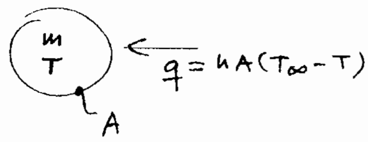
3A6 HEAT AND MASS TRANSFER 2006

PART IIA 2006
3A6: Heat and mass transfer
Principal Assessor: Prof S Hochgreb

Datasheet: None

Q1 (a) · ASSUME UNIFORM PARTICLE TEMPERATURE
· NEGLIGIBLE RADIATION.

(i) FROM ENERGY CONSERVATION:



$$mc\frac{dT}{dt} = hA\,(T_\infty - T)$$

$$\frac{d(T_\infty - T)}{(T_\infty - T)} = -\frac{hA}{mc}\,dt$$

$$\ln\frac{T_\infty - T(t)}{T_\infty - T_0} = -\frac{hA}{mc}\,t$$

$$\boxed{t_1 = \frac{mc}{hA}\ln\frac{(T_\infty - T_0)}{(T_\infty - T_{mp})}} \qquad \boxed{t_1 = \frac{1}{6}\frac{\rho D c}{h}\ln\frac{(T_\infty - T_0)}{(T_\infty - T_{mp})}}$$

(ii) DURING FUSION, THE PARTICLE REMAINS AT THE MELTING POINT TEMPERATURE T_{mp}, AND THE ENERGY EQUATION READS

$$\frac{dm_\ell}{dt}h_{sf} = hA\,(T_\infty - T_{mp})$$

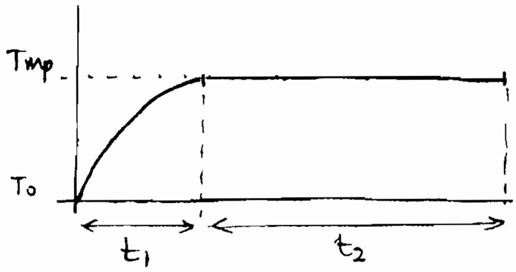
WHERE A IS THE AREA OF THE PARTICLE, ASSUMED TO REMAIN CONSTANT AS THE DROPLET MELTS. INTEGRATING

$$\underbrace{m_{\ell 2}}_{m} - \underbrace{\cancel{m_{\ell 1}}}_{0} = \frac{hA\,(T_\infty - T_{mp})}{h_{sf}}\,t_2$$

$$t_2 = \frac{m}{A}\frac{h_{sf}}{h(T_\infty - T_{mp})} = \frac{\rho D}{6}\frac{h_{sf}}{h(T_\infty - T_{mp})}$$

$$\boxed{t_2 = \frac{\rho D}{6}\frac{h_{sf}}{h(T_\infty - T_{mp})}}$$

(ii)



Substituting:

$$t_1 = \frac{1}{6}\frac{(3970\ kg/m^3)(50\times10^{-6}\ m)(1560\ J/kg\ K)}{(30{,}000\ W/m^2K)}\ln\frac{(10{,}000-300)}{(10{,}000-2318)}$$

$$\boxed{t_1 = 0.40\ ms}$$

$$t_2 = \frac{1}{6}\frac{(3970\ kg/m^3)(50\times10^{-6}\ m)\ (3577\times10^3\ J/kg)}{(30{,}000\ W/m^2K)(10{,}000K - 2318\ K)}$$

$$\boxed{t_2 = 0.51\ ms}$$

(b) IN ORDER FOR THE UNIFORM TEMPERATURE ASSUMPTION TO BE VALID THROUGHOUT, WE MUST HAVE $h \ll k/D$, ie

$$Bi = \frac{hD}{k} \ll 1$$

IF WE ASSUME THAT THE CONDUCTIVITY OF THE LIQUID IS THE SAME OR SIMILAR TO THAT OF THE SOLID, THIS ASSUMPTION WILL HOLD THROUGHOUT THE PROCESS. WE HAVE:

$$Bi = \frac{(30{,}000\ W/m^2K)(50\times10^{-6}\ m)}{(10.5\ W/mK)} \approx 0.14$$

THIS IS NOT A VERY LOW VALUE, BUT ACCEPTABLY LOW.

(c) VALIDITY OF NEGLECTING RADIATION: IN THIS CASE, WE MUST HAVE;
DURING WARM-UP:

$$hA(T_\infty - T) \gg \varepsilon \sigma A (T^4 - T_0^4)$$

THE HIGHEST RADIATION WILL TAKE PLACE WHEN THE PARTICLE IS AT THE MELTING POINT. THEREFORE, IF THE CRITERION HOLDS AT THAT POINT, IT SHOULD HOLD THROUGHOUT. WE HAVE

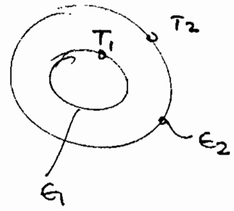
$$h(T_\infty - T_{mp}) = (30{,}000\ W/m^2K)(10{,}000 - 300)K = 2.91\times10^{8}\ W/m^2$$

$$\varepsilon \sigma (T_{mp}^4 - T_0^4) = (0.4)\left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(2318^4 - 300^4)$$
$$= 6.5\times10^{5}\ W/m^2$$

THEREFORE, RADIATION FROM THE PARTICLES CAN COMFORTABLY BE NEGLECTED.

Q2 (a) EQUIVALENT CIRCUIT — NO SHIELD

$$\xrightarrow{q} \; E_{b1} \;\text{—}\!\underset{\frac{1-\epsilon_1}{A\epsilon_1}}{\text{ww}}\!\text{—}\; J_1 \;\text{—}\!\underset{\frac{1}{A_1F_{12}}}{\text{ww}}\!\text{—}\; J_2 \;\text{—}\!\underset{\frac{1-\epsilon_2}{A_2\epsilon_2}}{\text{ww}}\!\text{—}\; E_{b2}$$



$$q_1'' = \frac{E_{b1}-E_{b2}}{\frac{1-\epsilon_1}{A_1\epsilon_1}+\frac{1-\epsilon_2}{A_2\epsilon_2}+\frac{1}{A_1}} \qquad F_{12}=1$$

$$q_1 = \frac{A_1\sigma(T_1^4-T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1}+\frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}+1} = \frac{A_1\sigma(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}}$$

$$\frac{q_1}{L} = \frac{\sigma\pi D_1(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{1-\epsilon_2}{\epsilon_2}\frac{D_1}{D_2}} = \frac{\pi(20\times10^{-3}\,m)(500^4-300^4)K^4\,5.67\times10^{-8}\,\frac{W}{m^2K^4}}{\frac{1}{0.1}+\frac{1-0.02}{0.02}\frac{20}{50}}$$

$$\boxed{\frac{q_1}{L} = 6.55\ W/m}$$

(b) WITH SHIELD

$$\xrightarrow{q} \; E_{b1} \;\text{—}\!\underset{\frac{1-\epsilon_1}{A_1\epsilon_1}}{\text{ww}}\!\text{—}\; J_1 \;\text{—}\!\underset{\frac{1}{A_1F_{13}}}{\text{ww}}\!\text{—}\; J_{3i} \;\text{—}\!\underset{\frac{1-\epsilon_3}{A_3\epsilon_3}}{\text{ww}}\!\text{—}\; E_{b3} \;\text{—}\!\underset{\frac{1-\epsilon_3}{A_3\epsilon_3}}{\text{ww}}\!\text{—}\; J_{3o} \;\text{—}\!\underset{\frac{1}{A_3F_{32}}}{\text{ww}}\!\text{—}\; J_2 \;\text{—}\!\underset{\frac{1-\epsilon_2}{\epsilon_2A_2}}{\text{ww}}\!\text{—}\; E_{b2}$$

$$q_1'' = \frac{E_{b1}-E_{b2}}{\frac{1-\epsilon_1}{\epsilon_1}\frac{1}{A_1}+\frac{1}{A_1}+2\left(\frac{1-\epsilon_3}{\epsilon_3}\frac{1}{A_3}\right)+\frac{1}{A_3}+\frac{1-\epsilon_2}{\epsilon_2}\frac{1}{A_2}}$$

$$q_1'' = \frac{A_1\sigma(T_1^4-T_2^4)}{\underbrace{\left[\frac{1-\epsilon_1}{\epsilon_1}+1\right]}_{\frac{1}{\epsilon_1}-1+1}+\underbrace{\left[2\left(\frac{1-\epsilon_3}{\epsilon_3}\right)+1\right]}_{\frac{2}{\epsilon_3}-2+1=\left(\frac{2}{\epsilon_3}-1\right)\frac{A_1}{A_3}}\frac{A_1}{A_3}+\frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}} = \frac{A_1\sigma(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{2-\epsilon_3}{\epsilon_3}\frac{A_1}{A_3}+\frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}}$$

$$\frac{q_1}{L} = \frac{\pi D_1\sigma(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{2-\epsilon_3}{\epsilon_3}\frac{D_1}{D_3}+\frac{1-\epsilon_2}{\epsilon_2}\frac{D_1}{D_2}} = \frac{\pi(20\times10^{-3}m)(5.67\times10^{-8}W/m^2K^4)(500^4-300^4)K^4}{\underbrace{\underbrace{\frac{1}{0.1}}_{10}+\underbrace{\frac{2-0.03}{0.03}\frac{20}{35}}_{37.5}+\underbrace{\frac{1-0.02}{0.02}\frac{20}{50}}_{19.6}}_{67.1}}$$

$$\boxed{\frac{q_1}{L} = 2.88\ W/m}$$

(56 % REDUCTION)

(c) WITH INSULATION (IN AIR)

$$\frac{q_1}{L} = \frac{T_1 - T_2}{\frac{\ln D_2/D_1}{2\pi k}} = \frac{500 - 300\ K}{\frac{\ln (50/20)}{2\pi (0.03\ W/m\ K)}} = 41.1\ \frac{W}{m}$$

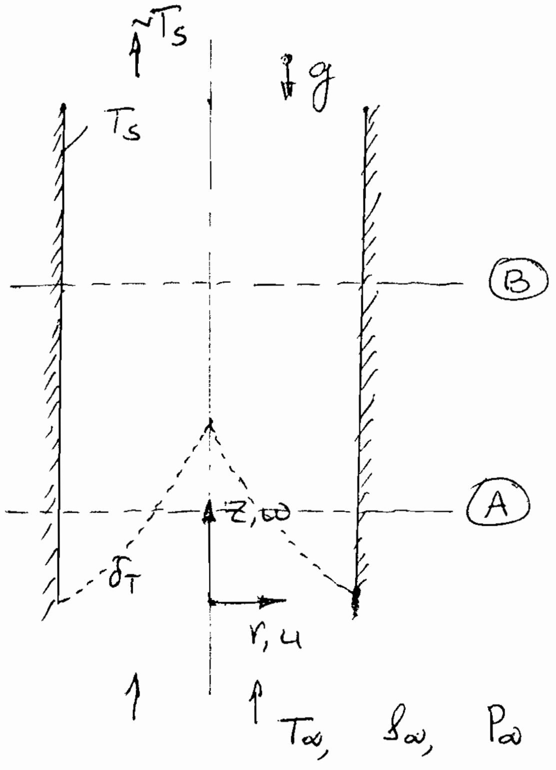
(d) CLEARLY, THE SHIELD PROVIDES BETTER INSULATION THAN THE MATERIAL, PROVIDED THE OUTER ANNULUS REMAINS UNDER VACUUM. THIS CAN OF COURSE BE A VERY costly SOLUTION.
IF NOT UNDER VACUUM, AIR will provide conductivity similar TO THAT OF THE INSULATION – with additional EFFECTS OF NATURAL convection.

(e) THE SHIELD SHOULD BE POSITIONED AS close AS POSSIBLE TO THE INNER TUBE, SINCE THE RESISTANCE DECREASES WITH INCREASING D_3.

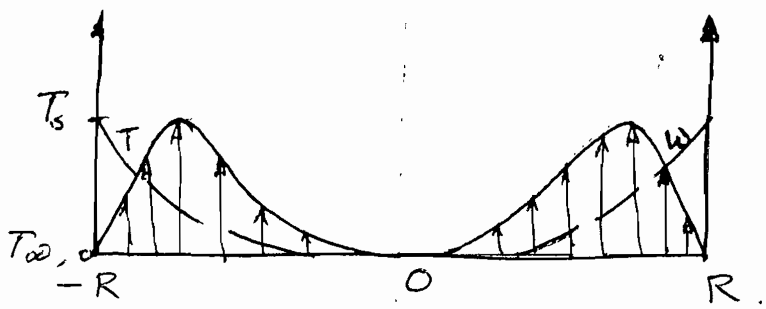
$$\frac{dq}{d(D_3)} = \frac{-\pi D_1 \sigma (T_1^4 - T_2^4)}{\left[\frac{1}{\epsilon_1} + \frac{2-\epsilon_3}{\epsilon_3}\frac{D_1}{D_3} + \frac{1-\epsilon_2}{\epsilon_2}\frac{D_1}{D_2}\right]^2} \frac{2-\epsilon_3}{\epsilon_3}\left(-\frac{D_1}{D_3^2}\right) > 0$$

ie THE HEAT TRANSFER INCREASES FOR LARGER SHIELD DIAMETERS. THE PRACTICAL LIMIT FOR AN EVACUATED TUBE IS THEN THE CLEARANCE BETWEEN THE TWO TUBES, WHICH SHOULD NOT TOUCH.

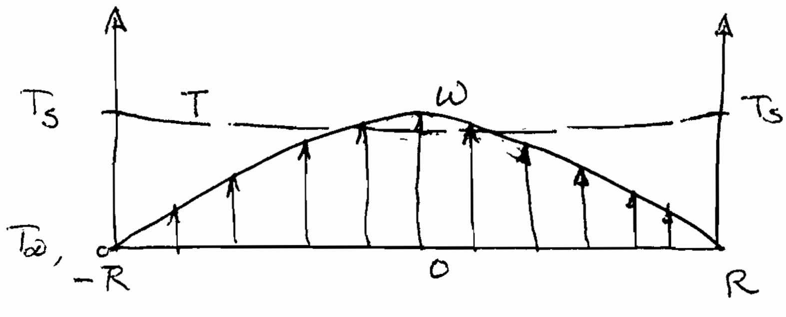
3)



a) @ section A



@ Section B



b) The Governing equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = -\frac{g\beta}{\nu}\left(T - T_\infty\right)$$

Physical meaning:

viscous forces are balanced by the bouyancy forces.

B.C: From fig in (a)

@ $r=0$ $\quad \frac{dw}{dr} = 0$

@ $r = \pm R$ $\quad w = 0$

c) $T - T_\infty = (T_s - T_\infty) - (T_s - T)$

in the fully developed region

$(T_s - T) \ll (T_s - T_\infty)$ (See figs in (a))

$\Rightarrow (T - T_\infty) \approx (T_s - T_\infty)$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = -\frac{g\beta}{\nu}(T_s - T_\infty) \quad \text{const}$$

$$r\frac{dw}{dr} = -\frac{g\beta(T_s - T_\infty)}{\nu}\frac{r^2}{2} + \cancel{C_1}^{\,0} \quad \text{(from B.C)}$$

$$\therefore w(r) = \frac{g\beta(T_s - T_\infty)R^2}{4\nu}\left(1 - r^2/R^2\right)$$

Therefore $\dot{m} = \int_A \rho_\infty w \, dA = \rho_\infty 2\pi \int_0^R w(r)\, r\, dr$

$$\int_0^R w(r)\, r\, dr = \frac{g\beta(T_s - T_\infty)R^2}{4\nu}\int_0^R r\left(1 - r^2/R^2\right) dr$$

$$= \frac{g\beta(T_s - T_\infty)R^4}{16\nu}$$

$r - r^3/R^2$

$\left.\frac{r^2}{2} - \frac{r^4}{4R^2}\right|_0^R$

$\frac{R^2}{2} - \frac{R^2}{4}$

$$\therefore \boxed{\dot{m} = \frac{\pi \rho_\infty g\beta(T_s - T_\infty)R^4}{8\nu}}$$

d) $$Nu_L \overset{?}{=} \frac{Ra_R}{16}$$

$$Nu_L = \frac{hL}{k_f} = \frac{\dot{q}_s}{(T_s - T_\infty)} \frac{L}{k_f}.$$

$$\dot{q}_s = \frac{\text{total heat transferred}}{\text{Area}} = \frac{\dot{Q}}{2\pi RL}$$

$$\dot{Q} = \dot{m} c_p (T_s - T_\infty)$$

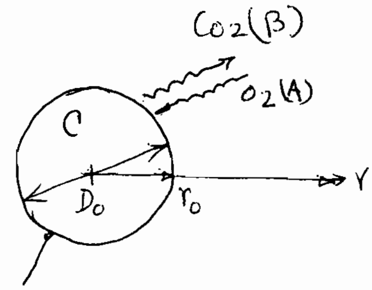
$$\dot{q}_s = \frac{\cancel{\pi}(\rho_\infty c_p)\, g\beta(T_s - T_\infty) R^4}{8\nu \; 2\cancel{\pi} RL} (T_s - T_\infty)$$

$$= \frac{g\beta(T_s - T_\infty) R^3}{16 \nu L} (\rho_\infty c_p)(T_s - T_\infty)$$

$$Nu_L = \left(\frac{g\beta(T_s - T_\infty) R^3}{\nu\alpha}\right) \frac{1}{16} = \frac{Ra_R}{16}$$

$$\boxed{Nu_L = \frac{Ra_R}{16}}$$

4



$\dot{\Omega}$, $C + O_2 \rightarrow CO_2$

$\mathcal{D} = 1.71 \times 10^{-4}\ m^2/sec.$

O_2, $T = 1500k$

$P = 1 atm.$

$[C_T] = \dfrac{P}{RT}$; $R = 82.05 \dfrac{cm^3 atm}{gmol\ K}$

$= \dfrac{1.0}{82.05 \times 1500} = 8.125 \times 10^{-6} \dfrac{mol}{cc}.$

$$\boxed{[O_2]_\infty = 8.125 \times 10^{-3} \dfrac{Kmol}{m^3}}$$

a) Conservation of $[O_2]$, oxygen concentration is, in spherically symmetric system,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \mathcal{D} \frac{d[O_2]}{dr}\right) = 0$$

$$\Rightarrow r^2 \frac{d[O_2]}{dr} = C_1 \Rightarrow \boxed{[O_2] = -\frac{C_1}{r} + C_2.}$$

When $r \rightarrow \infty$ $[O_2] \rightarrow [O_2]_\infty \Rightarrow C_2 = [O_2]_\infty$

$$\therefore [O_2] = -\frac{C_1}{r} + [O_2]_\infty$$

to determine C_1 we require another B.C. which is the surface condition.

Also $[O_2] + [CO_2] = [C_T]$ $[C_T]$ total concentration.

$$\Rightarrow [CO_2] = [C_T] - [O_2]$$

$[CO_2]_\infty = 0 \Rightarrow [C_T] = [O_2]_\infty$

$$\therefore [CO_2] = [O_2]_\infty - [O_2]$$

$$[O_2] = -\frac{C_1}{r} + [O_2]_\infty .$$

i) When the rate is infinitely fast.

$$[O_2]_s = 0 \quad \text{ie} \quad @ \; r = r_o$$

$$\therefore [O_2]_s = 0 = -\frac{C_1}{r_o} + [O_2]_\infty$$

$$\Rightarrow \quad C_1 = r_o [O_2]_\infty$$

$$\boxed{\begin{aligned} \therefore [O_2] &= [O_2]_\infty \left(1 - \frac{r_o}{r}\right) = 8.125 \times 10^{-3} \left(1 - \frac{r_o}{r}\right) \text{ Kmol/m}^3 \\ [CO_2] &= \frac{r_o}{r} [O_2]_\infty . = \frac{r_o}{r} \, 8.125 \times 10^{-3} \text{ Kmol/m}^3 \end{aligned}}$$

(ii) Surface condition:

$$-\dot{\imath}_{O_2} = D \left. \frac{d[O_2]}{dr} \right|_{r = r_o}$$

$$+ k_1 [O_2]_s = -\frac{C_1}{r_o} k_1 + k_1 [O_2]_\infty = D \frac{C_1}{r_o^2}$$

$$C_1 \left\{ \frac{D}{r_o^2} + \frac{k_1}{r_o} \right\} = k_1 [O_2]_\infty$$

$$\therefore \quad C_1 = \frac{k_1 [O_2]_\infty}{(k_1 / r_o) + (D / r_o^2)}$$

$$\therefore [O_2] = -\frac{K_1 [O_2]_\infty}{\left[\left(K_1/r_0\right) + \left(\mathcal{D}\ /r_0^2\right)\right] r} + [O_2]_\infty$$

$$[C_{O_2}] = \frac{K_1 [O_2]_\infty}{\left[\left(K_1/r_0\right) + \left(\mathcal{D}\ /r_0^2\right)\right] r}$$

b) Conservation of Carbon particle.

$$\frac{dm}{dt} = -W_c \dot{\Omega}_c A_s =$$

$\Omega_c = kg/m^2 \cdot sec.$

$C + O_2 \rightarrow CO_2$

$$= -W_c K_1 [O_2]_s \pi D^2$$

$\Omega_c = -\Omega_{O_2} = \Omega_{CO_2}$

$\Omega_{CO_2} = K_1 [O_2]_s$

$$m = \rho_c V = \rho_c \frac{4}{3}\pi r^3$$

$$= \rho_c \frac{\pi}{6} D^3$$

$\dot{\Omega}_c = \dot{\Omega}_{O_2}$

$A_s = 4\pi r^2 \quad \frac{D^2}{4}$

$= \pi D^2$.

$\frac{D^3}{8}$

$$\rho_c \frac{\cancel{\pi}}{6} \frac{dD^3}{dt} = -\cancel{\pi} K_1 D^2 [O_2]_s W_c$$

$$\frac{dD^3}{dt} = -\left(\frac{6K_1}{\rho_c}\right) D^2 W_c [O_2]_s$$

$$[O_2]_s = -\frac{K_1 [O_2]_\infty}{\frac{K_1}{\cancel{r_0}}\cancel{r_0} + \frac{\mathcal{D}}{r_0^{\cancel{2}}}\cancel{r_0}} + [O_2]_\infty.$$

$$= -\frac{K_1 [O_2]_\infty}{K_1 + 2\mathcal{D}/D} + [O_2]_\infty.$$

$$\therefore [O_2]_s = [O_2]_\infty \left\{ 1 - \frac{1}{1 + \frac{2\mathcal{D}}{k_1 D}} \right\}$$

if $\frac{2\mathcal{D}}{k_1 D} \gg 1$ $\quad [O_2]_s = [O_2]_\infty$

$\Rightarrow D \ll \left(\frac{2\mathcal{D}}{k_1}\right) \quad R \ll \left(\frac{\mathcal{D}}{k_1}\right)$

$$\therefore \frac{dD^3}{dt} = -\left(\frac{6 k_1}{\rho_c}\right) D^2 W_c [O_2]_\infty$$

$$\Rightarrow \frac{dD}{dt} = -\left(\frac{2 k_1 W_c}{\rho_c}\right) [O_2]_\infty$$

$$\therefore D - D_0 = -\left(\frac{2 k_1 W_c}{\rho_c}\right) [O_2]_\infty t$$

$$\therefore \boxed{t = \frac{D_0 - D}{(2 W_c k_1/\rho_c)[O_2]_\infty}} = \frac{\rho_c (R_0 - R)}{W_c k_1 [O_2]_\infty}$$

c) for $D_0 = 0.1$ mm, $D = 0.05$ mm

$k_1 = 0.1$ m/sec $\quad \rho_c = 1950$ kg/m^3

$[O_2]_\infty = 8.125 \times 10^{-3}$ kmol/m^3 (from above)

$$t = \frac{0.05 \times 10^{-3}}{\left(\frac{2 \times 12 \times 0.1}{1950}\right) * 8.125 \times 10^{-3}} = 5 \text{ sec}$$