w}}{\text{mK}}\text{x15x}\left(\frac{12\text{x}19\text{x}10^{-6}}{1}\right)^{-\frac{1}{2}}\text{x2x}(2)^{\frac{1}{2}} = 404.5 \text{ watt}$$

$$\text{Cost} = \left(\frac{404.5}{1000}\right)\text{x}(90\text{x}24\text{hours})\text{x}(12\text{p per kw} - \text{hr}) = £104.80\text{p}$$

7



Roughly Δp+ρuΔu~0
for an adverse pressure gradient $\Delta p > 0 \therefore \Delta u < 0$ the flow must decelerate to support the Δp and $\Delta u \sim -\frac{\Delta p}{\rho u}$; hence where u is **low**, near the wall, a larger Δu is needed; eventually $|\Delta u| > |u|$ and so reverse flow occurs.

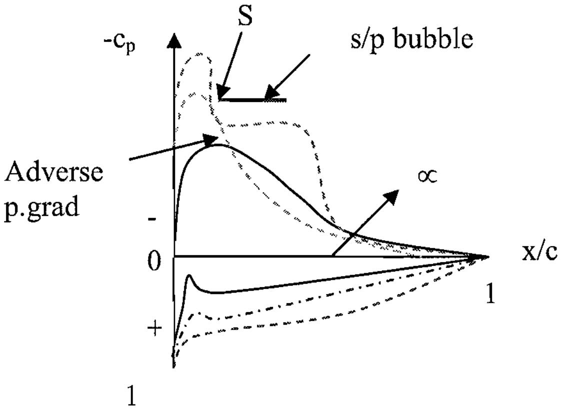
(b) on an airfoil there are **two** main regions susceptible to boundary layer separation (on the upper surface):

(i) downstream of the LE suction peak (which may result in a laminar separation bubble followed by turbulent re-attachment, or the bubble may burst).

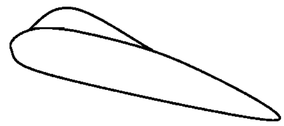
(ii) towards the TE after the boundary layer has experienced a long stretch of adverse pressure gradient.

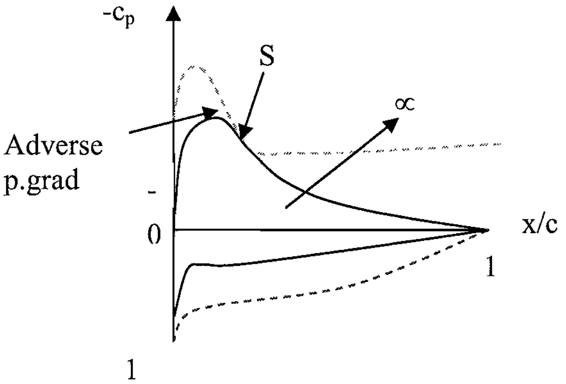
This behaviour is seen in $c_p(x)$ plots as:

LE sep

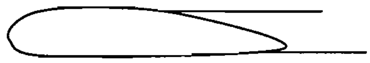


LE seperation with reattachment

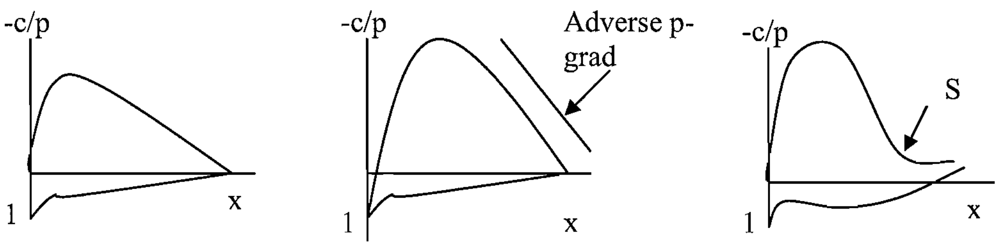




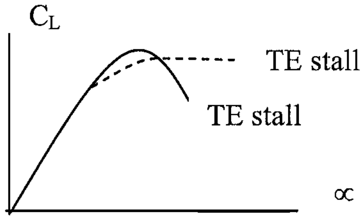
LE separation but NO reattachment



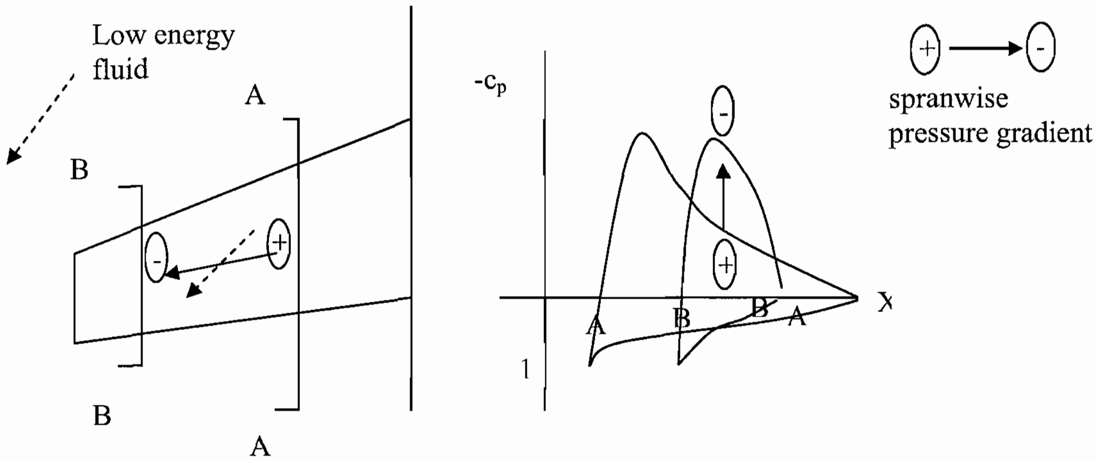
TE sep

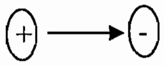


Hence, stalling behaviour of section differs as LE separation is sudden, TE separation is more gradual

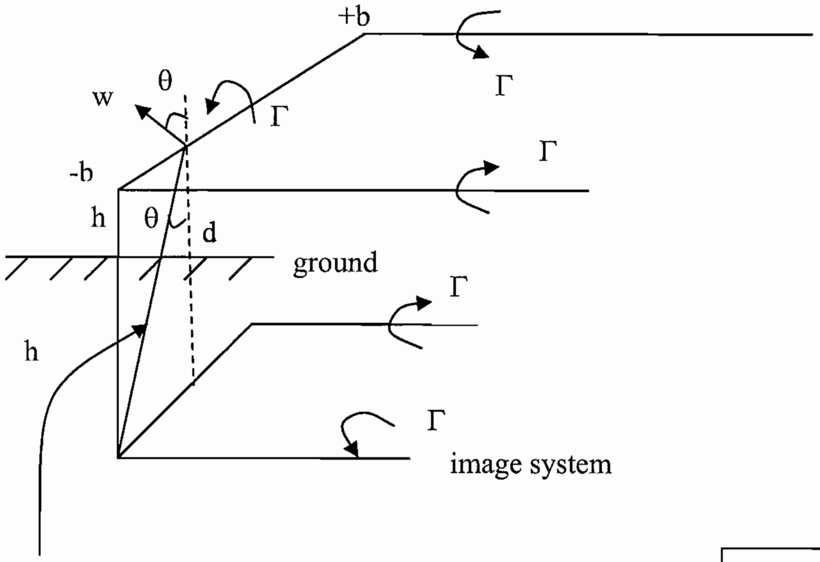


(c) Stall on **swept** wings – and boundary layer behaviour in general – is influenced strongly by spanwise migration of low energy fluid (boundary fluid) driven by the spanwise pressure gradient; this also impacts significantly on the transitional behaviour of laminar boundary layers.



The  and p. grad. Tends to drive low energy fluid towards the tips - this benefits the stalling behaviour of the wing root but is detrimental to the performance of the tips which may stall early.

8



in groud effect
$h<<b \therefore \cos\theta \sim 1$

$$\sqrt{(2h)^2 + b^2} = b\sqrt{1 + 4(h/b)^2}$$

The **up**wash from the image system, $w = 2x\frac{1}{2}x\frac{\Gamma}{2\pi b\sqrt{1+4(k/b)^2}}$

2 off

semi-infinite

"simple approximations"

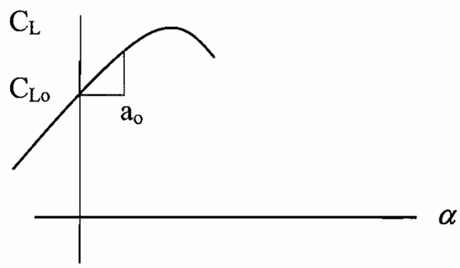
$\cos\theta \sim 1$

The wing lift, $C_L = C_{L_o} + a_o\left(\alpha - \frac{w}{U}\right)$

downwash

2π for thin airfoils in 2D

$$\therefore \Delta C_L = \frac{a_o}{U}\cdot\frac{\Gamma}{2\pi b}\cdot\frac{1}{\sqrt{1+4(\ )^2}}$$



$$\left.\begin{aligned} L &= \rho U \Gamma_x 2b \\ AR &= 4b^2 / A \\ C_L &= \frac{L}{\frac{1}{2}\rho u^2 A} \end{aligned}\right\} \quad \therefore \quad \Delta C_L = \frac{a_o}{2\pi b} \cdot \frac{1}{U} \cdot \frac{L}{\rho U 2b} \cdot \frac{1}{\sqrt{\ldots}}$$

$$= \frac{L}{\rho U^2} \cdot \frac{a_o}{\pi 4b^2} \cdot \frac{1}{\sqrt{\ldots}} \cdot \frac{1}{\frac{1}{2}A} \cdot \frac{A}{2}$$

$$\therefore \frac{\Delta C_L}{C_L} = \frac{a_0}{2\pi} \cdot \frac{1}{AR} \cdot \frac{1}{\sqrt{1 + 4(h/b)^2}}$$

For, say h/b ~ 0.5 (typical high downforce configuration) with AR~3, $\frac{\Delta C_L}{C_L} \sim 24\%$!!!

Extreme pitch-sensitivity makes the car difficult to drive!

WND
June '06