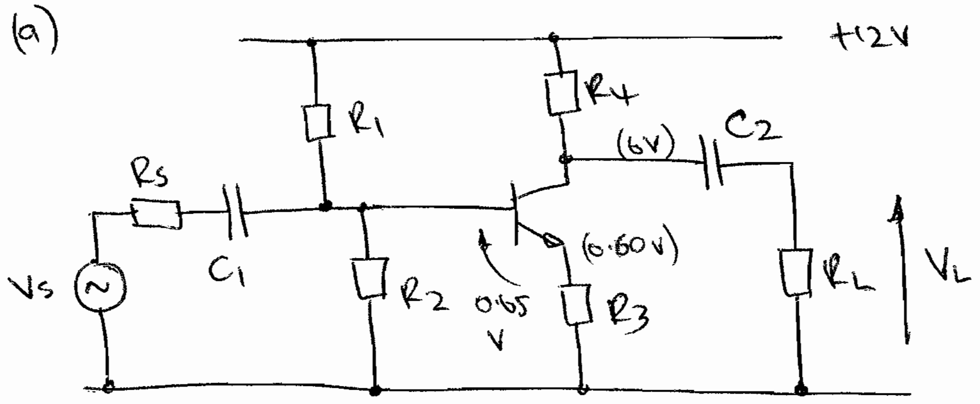
PART IIA 2006
3B1: Radio frequency electronics
Principal Assessor: Dr P A Robertson

3B1 *Datasheet: None*

1) (a)



Bias resistors R_1, R_2 Gain $\simeq -R_4/R_3$
Load R_L
collector load R_4

B/width @ input (high-pass) $= \dfrac{1}{2\pi C_1 (R_s + R_1 \| R_2 \| h_{fe}.R_3)}$

B/width @ output (high-pass) $= \dfrac{1}{2\pi C_2 (R_4 + R_L)}$

(b) $V_s = 75\ mV_{pp}$, $R_s = 100k\Omega$
$V_L = 1\ V_{pp}$, $R_L = 10k\Omega$

Assume $h_{fe} = 200$

$$\therefore \frac{P_{in}}{P_{out}} = \frac{\left(\frac{75}{2}\times 10^{-3}\right)^2 / 10^5}{1^2/10^4} = 1.46\times 10^{-4} = -38.5\ dB$$

$\therefore$ allowing 20dB gain/stage, we need 2 stages (40dB)

linear gain (voltage) per stage $= \sqrt[2]{\dfrac{1}{75\times 10^{-3}} \Big/ \dfrac{1}{2^3}}$ as 3 couplings $= 10.3$ $\therefore$ 10 say

and input/output impedance ratio $= \sqrt[2]{\dfrac{100}{10}} \simeq 3.3$

1(b) cont.

Setting base bias @ 1·4 V dc. $\therefore \dfrac{R_2}{R_1+R_2} \times 12 = 1{\cdot}4 \text{ V}$

$\sim (0{\cdot}6 + 0{\cdot}65)\text{V} + 10\%$

[and choose $R_2 \sim 1{\cdot}5 \times$ reqd R_{in}] $\therefore R_1 \simeq 8R_2$

(100k source)	R_1	R_2	R_3	R_4
stage 1:	1M2	150K	3K3	33K
stage 2:	390K	47K	1k	10K
(10K load)				

[R_4 = reqd. R_{out}]

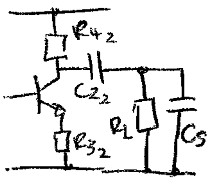
near standard values.

For 50Hz : $\dfrac{1}{2\pi \cdot 214 \times 10^3 \cdot C_1} = 50 \quad \therefore C_{1_1} = 15\text{nF}$

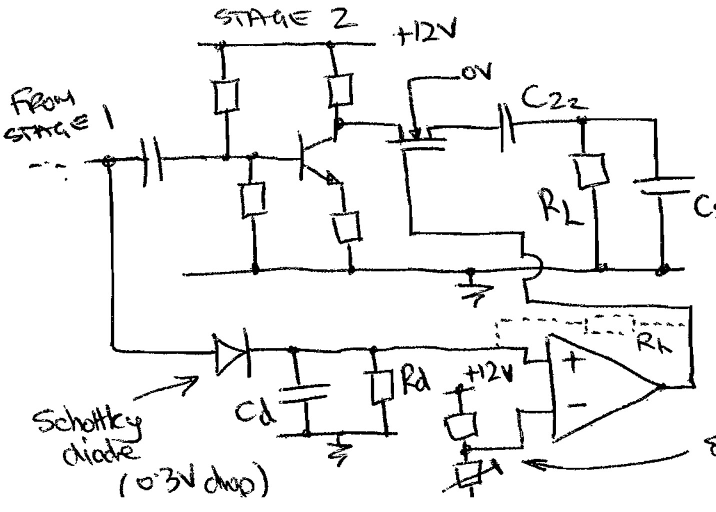
$C_{2_1} \equiv C_{1_2} > 47\text{nF} \quad \therefore$ say 100nF and $C_{2_2} > 150\text{nf}$ $\therefore$ say 330nf

And for 2kHz high-freq roll-off, we shall need to shunt the load resistance with a capacitor :-

$$2000 = \frac{1}{2\pi \left(\frac{10\times10^3}{2}\right) C_S} \quad \therefore \underline{C_S = 15\text{nF}}$$



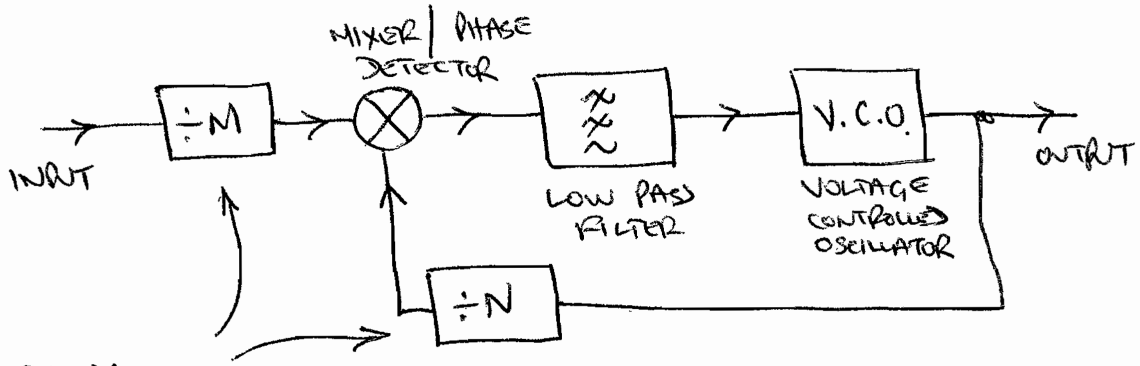
(c) We need to monitor the audio signal amplitude $\therefore$ use a diode demodulator on the output of stage 1 and use this to control a MOSFET analogue switch to control the signal path to R_L (note we need stage 2 as a buffer or switching R_L will alter the output level being monitored)



- Choose $C_d \times R_d \sim$ seconds
  eg: $R_d = 1\text{M}\Omega$
  $C_d = 3.3\mu\text{F}$
- Can also add hysteresis with R_h eg: 22 MΩ

set for $\left(8 \times \frac{75\times10^{-3}}{2}\right) + 6 - 0.3\text{V} \simeq \underline{6{\cdot}0\text{V threshold.}}$

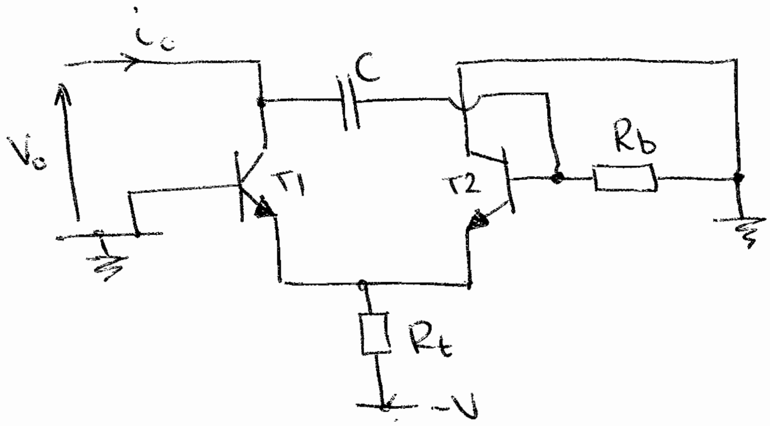
2(a)



P.L.L. SCHEMATIC

The output from a V.C.O (perhaps divided down in freq.) is compared to the input signal (perhaps also divided down by a different factor) by a phase detector. The output of the phase detector controls the V.C.O. through a low-pass filter such that it pushes the V.C.O. output to match the input in frequency and phase. The low-pass filter gives a 'flywheel' effect to the V.C.O. output freq. and with dividers M and N, it is possible to create an output freq., f_{out}, such that $\frac{f_{out}}{N} = \frac{f_{in}}{M}$ - hence a fixed reference freq., f_{in}, may be scaled to a digitally selectable output frequency from the V.C.O.

(b)

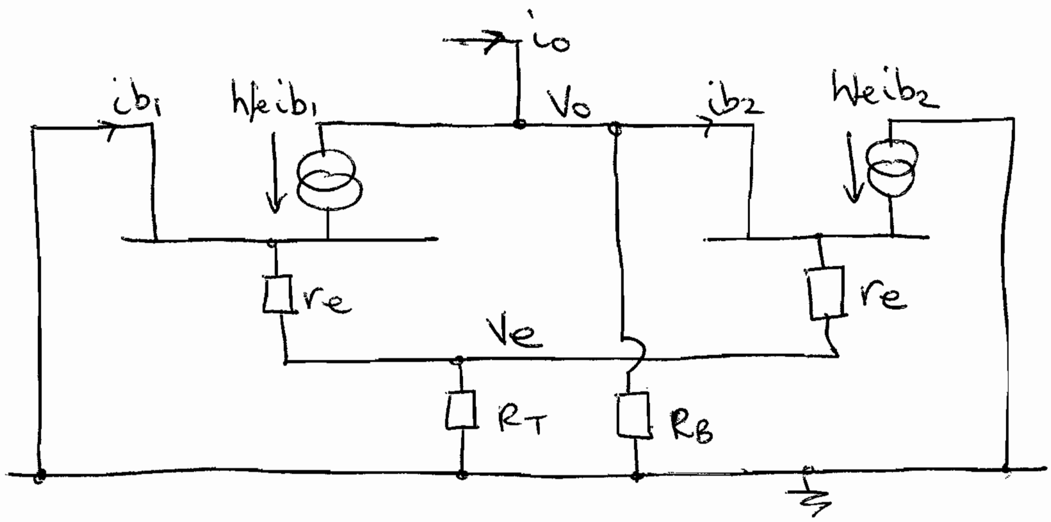


Consider the transistor pair connected above.

Small signal model :-

(using re transistor model; or could use hie as in lecture notes)

2b)



① — $V_e = h_{fe}(i_{b_1} + i_{b_2}) R_T$, $V_e = -h_{fe} i_{b_1} r_e$ — ②

③ — $i_o = h_{fe} i_{b_1} + \dfrac{V_o}{R_B} + i_{b_2}$

④ — $V_o - V_e = r_e h_{fe} i_{b_2}$

Sub. for i_{b_1} from ② in ③ and for i_{b_2} from ④ in ③ } $i_o = \dfrac{-V_e}{r_e} + \dfrac{V_o}{R_B} + \dfrac{V_o - V_e}{h_{fe} r_e}$

⑤ — ∴ $i_o = \dfrac{-V_e}{r_e} + \dfrac{V_o}{R_B} + \dfrac{V_o}{h_{fe} r_e} - \cancel{\dfrac{V_e}{h_{fe} r_e}}$ small compared to 1st term as $h_{fe} \gg 1$

Sub. for i_{b_1} from ② in ① and for i_{b_2} from ④ in ① } $V_e = -\dfrac{V_e R_T}{r_e} + (V_o - V_e)\dfrac{R_T}{r_e}$

⑥ — ∴ $V_e = \dfrac{V_o R_T}{r_e(1 + 2R_T/r_e)} = V_o \cdot \dfrac{R_T}{(r_e + 2R_T)}$

$\simeq V_o/2$ as $r_e \ll R_T$

Sub. ⑥ into ⑤ to elliminate V_e : $i_o = V_o \left[\dfrac{-1}{2r_e} + \dfrac{1}{R_B} + \dfrac{1}{h_{fe} r_e}\right]$

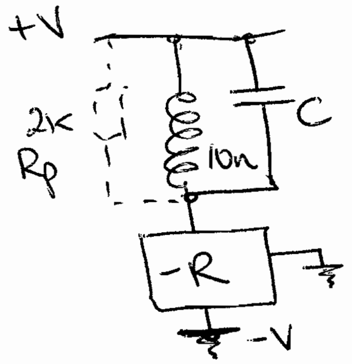
Hence the input impedance = $\left[\dfrac{-1}{2r_e} + \dfrac{1}{R_B} + \dfrac{1}{h_{fe} r_e}\right]^{-1}$ which is ∴ equivalent to $-2r_e \,||\, R_B \,||\, \underbrace{h_{fe} r_e}_{\text{this } = h_{ie}}$

So, we have a negative resistance (r_e is small)

(5)

2(b) If $L = 10nH$ @ 1GHz $j\omega L = j63\,\Omega$

so a Q factor of 30 gives an effective parallel loss resistor of $30 \times 63 \simeq 2k\Omega$ — hence the −ve resistance must be less than this



$$\frac{1}{2\pi\sqrt{LC}} = 10^9\ Hz$$

10×10^{-9} $\therefore C = 2.53\,pF$

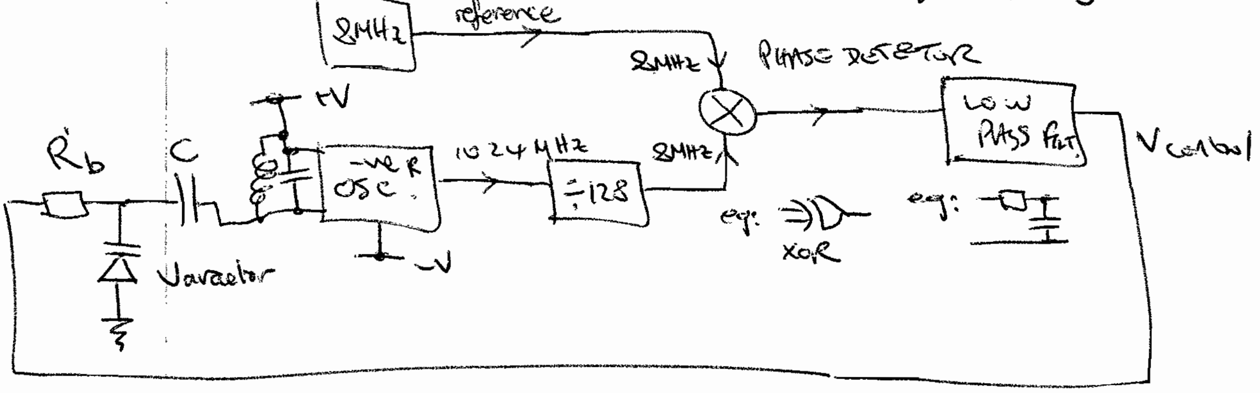
For the −ve resistance circuit, we need $r_e \simeq 100$'s Ω at most. $\therefore$ choose $R_b = 10k\Omega$ say $(\gg R_p)$

$r_e = \dfrac{0.025}{I_c}$ where I_c is the collector bias current in mA

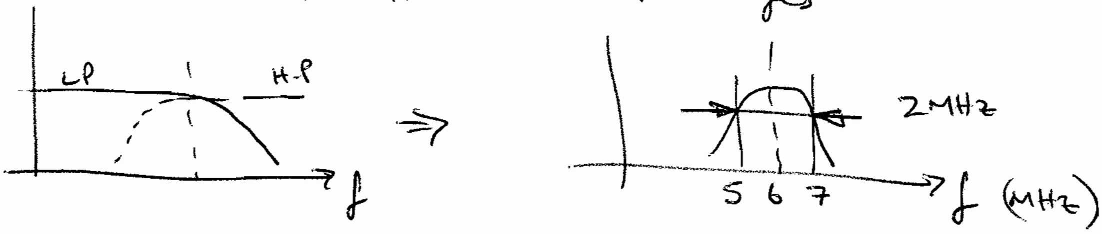
$\therefore$ set $I_c = 1mA$ say $\therefore r_e = 25\,\Omega$
giving a negative resistance $\sim -50\,\Omega$

eg: for a supply of $\pm 5V$, $R_T = 2k2\,\Omega$ will bias each transistor with approx. 1mA.

2(d) Putting a varactor in parallel with the inductor L allows voltage control of frequency.

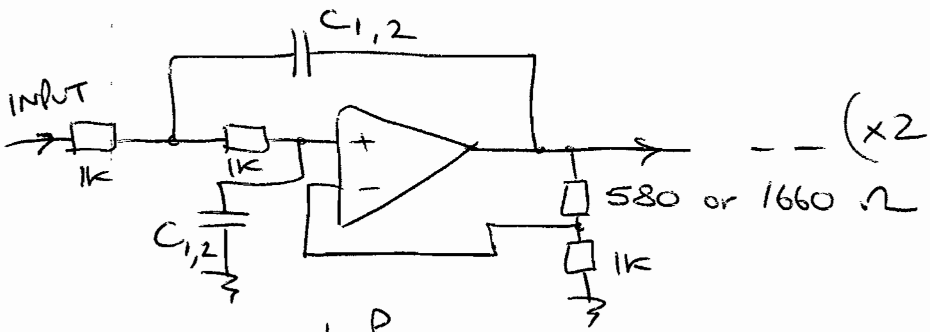


3(a). VCVS – choose Chebyshev for sharpest band edges
– cascade H.P and L.P. stages



H.P. : $f_c = 5$ MHz
L.P. : $f_c = 7$ MHz

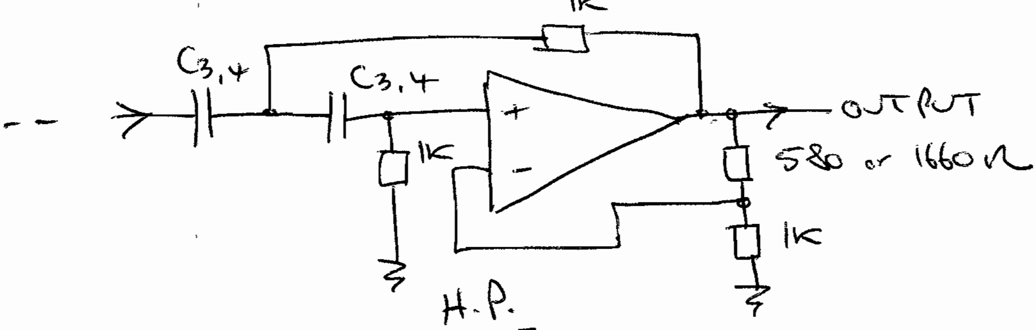
$f_n = 0.597$ $A = 1.582$
1.031 $= 2.660$



(x2 cascaded)

$Z = (A-1)R$ where $R = 1k\Omega$.

L.P.



(x 2 cascaded)

H.P.

For L.P. $f_c = \dfrac{1}{2\pi f_n R C_{1,2}} = 7\times10^6$, $R = 1k\Omega$ throughout

H.P. $f_c = \dfrac{1}{2\pi \frac{1}{f_n} R C_{3,4}} = 5\times10^6$

$\therefore C_1 = 38$ pF $C_3 = 19$ pF
$C_2 = 22$ pF $C_4 = 33$ pF

(b)

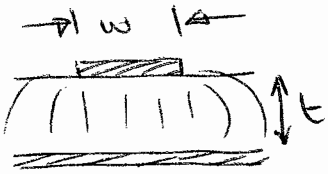
- Easy to attach components & make connections
- Z_0 can be higher for given board thickness

- Lower losses + radiation
- Lower cross-coupling to other lines
- Z_0 can be lower for given board thickness.

3(b) cont.

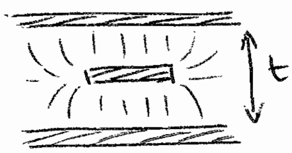
$$Z_0 = \sqrt{\frac{L}{C}} \qquad v = \frac{1}{\sqrt{LC}} = \frac{C_0}{\sqrt{\varepsilon_r}}$$

For microstrip:



$$C \simeq \frac{(w+2t)\,\varepsilon_0\varepsilon_r}{t}$$

For stripline:



$$C \simeq \frac{2(w+t)\,\varepsilon_0\varepsilon_r}{t/2}$$

For $\varepsilon_r = 3.8 \Rightarrow v = 154\times10^6$ m/s $= \dfrac{1}{\sqrt{LC}}$

with $Z_0 = 33 = \sqrt{L/C} \quad \therefore \; L = 1089\,C$

$$(154\times10^6)^2 = \frac{1}{LC} = \frac{1}{1089\,C^2} \quad \therefore \; C = 197\times10^{-12} \text{ F/m}$$

For microstrip: $197\times10^{-12} = \dfrac{(w+2\times10^{-3})\,.\,3.8\,.\,8.854\times10^{-12}}{10^{-3}}$

$\therefore \; w = 3.85\times10^{-3}$ m $= \underline{3.85 \text{ mm}}$

For stripline: $197\times10^{-12} = \dfrac{4\,(w+10^{-3})\,.\,3.8\,.\,8.854\times10^{-12}}{10^{-3}}$

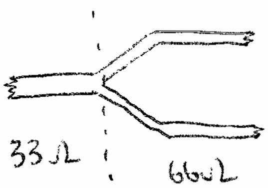
$\therefore \; w = 4.6\times10^{-4}$ m $= \underline{0.46 \text{ mm}}$

3(c) $f = 1030\times10^6$ Hz $\quad \therefore \; 154\times10^6 = 1030\times10^6\,.\,\lambda$

$\qquad v = f\,.\,\lambda$

Hence $\lambda = 0.15$ m (15cm)

For 90° phase shift $\equiv \lambda/4 = \underline{3.7 \text{ cm track difference}}$

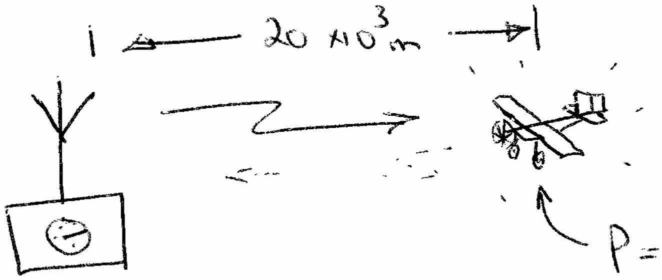


For matching $Z_0 = 66\Omega \quad \therefore \; w = 0.92$ mm in microstrip

But for $Z_0 = 66\Omega$ in stripline, $w = -0.26$ mm!

$\therefore$ not possible

4(a)



$$P = \frac{P_{radar}}{4\pi R^2} \cdot \text{GAIN} \quad W/m^2$$

$$P = \frac{25\times10^3}{4\pi\,(20\times10^3)^2} \cdot 500 = 2.5\times10^{-3}\ w/m^2$$

$$P = \tfrac{1}{2}\eta H^2 = \tfrac{1}{2} E^2/\eta \quad \text{with} \quad \eta = 120\pi$$

$$\therefore\ H = 3.6\times10^{-3}\ A/m\ , \quad E = 1.37\ V/m$$

(b) $P_{rad} = 2.5\times10^{-3}\times 20 = 0.05\ W$ isotropic

$$G = \frac{4\pi A_e}{\lambda^2} = 500 \quad \text{and} \quad \lambda = \frac{c}{f} = 0.291\ m$$

$$\therefore\ A_e = \frac{500 \cdot 0.29^2}{4\pi} = 3.35\ m^2$$

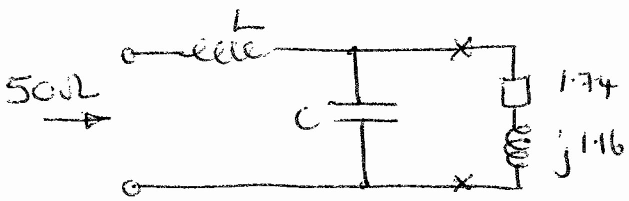
$$\therefore\ P_{rec} = \frac{50\times10^3}{4\pi\,(20\times10^3)^2} \cdot 3.35 = 33\ pW$$

$$\therefore\ 33\times10^{-12} = \tfrac{1}{2} V^2/R = V^2/100 \Rightarrow V = 58\ \mu V$$

with $R = 50\Omega$

(c) $87 + j58\ \Omega\ /\ 50\Omega \rightarrow 1.74 + j1.16$

From Smith chart, volt. ref. coef. $= 0.46$



4(c) SMITH CHART

- $A = 1{\cdot}74 + j1{\cdot}16$

  ↓

  admittance $B = 0{\cdot}4 - j0{\cdot}26$

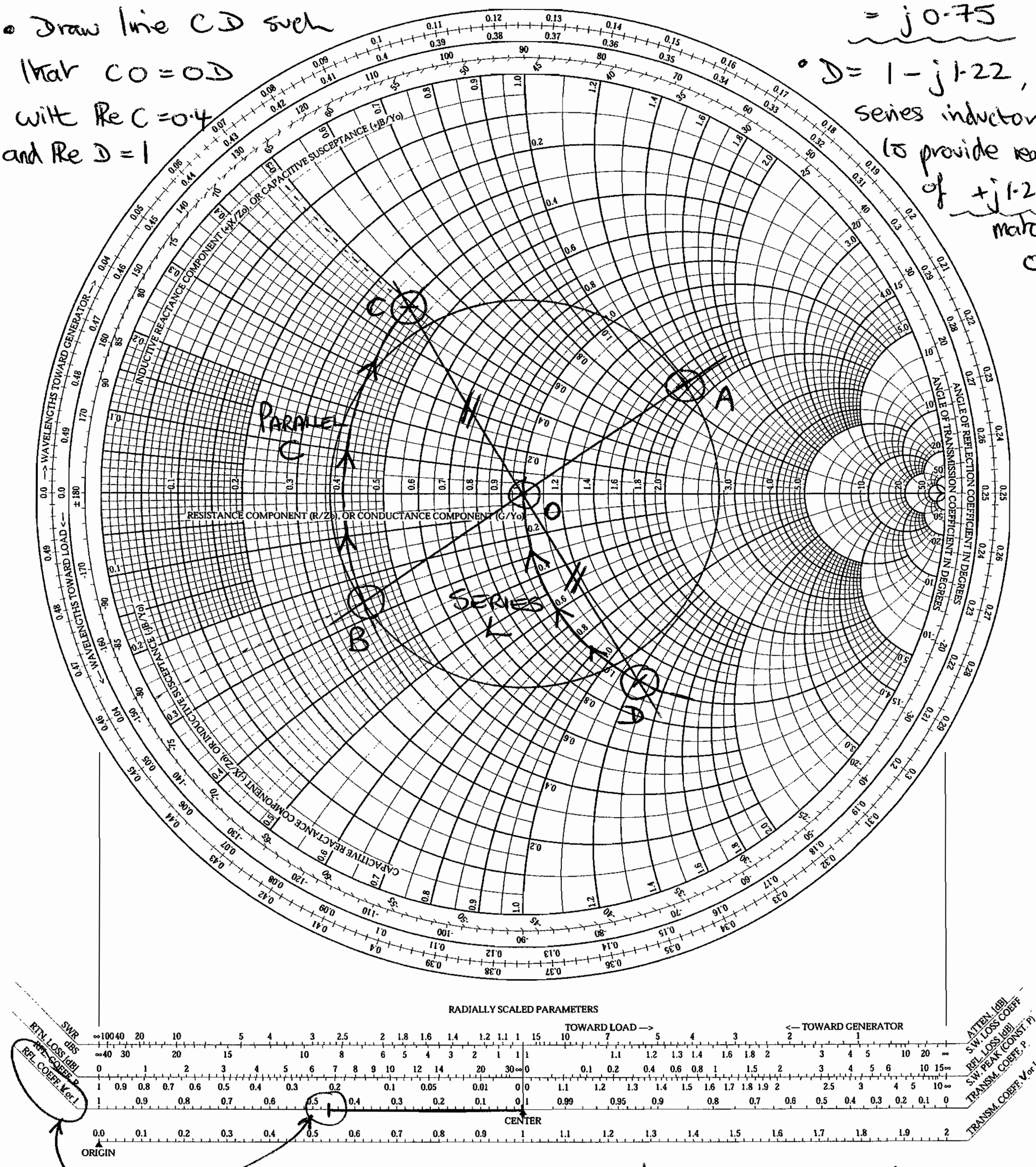
- $C = 0{\cdot}4 + j0{\cdot}49$

  hence parallel capacitor gives susceptance of $j(0{\cdot}49 - {}^{-}0{\cdot}26)$

  $= j0{\cdot}75$

- Draw line CD such that CO = OD with Re C = 0·4 and Re D = 1

- $D = 1 - j1{\cdot}22$, so series inductor needs to provide reactance of $+j1{\cdot}22$ to match at O.



0·46

A, B
impedance → D, C ← admittance

Hence: $j\omega C = j0{\cdot}75/50$

$j\omega L = j1{\cdot}22 \times 50$

with $\omega = 2\pi \times 1030 \times 10^6$

$\Rightarrow C = 2{\cdot}3\,pF, \quad L = 9{\cdot}4\,nH$

ENGINEERING TRIPOS PART IIA 2006

Module 3B1, Radio Frequency Electronics - Assessors' Report

Q1. Transistor amplifier

This was a fairly popular question and generally well attempted. All candidates knew the basic circuit layout for an amplifier and many made good attempts at the design, although some chose impractical component values. The final part, on a signal switching circuit, was not answered very well in many cases; with few attempts including the required demodulator circuit.

Q2. Phase locked loop & oscillator

Most candidates could draw the PLL schematic and knew how it worked. Also, many candidates correctly derived the negative impedance part (which was standard book-work) although some had remembered the final result and contrived to get there with an incorrect circuit. Several candidates made a good attempt at the final PLL design, locking the 1.024 GHz signal to an 8 MHz quartz crystal reference.

Q3. VCVS filter & microstrip design

The most popular question and well answered in most cases. Only a few candidates did not recall the VCVS circuits accurately and almost all selected the most appropriate filter type. The second half on microstrip design was also generally well answered, although only a few worked out the impedance of a stripline geometry and realised that the final part, needing a 66 ohm characteristic impedance, could only be realised in microstrip.

Q4. Antenna theory & Smith chart

This was also a fairly popular question attracting a wide range of attempts. The first part was quite straightforward and most candidates managed to work out the radar signal levels quite well. The section on impedance matching was well answered in most cases, although a number of attempts used a different design method to the graphical method elicited in the question and some candidates used an extra series capacitor to simplify the Smith chart plot.

Dr P.A. Robertson, May 2006

3B1 – Radio Frequency Electronics 2006

(some numerical answers etc.)

1(b)

	R1	R2	R3	R4	
	1M2	150k	3k3	33k	stage 1
	390k	47k	1k	10k	stage 2 100nF coupling caps, 15nF shunt cap.

1(c) Use diode demodulator and comparator to monitor signal level and MOSFET analogue switch to mute the signal when at low levels.

2(b) With L = 10 nH @ 1 GHz = j63 Ω, so Q factor = 30 gives shunt resistance of ~2 k Ω, hence choose transistor bias to give –R of LESS than this eg. -50 Ω to guarantee oscillation. C = 2.53 pF for LC resonance at 1 GHz.

3(a) Choose Chebyshev for sharp frequency cut-off. Low pass up to 7 MHz, high pass from 5 MHz gives cascaded bandpass 5-7 MHz.
C1 = 38 pF, C2 = 22 pF (low pass stages), (A-1)R = 580 or 1660 Ω
C3 = 19 pF, C4 = 33 pF (high pass stages), ditto

3(b) w = 3.85 mm microstrip, 0.46 mm stripline for 33 Ω.

3(c) 37 mm track length difference = 90 degrees.
For 66 Ω track, use microstrip 0.92 mm wide; not possible in stripline (-0.26 mm).

4(a) P = 2.5 mW/m^2, H = 3.6 x 10^{-3} A/m, E = 1.37 V/m.

4(b) P rec. = 33 pW, Vrec. = 58 μV into R = 50 Ω.

4(c) Normalise to 1.74 + j1.16. Voltage refl. coeff. = 0.46
Match with C = 2.3 pF and L = 9.4 nH.