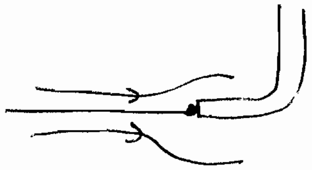
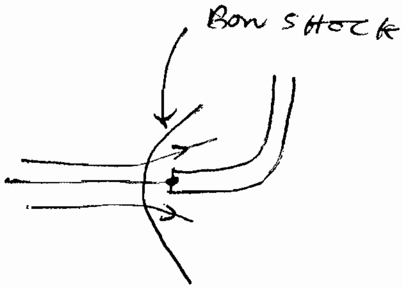
Model Solutions 3A3 2006
Dr R J Miller

PART IIA 2006
3A3: Fluid mechanics II (double module)
Principal Assessor: **Dr R Miller**

Datasheet: ***Compressible Flow Data Book***

1a





SUBSONIC

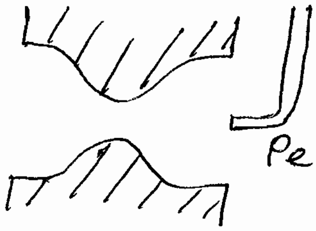
SUPERSONIC

P_o CONSTANT ALONG STREAMLINE

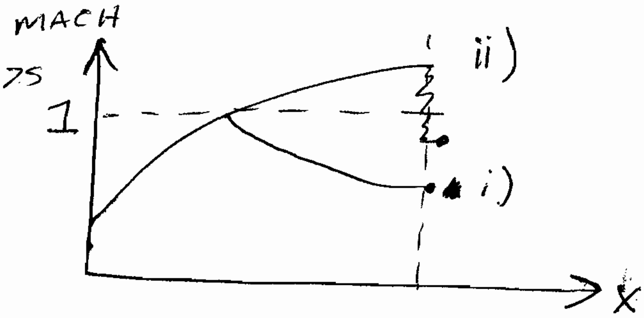
P_o DROP ACCROSS SHOCK DEPENDANT ON UPSTREAM MACHNUMBER

B

P_o



$\frac{A^*}{A_e} = 0.75$



i) $M = 1$ AT THROAT $\quad \frac{\dot m \sqrt{C_p T_o}}{A^* P_o} = 1.281$

AT EXIT $\quad \frac{\dot m \sqrt{C_p T_o}}{A_e P_o} = 1.281 \times \frac{A^*}{A_e}$

$= 0.9607$

CUED $\quad$ MACH $= 0.503$

OR MACH $= 1.696$

MACH 0.503

WITHOUT SHOCK AT EXIT $\frac{P_E}{P_o} = 0.2067$

MACH 1.696 $\quad \cancel{\frac{P_E}{P_o} = 0.2067}$

NORMAL SHOCK $\quad \frac{P_S}{P} = 3.1655$

$\frac{P_E}{P_o} = 0.2067 \times 3.1655 = \underline{\underline{0.6543}}$

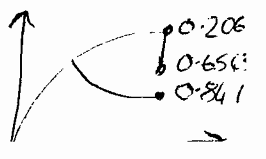
$\frac{P_{OS}}{P} = \underline{\underline{0.8575}} = \frac{P_{O PITOT}}{P_o}$

MACH 0.503 $\quad \frac{P_E}{P_o} = \underline{\underline{0.841}}$

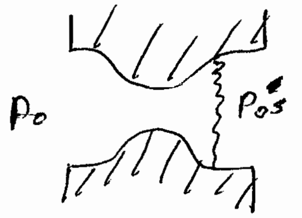
$\frac{P_{O PITOT}}{P_o} = \underline{\underline{1}}$

CASE ii) PITOT MEASURES 85.8% OF P_o

CASE i) PITOT MEASURES 100% OF P_o



1 (c) $\frac{P_E}{P_0} = 0.75$



NEED TO CALCULATE ΔP_0 SHOCK
SO NEED SHOCK STRENGTH

$$① \quad \frac{P_{03}}{P_0} = \frac{P_{0E}}{P_0} = \frac{\dot m\sqrt{C_pT_0}}{A^* P_0} \times \frac{A_E P_{0E}}{\dot m\sqrt{C_pT_0}} \times \frac{A}{A}$$

NEED TO KNOWN MACH EXIT

NEED MACH No AT EXIT

$$\frac{\dot m\sqrt{C_pT_0}}{A_E P_E} = \frac{\dot m\sqrt{C_pT_0}}{A^* P_0} \times \frac{A^*}{A_E} \times \frac{P_0}{P_E}$$

$$= 1.281 \times 0.75 \times \frac{1}{0.75} = 1.281$$

CALCD MACH EXIT $= 0.562$

CALCD $\frac{A_E P_{0E}}{\dot m\sqrt{C_pT_0}} = \frac{1}{1.034}$

FROM ① $\frac{P_{03}}{P_0} = 1.281 \times \frac{1}{1.034} \times 0.75$

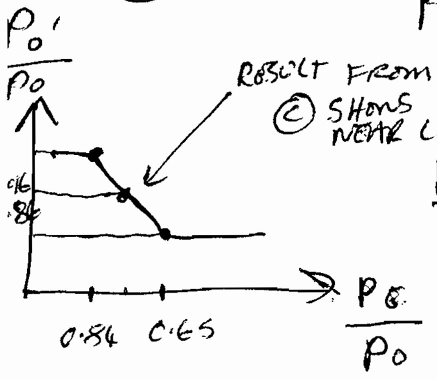
$= 0.93$

FLOW SUBSONIC AT EXIT SO $\frac{P_{0PITOT}}{P_0} = 0.93$

(D) FROM $\frac{P_E}{P_0} = 1 \rightarrow 0.84$

EXIT SUBSONIC SO

$\frac{P_{0PITOT}}{P_0} = 1$



$\frac{P_E}{P_0} = 0.84 \longrightarrow 0.65$

$\frac{P_{0PITOT}}{P_0}$ DROPS TO 0.86

THEN P_{0PITOT} FROZEN BECAUSE UPSTREAM MACH CONSTANT

(2a) $\frac{P_o}{P_E} = 1{\cdot}25$ $\quad\quad \frac{P_E}{P_o} = 0{\cdot}8$

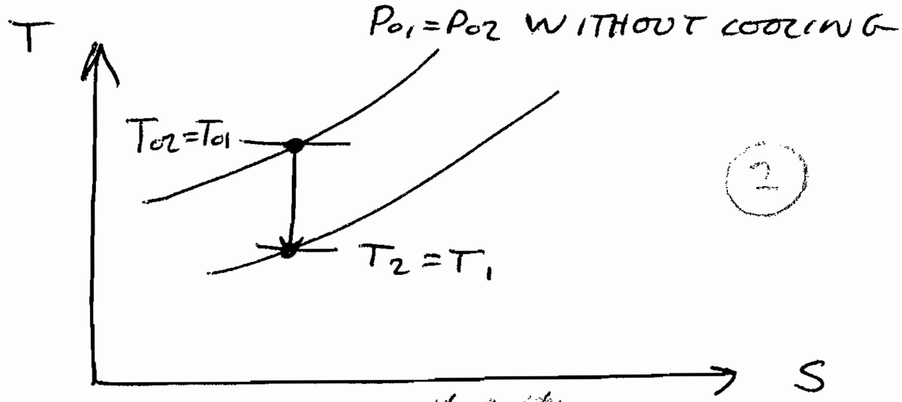
$M_1 = 0{\cdot}572$ CUED (1)

SUBSONIC FLOW COOLED DROPS IN MACH No SO COOLING OF PIPE DROPS MACH NUMBER. (2)

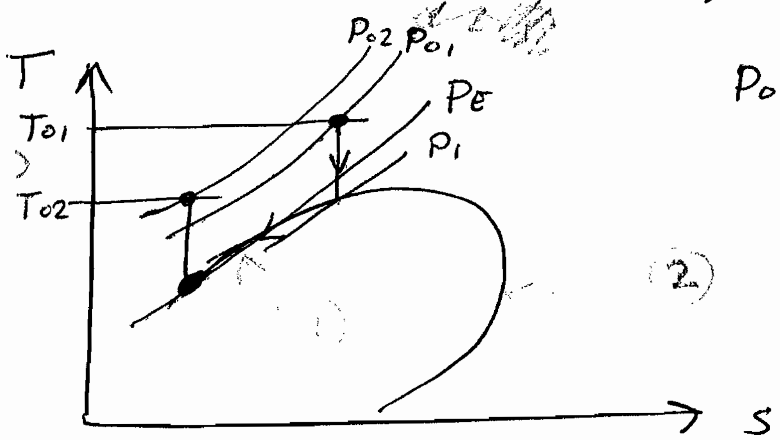
$M_2 < M_1$

[3]

(b)



(2)

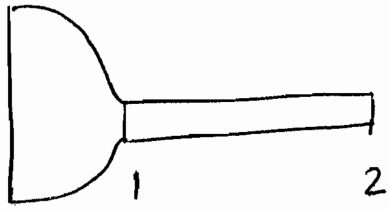


P_o RISES WITH COOLING

(2)

[6]

2 c



$$\frac{T_{02}}{T_{01}} = 0.85$$

$$M_2 = 0.572$$

AT 2 $\qquad \dfrac{F}{\dot{m}\sqrt{T_{02}C_p}} = 1.11$

F CONSTANT BECAUSE NO FRICTION

$$\frac{F}{\dot{m}\sqrt{T_{01}C_p}} = 1.11 \times \sqrt{\frac{T_{02}}{T_{01}}} = 1.11 \times \sqrt{0.85}$$
$$= 1.023$$

$$\underline{\underline{M_1 = 0.74}}$$

(1)

$$\frac{\dot{m}\sqrt{C_p T_{01}}}{A P_{01}} = 1.199 \qquad \frac{\dot{m}\sqrt{C_p T_{02}}}{A P_{02}} = 1.30$$

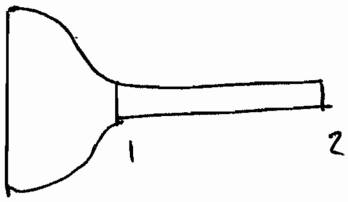
(2)

$$\Pi m_1 = \Pi m_2 \times \frac{P_{02}}{P_{01}} \times \sqrt{\frac{T_{01}}{T_{02}}}$$

$$\frac{P_{01}}{P_{02}} = \frac{\Pi m_2}{\Pi m_1} \times \sqrt{\frac{T_{01}}{T_{02}}} = \frac{1.30}{1.199} \times \sqrt{0.85} = 1.176$$

$$\underline{\underline{P_{01} = 1.17 \times P_{ATMOS}}}$$

2 d



$M_2 = 0.572$

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{A P_2} = 1.30$$

T_0 CHANGES WITH COOLING

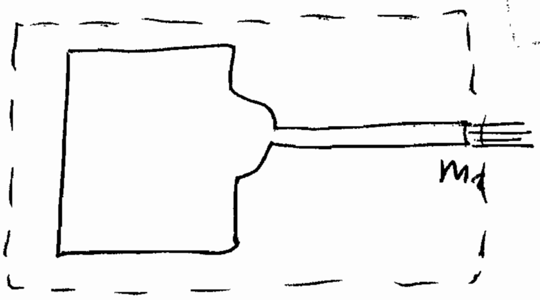
$$\left(\dot{m}\sqrt{T_{02}}\right)_{\text{COOLED}} = \left(\dot{m}\sqrt{T_{02}}\right)_{\text{UNCOOLED}}$$

$$\frac{\dot{m}_{\text{UNCOOLED}}}{\dot{m}_{\text{COOLED}}} = \sqrt{\frac{T_{02\,\text{COOLED}}}{T_{02\,\text{UNCOOLED}}}} = \sqrt{\frac{T_{02}}{T_{01}}}$$

8.5% CHANGE

$$= \underline{\underline{0.9220}}$$

3



$$\text{THRUST} = F - A P_a$$

$$\frac{F}{\dot{m}_2\sqrt{c_p T_{02}}} = f(M_2)$$

$\dot{m}\sqrt{T_{02}}$ = CONSTANT WHEN COOLED

2

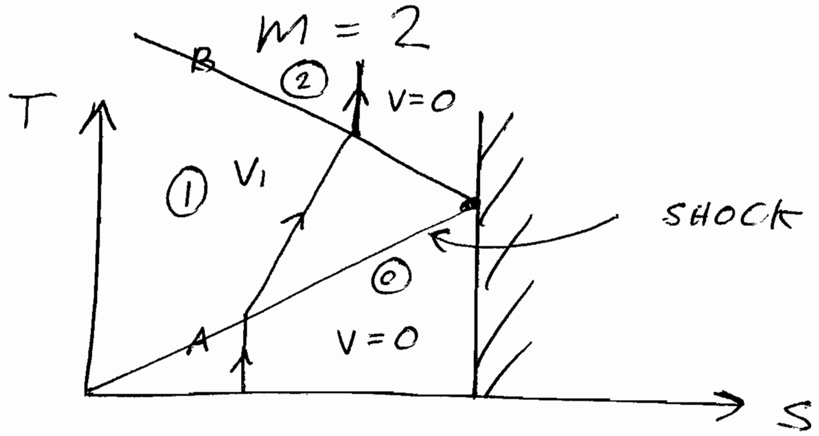
SO F_2 CONSTANT WHEN COOLED

SO THRUST SAME FOR BOTH CASES.

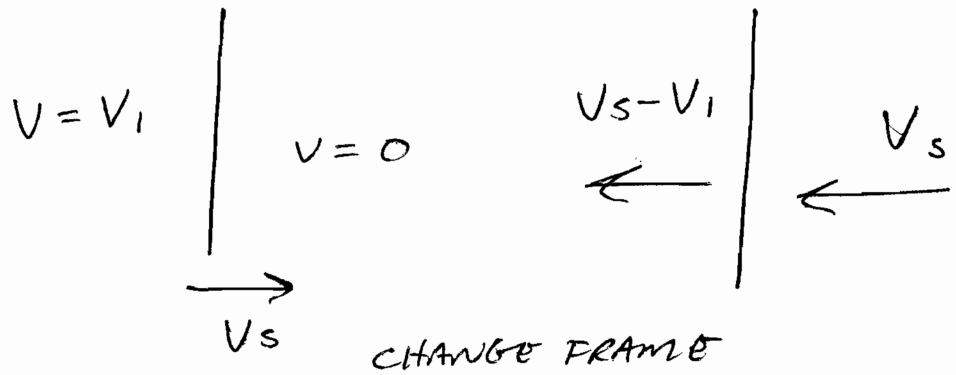
5

(3A)

$$\frac{P_s}{P} = 4.5 \quad \text{CUED}$$



(B)



CHANGE FRAME

$$\rho_0 V_s = (V_s - V_1)\,\rho_1 \quad \text{CONTINUITY}$$

$$\frac{\rho_0}{\rho_1} = \frac{V_s - V_1}{V_s} = \frac{P_0 T_1}{P_1 T_0} = \frac{1}{4.5} \times 1.6875$$
$$= 0.375$$

$$\therefore \quad V_s - V_1 = 0.375 V_s$$

$$V_1 = V_s - 0.375 V_s \quad (1)$$

MACH 2 $\quad V_s = 2a_0$

SUB INTO (1)

$$V_1 = (1 - 0.375) \times 2a_0 = \underline{1.25a}$$

CUED $M = 2$ $\quad \frac{T_s}{T} = 1.6875 \quad a_1 = a_0 \times \sqrt{1.687}$

$$a_1 = 1.299\,a_0$$

(3c) $$\boxed{\frac{\rho}{\rho_s} = \frac{2}{\gamma+1} \times \frac{1}{M_1^2} + \frac{\gamma-1}{\gamma+1}}$$

FLOW STOPS AFTER SHOCK

CHANGE FRAME

$$\rho_s V_{SB} = \rho (V_1 + V_{SB})$$

$$\frac{\rho}{\rho_s} = \frac{V_{SB}}{V_1 + V_{SB}} = \left(\frac{2}{\gamma+1}\right)\frac{1}{M^2} + \left(\frac{\gamma-1}{\gamma+1}\right)$$

$$M_{SB} = \frac{V_{SB} + V_1}{a_1}$$ IS MACH RELATIVE TO REFLECTED SHOCK

$$\frac{M_{SB} - V_1/a_0}{M_{SB}} = \left(\frac{2}{\gamma+1}\right)\frac{1}{M_{SB}^2} + \left(\frac{\gamma-1}{\gamma+1}\right)$$

$$\frac{2}{\gamma+1} M_{SB}^2 - \frac{V_1}{a_1} M_{SB} - \frac{2}{\gamma+1} = 0$$

$$M_{SB} = \left(\frac{V_1}{a_1} \pm \sqrt{\frac{V_1^2}{a_1^2} + \frac{16}{(\gamma+1)^2}}\right) \Big/ \frac{4}{\gamma+1}$$

FROM (b)
$$\frac{V_1}{a_1} = 0.9623$$

$$= \frac{0.9623 \pm \sqrt{(0.9623)^2 + \frac{16}{(2.4)^2}}}{\left(\frac{4}{\gamma+1}\right)}$$

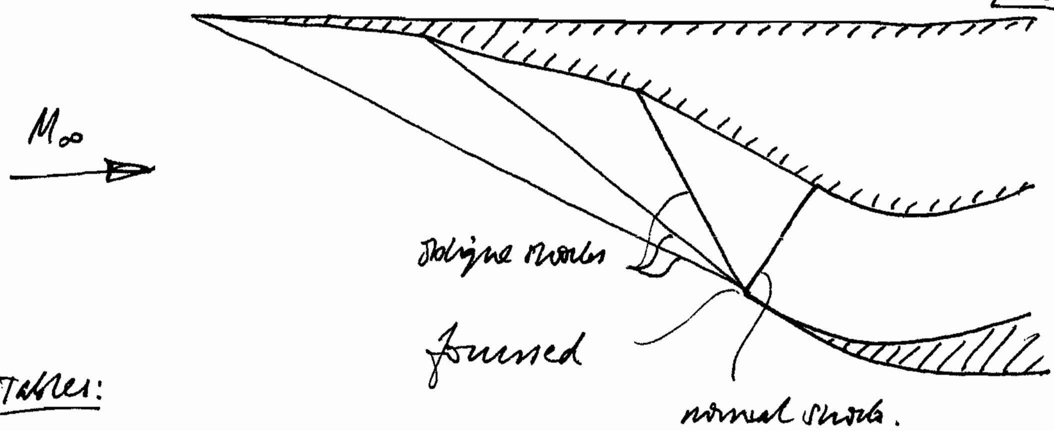
$$M_{SB} = 1.73$$

SHOCK STRENGTH $= \underline{1.73}$

SAS - 2D COMPRESSIBLE
Flow 2005 - 2006
J.P. JARRETT

4)

a)



From Tables:

b)

	δ	β	M_2	p_0/p
1	8	33.8	1.90	0.990
2	8	39.3	1.62	0.993
3	8	47.3	1.34	0.994
4			0.77	0.972

Mach No. downstream of normal shock = 0.77

c) relate oblique shock strength to equivalent normal shock

with u/stream Mach no = $M \sin\beta$

and relate to $p_{0\infty}$ via $p_0 = p\left[1 + \frac{1}{2}(\gamma-1)M^2\right]^{\frac{\gamma}{\gamma-1}}$

to give values in table above:

$$\therefore \; p_{0f}/p_{0\infty} = 0.990 \times 0.993 \times 0.994 \times 0.972 = 0.95$$

d) remove final oblique & normal & replace with a normal with

u/stream $M = 1.62$

$$\therefore \; p_{0f}/p_{0\infty} = 0.990 \times 0.993 \times 0.889 = 0.87$$

(4e) $M_\infty = 2$

From tables

	δ	β	M_2	$p_0/p_{0\infty}$
1	δ	37.2	1.71	0.993
2	δ	44.2	1.44	0.990
3				0.947

$$\Rightarrow \quad p_{0f}/p_{0\infty} = 0.993 \times 0.990 \times 0.947 = 0.93$$

The significant loss in pressure recovery resulting from simplifying the intake design is almost entirely regained by a relatively modest reduction in flight speed.

3A3 – 2D COMPRESSIBLE FLOWS 2005 – 2006
J. P. JARRETT

5) Flow is irrotational $V = (u, v)$

Mass: $\nabla \cdot (\rho V) = 0 \Rightarrow \nabla \cdot V + \frac{1}{\rho} V \cdot \nabla \rho = 0$

Energy: $\text{const} = h_0 = c_p T + \frac{1}{2}(u^2+v^2) = \frac{a^2}{\gamma-1} + \frac{1}{2}(u^2+v^2)$

and flow is isentropic : $p = k\rho^\gamma \Rightarrow dp = k\gamma \rho^{\gamma-1} d\rho \Rightarrow \nabla p = a^2 \nabla \rho$

Consider Euler (momentum): $-\frac{1}{\rho}\nabla p = V \cdot \Delta V$

Eliminate ∇p & $\Delta \rho$ $\Rightarrow \nabla \cdot V = -\frac{1}{\rho} V \cdot \nabla \rho = -\frac{1}{\rho a^2} V \cdot \nabla p = \frac{1}{a^2} V \cdot (V \cdot \nabla V)$

Rearranging gives $a^2 \nabla \cdot V - V(V \cdot \nabla V) = 0.$

In terms of components $V = (u, v)$ in x & y dirn.

$$a^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - [u\underline{e_x} + v\underline{e_y}] \cdot \left[\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\underline{e_x} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\underline{e_y}\right] = 0$$

$$a^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - u\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - v\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = 0$$

$$(a^2 - u)\frac{\partial u}{\partial x} - uv\frac{\partial u}{\partial y} - vu\frac{\partial v}{\partial x} + (a^2 - v^2)\frac{\partial v}{\partial y} = 0$$

Flow is irrotational $\therefore \nabla \times V = 0 \Rightarrow \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

$$\therefore (a^2 - u^2)\frac{\partial u}{\partial x} - 2uv\frac{\partial u}{\partial y} + (a^2 - v^2)\frac{\partial v}{\partial y} = 0$$

2 cont

Subs: $V = \left(U_\infty + \frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y} \right)$

$$\left[a^2 - \left(U_\infty + \frac{\partial\phi}{\partial x}\right)^2\right]\frac{\partial^2\phi}{\partial x^2} - 2\left[U_\infty + \frac{\partial\phi}{\partial x}\right]\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y} + \left[a^2 - \left(\frac{\partial\phi}{\partial y}\right)^2\right]\frac{\partial^2\phi}{\partial y^2} = 0$$

$$\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y} << U_\infty$$

$\therefore$ simplify to $\left[a_\infty^2 - U_\infty^2\right]\frac{\partial^2\phi}{\partial x^2} + a_\infty^2\left(\frac{\partial^2\phi}{\partial y^2}\right) = 0$

$$\Rightarrow \beta^2 \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

where $\beta = \sqrt{1 - M_\infty^2}$

This is the term which enables compressible C_p, C_L & C_m to be evaluated from the incompressible equivalents via the Prandtl-Glauert compressibility correction.

Num 6(a) Hyperbolic equations are 'wave-like' and solution information travels along specific directions with characteristic speeds. Upwind differences are used for spatial derivatives and use values at the current position and upstream (in the characteristic direction). When combined with forward time differencing, they preserve this information property – i.e. that information can only travel in the downstream characteristic direction.

(b)
$$u_i^{n+1} = u_i^n + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2!}\frac{\partial^2 u}{\partial t^2} + O(\Delta t^3)$$

[All derivs evaluated at i, n]

$$u_{i-1}^n = u_i^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 u}{\partial x^2} + O(\Delta x^3)$$

Substituting into (1)

$$\Rightarrow u_i^n + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2!}\frac{\partial^2 u}{\partial t^2} + O(\Delta t^3)$$
$$= u_i^n - c\left(u_i^n - u_i^n + \Delta x \frac{\partial u}{\partial x} - \frac{\Delta x^2}{2!}\frac{\partial^2 u}{\partial x^2} + O(\Delta x^3)\right)$$

$$\Rightarrow \frac{\partial u}{\partial t} = -\frac{\Delta t}{2}\frac{\partial^2 u}{\partial t^2} + O(\Delta t^2) - \frac{A}{\Delta x}\left[\Delta x \frac{\partial u}{\partial x} - \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + O(\Delta x^3)\right]$$

$$\Rightarrow \frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = -\frac{\Delta t}{2}\frac{\partial^2 u}{\partial t^2} + \frac{A\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + O(\Delta t^2, \Delta x^2)$$

$$= -\frac{A^2 \Delta t}{2}\frac{\partial^2 u}{\partial x^2} + \frac{A\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + O(\Delta t^2, \Delta x^2)$$

using hint.

$$= \frac{A\Delta x}{2}\left[1 - c\right]\frac{\partial^2 u}{\partial x^2} + O(\Delta t^2, \Delta x^2)$$

i.e. $\alpha = \frac{A\Delta x}{2}(1-c)$ & scheme is 1st order

unless $c=1$ when 2nd order

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⑥ (c)
(iii) Scheme is unstable if perturbation at $n+1$ is bigger (in magnitude) than at n.

For a perturbation of the form $u_i^n = (-1)^i \varepsilon$ $\left(\Rightarrow u_{i-1}^n = -u_i^n\right)$

$$u_i^{n+1} = \varepsilon(-1)^i - c\,\varepsilon(-1)^i\left[1+1\right]$$

$$\therefore \left|\frac{u_i^{n+1}}{\varepsilon}\right| = |1-2c|$$

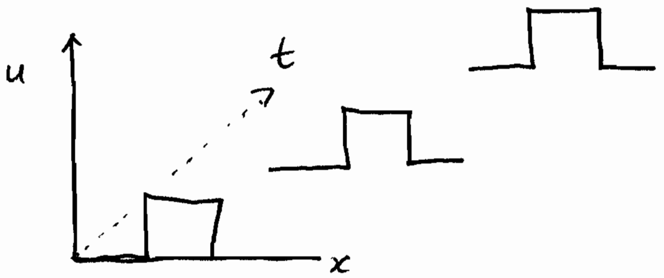
Stable only if $|1-2c| \le 1$ i.e. $-1 \le 1-2c \le 1$

$$\Rightarrow \quad 0 \le c \le 1$$

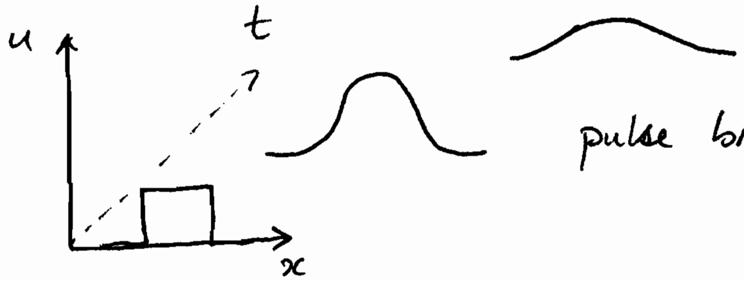
(c) (i) $c = 1$ $\quad \alpha \dfrac{\partial^2 u}{\partial x^2}$ is a diffusion term

and $\alpha = 0$ for $c = 1$

$\therefore$ Very little spread/reduction [in fact difference scheme is exact]



(ii) $c = 1/2$ Dominant error is diffusion



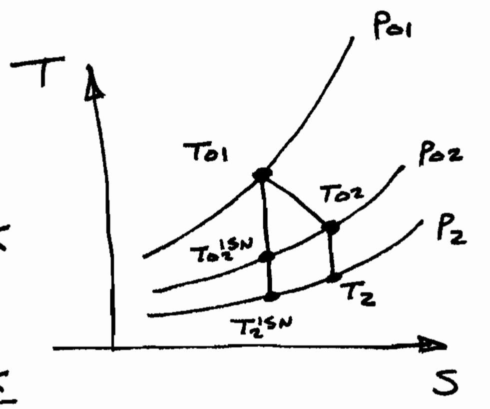
pulse broadens and reduces in height.

(7b)

HALF QUESTION

(a) $\text{POWER} = \dot{m}\Delta h_0$

$$\Rightarrow \Delta T_0 = \frac{\text{POWER}}{\dot{m} C_p} = \frac{15\times10^6}{50\times1005} = 298.5\text{K}$$



$$T_{02} = T_{01} - \Delta T_0 = 900 - 298.5 = \underline{601.5\text{K}}$$

$$P_{02} = P_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1} = 1.0\left(1+0.2\cdot0.7^2\right)^{3.5} = \underline{1.387 \text{ bar}}$$

TOTAL - TOTAL

$$\eta_{tt} = \frac{T_{01} - T_{02}}{T_{01} - T_{02}^{ISN}}$$

$$T_{02}^{ISN} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 900\left(\frac{1.387}{8.0}\right)^{1/3.5} = 545.5\text{K}$$

$$\eta_{tt} = \frac{900-601.5}{900-545.5} = 0.842 \qquad \eta_{tt} = \underline{\underline{84.2\%}}$$

TOTAL - STATIC

$$\eta_{ts} = \frac{T_{01} - T_{02}}{T_{01} - T_2^{ISN}}$$

$$T_2^{ISN} = T_{01}\left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 900\left(\frac{1.0}{8.0}\right)^{1/3.5} = 496.8\text{K}$$

$$\eta_{ts} = \frac{900-601.5}{900-496.8} = 0.740 \qquad \eta_{ts} = \underline{\underline{74.0\%}}$$

(b) η_{tt} — ASSUMES EXIT KE USEFUL

η_{ts} — REFLECTS CASE WHEN EXIT KE IS WASTED.

IMPROVE BY DROPPING M_2 — SAY EXIT DIFFUSER.

7 (b)

(i) Conservation form is

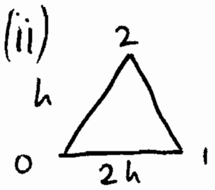
$$\frac{\partial \underline{U}}{\partial t} + \frac{\partial}{\partial x}\underline{F_x} + \frac{\partial}{\partial y}\underline{F_y} + \frac{\partial}{\partial z}\underline{F_z} = 0$$

where $\underline{U} = \begin{pmatrix} \rho V_x \\ \rho V_y \\ \vdots \end{pmatrix}$ and $\underline{F_x}, \underline{F_y}, \underline{F_z}$ represent the flux of a quantity.

It is called conservation form, because, when integrated over a control volume, the rate of change of stuff in the control volume is related directly to the flux across the surrounding surface.

Numerical schemes based on this formulation will conserve mass, momentum and energy for steady solutions, achieve the correct strength of any shocks, etc.

(ii)



$$\text{Area}\,\frac{d\rho_{cell}}{dt} = -\oint_c \rho \underline{V}\cdot\underline{n}\,dl \quad \text{using Gauss' theorem}$$

$$\int_A \frac{\partial\rho}{\partial t}\,dA = -\int_A \nabla\cdot(\rho\underline{V})\,dA$$

$$= -\oint_c (\rho V_x)\,dy + \oint_c \rho V_y\,dx$$

Using the trapezium rule

$$h^2\frac{d\rho_{cell}}{dt} = -\frac{(\rho V_x)_1 + (\rho V_x)_0}{2}\times(0) - \frac{(\rho V_x)_2 + (\rho V_x)_1}{2}(-h)$$

Area $= \frac{1}{2}\times 2h \times h$

$$- \frac{(\rho V_x)_0 + (\rho V_x)_2}{2}(-h)$$

$$+ \frac{(\rho V_y)_1 + (\rho V_y)_0}{2}(2h) + \frac{(\rho V_y)_2 + (\rho V_y)_1}{2}(-h)$$

$$+ \frac{(\rho V_y)_0 + (\rho V_y)_2}{2}(-h)$$

$$= -h\left\{\frac{(\rho V_x)_1 - (\rho V_x)_0}{2} + (\rho V_y)_2 - \frac{(\rho V_y)_1 + (\rho V_y)_0}{2}\right\}$$

(8)

(a) $U = r\Omega = 0{\cdot}3 \, . \, \frac{2\pi \, . \, 6500}{60} = \underline{204{\cdot}2 \text{ m/s}}$

$u_1 = \phi . U = 0{\cdot}5 \times 204{\cdot}2 = \underline{102{\cdot}1 \text{ m/s}}$

$T_1 = T_{01} - u_1^2/2c_p = 288 - 102{\cdot}1^2/(2 \times 1005) = \underline{282{\cdot}8 \text{ K}}$

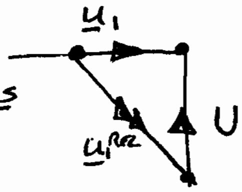
$P_1 = P_{01}(T_1/T_{01})^{\gamma/\gamma-1} = 1{\cdot}0 \left(\frac{282{\cdot}8}{288}\right)^{3{\cdot}5} = \underline{0{\cdot}9382 \text{ bar}}$

$\rho_1 = P_1/RT_1 = \frac{0{\cdot}9382 \times 10^5}{287 \times 282{\cdot}8} = \underline{1{\cdot}1560} \text{ kg/m}^3$

$A_1 = \frac{\dot{m}}{\rho_1 u_1} = \frac{25}{1{\cdot}156 \times 102{\cdot}1} = \underline{0{\cdot}2118 \text{ m}^2}$

$\Delta r = \frac{A_1}{2\pi r} = \frac{0{\cdot}2118}{2\pi . 0{\cdot}3} = \underline{\underline{0{\cdot}112 \text{ m}}}$ (BLADE HEIGHT = [redacted] 112 mm)

(b) $|\underline{u}_1^{REL}| = \sqrt{u_1^2 + U^2} = \sqrt{102{\cdot}1^2 + 204{\cdot}2^2} = \underline{228{\cdot}3 \text{ m/s}}$



$T_{01}^{REL} = T_1 + |\underline{u}_1^{REL}|^2/2c_p = 282{\cdot}8 + 228{\cdot}3^2/2 \times 1005 = \underline{\underline{308{\cdot}7 \text{ K}}}$

$P_{01}^{REL} = P_1 \left(\frac{T_{01}^{REL}}{T_1}\right)^{\gamma/\gamma-1} = 0{\cdot}9382 \left(\frac{308{\cdot}7}{282{\cdot}8}\right)^{3{\cdot}5} = \underline{\underline{1{\cdot}275 \text{ bar}}}$

$\alpha_1^{rel} = \tan^{-1}\left(\tan\alpha_1 - \frac{1}{\phi}\right) = \tan^{-1}\left(0 - \frac{1}{0{\cdot}5}\right) = \underline{\underline{-63{\cdot}4^\circ}}$

(c) $Y_p = \frac{P_{01}^{REL} - P_{02}^{REL}}{P_{01}^{REL} - P_1}$ (N.B. REL. QUANTITIES & FLOW AT FIXED RADIUS $T_{02}^{REL} = T_{01}^{REL}$)

$P_{02}^{REL} = P_{01}^{REL} - Y_p(P_{01}^{REL} - P_1) = 1{\cdot}275 - 0{\cdot}05(1{\cdot}275 - 0{\cdot}9382)$

$P_{02}^{REL} = \underline{\underline{1{\cdot}258 \text{ bar}}}$

(c cont.) $\dfrac{\dot{m}\sqrt{c_p T_{03}^{Rel}}}{A \cos\alpha^{Rel} p_0^{Rel}} = fn(M^{Rel})$ $A_2 = A_1$

$$\frac{\dot{m}\sqrt{c_p T_{02}^{Rel}}}{A_2 \cos\alpha_2^{Rel} p_{02}^{Rel}} = \frac{25\sqrt{1005 \cdot 308.7}}{0.2118\,(\cos -50^\circ)\, 1.258\times10^5} = 0.8131$$

TABLES $(\gamma = 1.4)$ $M_2^{Rel} = \underline{\underline{0.405}}$

$$P_2 = P_{02}^{Rel} \Big/ \left(1+\tfrac{\gamma-1}{2} M_2^{Rel\,2}\right)^{\gamma/\gamma-1} = \frac{1.258}{(1+0.2\times0.405^2)^{3.5}} = \underline{\underline{1.124 \text{ bar}}}$$

(d) $V_2^{REL} = M_2^{REL}\sqrt{\gamma R T_2}$

$$T_2 = T_{02}^{Rel} \Big/ \left(1+\tfrac{\gamma-1}{2} M_2^{Rel\,2}\right) = \frac{308.7}{(1+0.2\times0.405^2)} = \underline{298.9 \text{ K}}$$

$$V_2^{REL} = 0.405\sqrt{1.4 \cdot 287 \cdot 298.9} = 140.4 \text{ m/s}$$

$$u_{\theta2}^{REL} = V_2^{Rel} \sin\alpha_2^{Rel} = 140.4 \sin(-50^\circ) = \underline{-107.6 \text{ m/s}}$$

$$u_{\theta2} = u_{\theta2}^{Rel} + U = -107.6 + 204.2 = \underline{\underline{96.6 \text{ m/s}}}$$

$$\frac{\Delta h_0}{U^2} = \frac{U_2 u_{\theta2} - U_1 u_{\theta1}}{U^2} = \frac{u_{\theta2}}{U} = \frac{96.6}{204.2} = \underline{\underline{0.473}}$$

WOULD NORMALLY EXPECT $\Delta h_0/U^2 \sim 0.4$ to 0.45 SO THIS IS A CHALLENGING AERODYNAMIC DESIGN.

(e) CONSTANT BLADE HEIGHT MEANS U_x DECREASES SO SWIRL ANGLES BECOME HIGHER. BETTER TO DECREASE BLADE HEIGHT TO MAINTAIN CONSTANT AXIAL VELOCITY.

Answers to 3A3 2006

1. (b) (i) 0.84, 1, (ii) 0.65, 0.86.
   (c) 0.93.

2. (a) 0.572.
   (c) 0.74, 1.17.
   (d) 0.992.

3. (a) 2.
   (b) 1.25a_0, 1.30a_0.
   (c) 1.73.

4. (b) 0.77.
   (c) 0.95.
   (d) 0.87.
   (e) 0.93.

7. (a) 1.39 bar, 84.2%, 74.0%.

8. (a) 0.112m.
   (b) 308.7k, 1.275bar, -63.4°
   (c) 1.258 bar, 1,124bar.
   (d) 0.473.