

PART IIA 2006

3A5: Advanced thermodynamics and power generation

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Datasheet: None

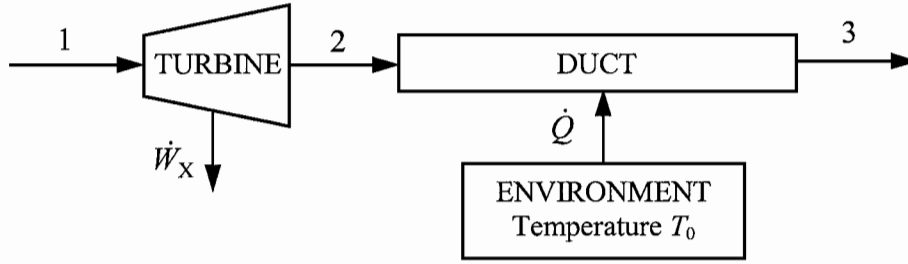
ENGINEERING TRIPOS PART IIA 2006

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

SOLUTIONS TO TRIPOS QUESTIONS

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1 (a)



(i) From availability theory, the maximum power output from a steady-flow process between states 1 and 3, exchanging heat with the environment at temperature T_0 is,

$$[\dot{W}_X]_{\text{MAX}} = \dot{m}[(h_1 - T_0 s_1) - (h_3 - T_0 s_3)]$$

The working fluid may be treated as a perfect gas. Hence,

$$\begin{aligned} [\dot{W}_X]_{\text{MAX}} &= \dot{m} \left[c_p (T_1 - T_3) - c_p T_0 \ln \left(\frac{T_1}{T_3} \right) + R T_0 \ln \left(\frac{p_1}{p_3} \right) \right] \\ &= 10 \times \left[1.25 \times (1600 - 780) - 1.25 \times 300 \times \ln \left(\frac{1600}{780} \right) + 0.29 \times 300 \times \ln \left(\frac{30.0}{1.0} \right) \right] \\ &= 10.52 \times 10^3 \text{ kW} \equiv 10.52 \text{ MW} \end{aligned} \quad [15 \%]$$

(ii) The turbine power output is,

$$\begin{aligned} \dot{W}_X &= \dot{m}(h_1 - h_2) = \dot{m} c_p (T_1 - T_2) \\ &= 10 \times 1.25 \times [1600 - 830] = 9.63 \times 10^3 \text{ kW} \equiv 9.63 \text{ MW} \end{aligned}$$

The expansion is adiabatic so the entropy increase in the turbine is caused solely by the irreversibility of the expansion. Hence, the 'lost power' is given by,

$$\begin{aligned} [\dot{W}_L]_{\text{CR}} &= \dot{m} T_0 \left[c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right) \right] \\ &= 10 \times 300 \times \left[1.25 \times \ln \left(\frac{830}{1600} \right) - 0.29 \times \ln \left(\frac{1.2}{30.0} \right) \right] = 0.34 \times 10^3 \text{ kW} \equiv 0.34 \text{ MW} \quad [15 \%] \end{aligned}$$

(iii) For the duct, let $d\dot{Q}$ be the differential heat transfer rate to the duct from the environment. The lost power due to the heat transfer is then given by,

$$\begin{aligned}
[\dot{W}_L]_{HT} &= -\int_2^3 \left(1 - \frac{T_0}{T}\right) d\dot{Q} = -\int_2^3 \left(1 - \frac{T_0}{T}\right) \dot{m} c_p dT = -\dot{m} \left[c_p (T_3 - T_2) - c_p T_0 \ln \left(\frac{T_3}{T_2} \right) \right] \\
&= -10 \times \left[1.25 \times (780 - 830) - 1.25 \times 300 \times \ln \left(\frac{780}{830} \right) \right] = 0.39 \times 10^3 \text{ kW} \equiv 0.39 \text{ MW} \quad [20\%]
\end{aligned}$$

(iv) The lost power due to flow friction can be found by subtracting the actual power and lost power terms from the maximum power value calculated in part (i) :

$$[\dot{W}_L]_{CR} = 10.52 - 9.63 - 0.34 - 0.39 = 0.16 \text{ MW} \quad [10\%]$$

(b) The given characteristic equation of state is,

$$g = A(T - T_0) - AT \ln \left(\frac{T}{T_0} \right) + RT \ln \left(\frac{p}{p_0} \right) + b(p - p_0)$$

(i) From $dg = vdp - sdT$ we get the p - v - T equation state as follows:

$$\left(\frac{\partial g}{\partial p} \right)_T = v = \frac{RT}{p} + b \quad \rightarrow \quad p(v - b) = RT \quad [10\%]$$

(ii) From $dg = vdp - sdT$ we get an expression for the specific entropy as follows:

$$\left(\frac{\partial g}{\partial T} \right)_p = -s = A - A - A \ln \left(\frac{T}{T_0} \right) + R \ln \left(\frac{p}{p_0} \right) \quad \rightarrow \quad s = A \ln \left(\frac{T}{T_0} \right) - R \ln \left(\frac{p}{p_0} \right)$$

The specific enthalpy is then given by,

$$h = g + Ts = A(T - T_0) + b(p - p_0)$$

By definition,

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p = A$$

Hence, A is the isobaric specific heat capacity c_p which is constant for this fluid. [20%]

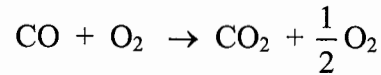
(iii) To prove $c_p - c_v = R$, we need an expression for c_v . The specific internal energy is,

$$u = h - pv = A(T - T_0) + b(p - p_0) - (RT + bp) = A(T - T_0) - RT - bp_0$$

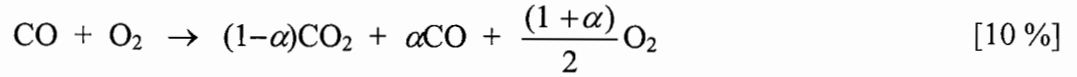
Then, by definition,

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v = A - R = c_p - R \quad \rightarrow \quad c_p - c_v = R \quad [10\%]$$

2 (a) The equation for complete combustion would be,



Allowing for dissociation, the actual reaction is,

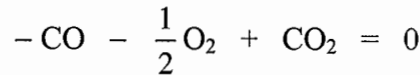


(b) Total number of moles of products = $n_2 = \frac{(3+\alpha)}{2}$

If the pressure after combustion is p_2 , then the partial pressures are given by,

$$p_{\text{CO}_2} = \frac{(1-\alpha)}{n_2} p_2, \quad p_{\text{CO}} = \frac{\alpha}{n_2} p_2, \quad p_{\text{O}_2} = \frac{(1+\alpha)}{2n_2} p_2.$$

Reaction (7) in the Thermofluids Data Book is,



so the equilibrium equation is,

$$\left(\frac{p_{\text{CO}_2}}{p_0}\right)^{+1} \left(\frac{p_{\text{CO}}}{p_0}\right)^{-1} \left(\frac{p_{\text{O}_2}}{p_0}\right)^{-0.5} = K_{p7}(T)$$

where $p_0 = 1$ bar is 'standard pressure'. Substituting the expressions for the partial pressures :

$$\left(\frac{1-\alpha}{n_2}\right)^{+1} \left(\frac{\alpha}{n_2}\right)^{-1} \left(\frac{1+\alpha}{2n_2}\right)^{-0.5} \left(\frac{p_2}{p_0}\right)^{-0.5} = K_{p7}(T)$$

From the Data Book at 2600 K, $\ln(K_{p7}) = 2.800$, so $K_{p7} = 16.445$. Substituting the expression for n_2 and using the given pressure of $p_2 = 6.625$ bar therefore gives,

$$\left(\frac{1-\alpha}{\alpha}\right) \left(\frac{3+\alpha}{1+\alpha}\right)^{0.5} = \left(\frac{p_2}{p_0}\right)^{0.5} K_{p7}(T) = 42.328$$

Using the given value of $\alpha = 0.03884$, LHS = 42.325. This verifies that $\alpha = 0.03884$. [30 %]

(c) No. of moles of reactants is $n_1 = 2$ and no. of moles of products is $n_2 = 1.5194$

For constant volume combustion, we have $p_1V = n_1\bar{R}T$ and $p_2V = n_2\bar{R}T$. Hence

$$p_2 = \frac{n_2}{n_1} \frac{T_2}{T_1} p_1 = \frac{1.5194}{2} \times \frac{2600}{298.15} \times 1.0 = 6.625 \text{ bar}$$

This agrees with the given pressure and confirms that the reaction is at constant volume. [20 %]

(d) Let Q be the heat transferred from the container. From the First Law for a constant volume process (*i.e.*, no work done) :

$$\begin{aligned} Q &= (U_{R1} - U_{P2}) = (H_{R1} - p_1V) - (H_{P2} - p_2V) \\ &= (H_{R1} - H_{P2}) - \bar{R}(n_1T_1 - n_2T_2) \end{aligned}$$

Expanding the enthalpy term :

$$\begin{aligned} (H_{R1} - H_{P2}) &= \bar{h}_{CO}(T_1) + \bar{h}_{O_2}(T_1) - (1-\alpha)\bar{h}_{CO_2}(T_2) - \alpha\bar{h}_{CO}(T_2) - \left(\frac{1+\alpha}{2}\right)\bar{h}_{O_2}(T_2) \\ &= (1-\alpha)\left[\bar{h}_{CO}(T_1) + \frac{1}{2}\bar{h}_{O_2}(T_1) - \bar{h}_{CO_2}(T_1)\right] - (1-\alpha)[\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)] - \\ &\quad \alpha[\bar{h}_{CO}(T_2) - \bar{h}_{CO}(T_1)] - \left(\frac{1+\alpha}{2}\right)[\bar{h}_{O_2}(T_2) - \bar{h}_{O_2}(T_1)] \end{aligned}$$

For accuracy, the enthalpy differences should be evaluated using the *Molar Enthalpies of Common Gases at Low Pressures*. The figures in curly brackets below are the values obtained assuming constant molar isobaric heat capacities (obtained from the Table *Properties of Perfect Gases*) :

$$\begin{aligned} (1-\alpha)\left[\bar{h}_{CO}(T_1) + \frac{1}{2}\bar{h}_{O_2}(T_1) - \bar{h}_{CO_2}(T_1)\right] &= (1-\alpha)[- \Delta \bar{H}_{298}^0] \\ &= 0.96116 \times 283.0 = 272.0 \text{ kJ} \quad \{272.0\} \\ (1-\alpha)[\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)] &= 0.96116 \times (137.80 - 9.37) = 123.4 \text{ kJ} \quad \{79.8\} \\ \alpha[\bar{h}_{CO}(T_2) - \bar{h}_{CO}(T_1)] &= 0.03884 \times (87.40 - 8.67) = 3.1 \text{ kJ} \quad \{2.6\} \\ \left(\frac{1+\alpha}{2}\right)[\bar{h}_{O_2}(T_2) - \bar{h}_{O_2}(T_1)] &= 0.51942 \times (90.92 - 8.66) = 42.7 \text{ kJ} \quad \{35.2\} \end{aligned}$$

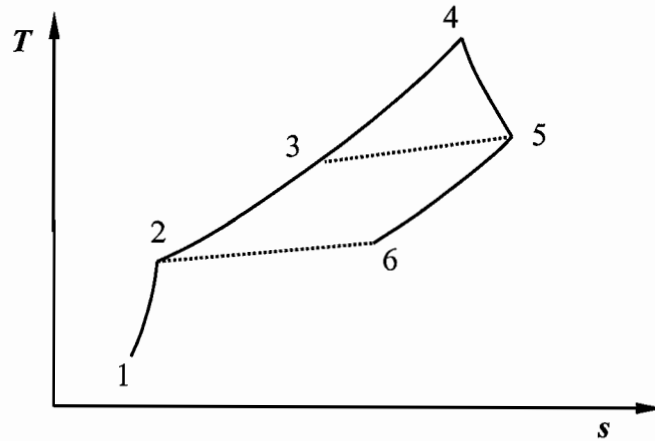
Hence :

$$\begin{aligned} (H_{R1} - H_{P2}) &= 272.01 - 123.44 - 3.06 - 42.73 = 102.8 \text{ kJ} \quad \{154.4\} \\ \bar{R}(n_1T_1 - n_2T_2) &= 8.3143 \times 10^{-3} \times (2 \times 298.15 - 1.5194 \times 2600) = -27.9 \text{ kJ} \quad \{-27.9\} \\ Q &= 102.78 + 27.89 = 130.7 \text{ kJ} \quad \{182.3\} \quad [40\%] \end{aligned}$$

Notes :

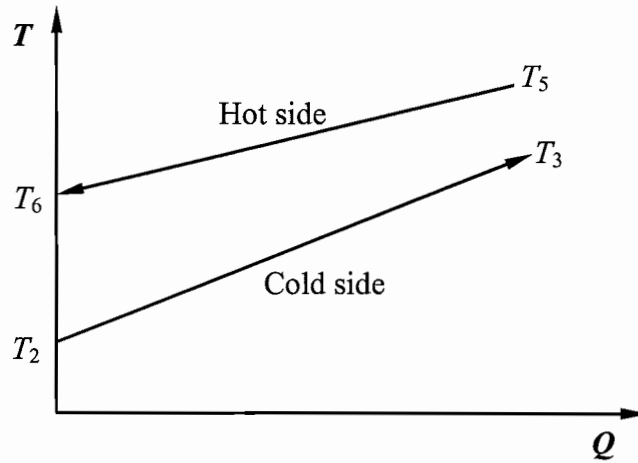
- (i) Using constant \bar{c}_p underestimates the enthalpy changes because \bar{c}_p increases with temperature for all species.
- (ii) The calculations can equally well be based around $\Delta \bar{H}_{2600}^0$. This will give the same result using the Molar Enthalpy Tables but a different value if constant \bar{c}_p is assumed.

3 (a)



[5 %]

(b)



Both lines are straight because $\dot{m}c_p$ is constant on each side. The cold side is steeper than the hot side because $(1/\dot{m}c_p)$ is greater.

[10 %]

(c) Let $r = (p_2/p_1) = (p_4/p_5)$ be the pressure ratio. We therefore have :

$$\left(\frac{T_2}{T_1}\right) = r^\alpha \quad \text{where} \quad \alpha = \frac{1}{\eta_c} \left(\frac{\gamma_a - 1}{\gamma_a} \right) = \frac{1}{0.9} \left(\frac{1.40 - 1.0}{1.40} \right) = 0.3175$$

$$\left(\frac{T_4}{T_5}\right) = r^\beta \quad \text{where} \quad \beta = \eta_t \left(\frac{\gamma_p - 1}{\gamma_p} \right) = 0.9 \left(\frac{1.35 - 1.0}{1.35} \right) = 0.2333$$

Maximum pressure ratio for a heat exchanger occurs when $T_5 = T_2$. Thus :

$$r_{max} = \left(\frac{T_4}{T_1}\right)^{[1/(\alpha+\beta)]} = \left(\frac{1600.0}{298.15}\right)^{(1/0.5508)} = 21.12 \quad [25 \%]$$

(d) (i) For a pressure ratio of 15.0 :

$$T_2 = T_1 r^\alpha = 298.15 \times 15.0^{0.3175} = 704.4 \text{ K}$$

$$T_5 = \frac{T_4}{r^\beta} = \frac{1600.0}{15.0^{0.2333}} = 850.6 \text{ K}$$

Heat exchanger effectiveness is defined by,

$$\varepsilon = \frac{T_3 - T_2}{T_5 - T_2}$$

Hence air temperature at combustor inlet is,

$$T_3 = T_2 + \varepsilon (T_5 - T_2) = 704.4 + 0.8 \times (850.6 - 704.4) = 821.4 \text{ K} \quad [15 \%]$$

(ii) SFEE for the combustion chamber :

$$(\dot{m}_a + \dot{m}_f) h_p(T_4) - \dot{m}_a h_a(T_3) - \dot{m}_f h_f(T_0) = 0$$

This can be written :

$$\begin{aligned} (\dot{m}_a + \dot{m}_f)[h_p(T_4) - h_p(T_0)] - \dot{m}_a[h_a(T_3) - h_a(T_0)] = \\ - [(\dot{m}_a + \dot{m}_f)h_p(T_0) - \dot{m}_a h_a(T_0) - \dot{m}_f h_f(T_0)] = -\dot{m}_f \Delta H_{298}^0 \end{aligned}$$

Writing $A = (\dot{m}_a / \dot{m}_f)$ the air/fuel ratio, we have :

$$(A+1)c_{pp}(T_4 - T_0) - A c_{pa}(T_3 - T_0) = -\Delta H_{298}^0$$

Hence,

$$A = \frac{-\Delta H_{298}^0 - c_{pp}(T_4 - T_0)}{c_{pp}(T_4 - T_0) - c_{pa}(T_3 - T_0)} = \frac{48.0 \times 10^3 - 1.10 \times (1600.0 - 298.15)}{1.10 \times (1600.0 - 298.15) - 1.01 \times (821.4 - 298.15)} = 51.5 \quad [25 \%]$$

(iii) Writing $f = 1/A = 0.0194$, the net work output per kg air entering the compressor is,

$$\begin{aligned} w_{net} &= (1+f)c_{pp}(T_4 - T_5) - c_{pa}(T_2 - T_1) \\ &= 1.0194 \times 1.10 \times (1600.0 - 850.6) - 1.01 \times (704.4 - 298.15) = 430.0 \text{ kJ/kg air} \quad [10 \%] \end{aligned}$$

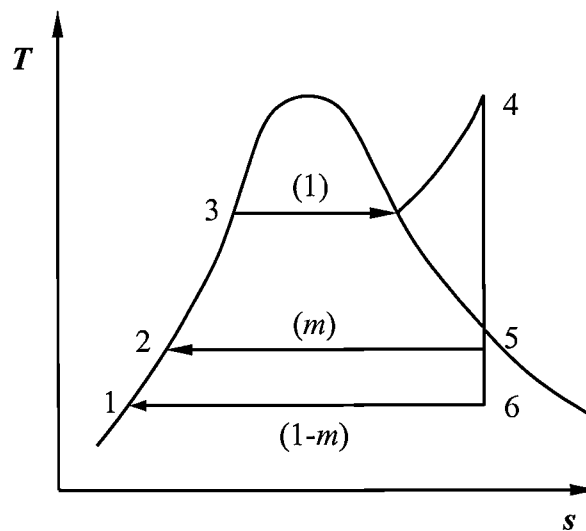
$$(iv) \text{ Overall efficiency} = \frac{w_{net}}{f[-\Delta H_{298}^0]} = \frac{430.0}{0.0194 \times 48.0 \times 10^3} = 0.462$$

$$\text{Rational efficiency} = \frac{w_{net}}{f[-\Delta G_{298}^0]} = \frac{430.0}{0.0194 \times 50.0 \times 10^3} = 0.443 \quad [10 \%]$$

4 (a) In a conventional cycle without feed heating, the *economiser* raises the temperature of the high pressure water delivered by the feed pump to the boiler saturation temperature. This means that the *mean temperature of heat addition* in the economiser is very low (around 200 °C). *Feed heating* is used to increase the temperature of the feed water entering the boiler. This has the effect of increasing the mean temperature of heat addition for the cycle, and hence the cycle efficiency. (Feed heating improves the cycle efficiency even when *direct contact feed heating* is used and the mixing process is inherently irreversible.)

In a combined-cycle, the mass flowrate of steam is fixed by the pinch-point condition in the HRSG and is the same whether or not steam-cycle feed heating is used. The power output from the steam turbine is therefore reduced by feedheating because the mass flowrate through the later stages is reduced. As the fuel input to the gas turbine is unchanged, this means that the cycle efficiency is always reduced if steam-cycle feed heating is employed. This change manifests itself as an increase in the turbine exhaust gas exergy flux. Hence, feed heating is never used with combined-cycles.

[20 %]



(b) No feed heating ($m = 0$) and neglect feed pump work :

From the superheated Steam Tables : $h_4 = 3450.4$ kJ/kg $s_4 = 6.523$ kJ/kg K.

At condenser pressure of 0.06 bar : $h_f = 151.5$ kJ/kg, $h_g = 2566.6$ kJ/kg
 $s_f = 0.521$ kJ/kg K $s_g = 8.329$ kJ/kg K

At turbine exit,

$$s_6 = s_4 = (1-x_6) s_f + x_6 s_g \rightarrow x_6 = \frac{s_6 - s_f}{s_g - s_f} = \frac{6.523 - 0.521}{8.329 - 0.521} = 0.7687$$

$$h_6 = (1-x_6) h_f + x_6 h_g = (1 - 0.7687) \times 151.5 + 0.7687 \times 2566.6 = 2008.0 \text{ kJ/kg}$$

$$\text{Work output} = (h_4 - h_6) = 3450.4 - 2008.0 = 1442.4 \text{ kJ/kg steam circulating}$$

$$\text{Cycle efficiency} = \frac{w_{out}}{q_{in}} = \frac{w_{out}}{h_4 - h_1} = \frac{1442.4}{3450.4 - 151.5} = 0.437$$

[25 %]

(c) With feed heating :

At bleed pressure of 5.0 bar : $h_f = 640.1 \text{ kJ/kg}$, $h_g = 2748.1 \text{ kJ/kg}$
 $s_f = 1.860 \text{ kJ/kg K}$ $s_g = 6.821 \text{ kJ/kg K}$

At turbine bleed point (assuming steam is wet) :

$$s_5 = s_4 = (1-x_5) s_f + x_5 s_g \rightarrow x_5 = \frac{s_5 - s_f}{s_g - s_f} = \frac{6.523 - 1.860}{6.821 - 1.860} = 0.9399$$

$$h_5 = (1-x_5) h_f + x_5 h_g = (1 - 0.9399) \times 640.1 + 0.9399 \times 2748.1 = 2621.4 \text{ kJ/kg}$$

SFEE applied to the feed heater gives :

$$m h_5 + (1-m) h_1 = h_2 \rightarrow m = \frac{h_2 - h_1}{h_5 - h_1} = \frac{640.1 - 151.5}{2621.4 - 151.5} = 0.1978$$

$$\begin{aligned} \text{Work output} &= (h_4 - h_5) + (1-m)(h_5 - h_6) \\ &= (3450.4 - 2621.4) + (1.0 - 0.1978) \times (2621.4 - 2008.0) \\ &= 1321.1 \text{ kJ/kg steam passing through boiler} \end{aligned}$$

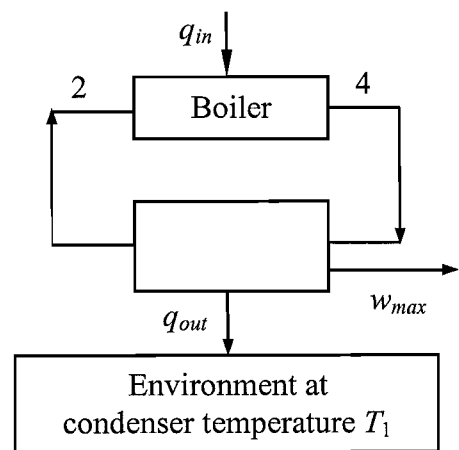
$$\text{Cycle efficiency} = \frac{w_{out}}{q_{in}} = \frac{w_{out}}{h_4 - h_2} = \frac{1321.1}{3450.4 - 640.1} = 0.470 \quad [30 \%$$

(d) The best cycle efficiency is achieved when the work output is a maximum. This can be found by applying availability theory between states 4 and 2, see diagram. The environment temperature must be taken as condenser temperature, $T_1 = 36.16 \text{ }^\circ\text{C} = 309.31 \text{ K}$ (so that there is no exergy loss associated with the heat rejection). Thus :

$$\begin{aligned} w_{max} &= (h_4 - T_1 s_4) - (h_2 - T_1 s_2) \\ &= (3450.4 - 640.1) - 309.31 \times (6.523 - 1.860) \\ &= 1368.0 \text{ kJ/kg} \end{aligned}$$

$$\text{Max cycle efficiency} = \frac{1368.0}{3450.4 - 640.1} = 0.487$$

The actual value is less than this because the cycle with a single feed heater is not reversible. There is irreversible entropy creation in the mixing process which reduces the cycle efficiency from 0.487 to 0.470.



[25 %]

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MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

ANSWERS

- 1 (a) (i) 10.52 MW (ii) 9.63 MW, 0.34 MW, 0.39 MW, 0.16 MW
- 2 (d) 130.7 kJ
- 3 (c) 21.12
(d) (i) 821.4 K (ii) 51.5 (iii) 430.0 kJ/kg air (iv) 0.462, 0.443
- 4 (b) 1442.3 kJ/kg, 0.437
(c) 0.198, 1321.1 kJ/kg, 0.470
(d) 1368.0 kJ/kg, 0.487

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