

1/ (a) Specific Magnetic Loading.

Datasheet: None

Average Flux Density over one pole pitch.

Related to the peak fundamental airgap flux and magnetising current and/or iron losses.

Specific Electrical Loading.

Total electrical current per unit length averaged around the airgap. ~ relates to the resistive losses on load.

(Clearly not the current density, although related, nor a true mmf)

$$(b) S = \frac{\pi}{\sqrt{2}} \cdot \pi \left( \frac{d}{2} \right)^2 L \frac{\omega}{P} \times \bar{B} \times \bar{J}$$

$$= \frac{\pi^2}{\sqrt{2}} \frac{0.03^2}{4} \cdot 0.035 \times 2\pi \times 50 \times 0.5 \times 30,000.$$

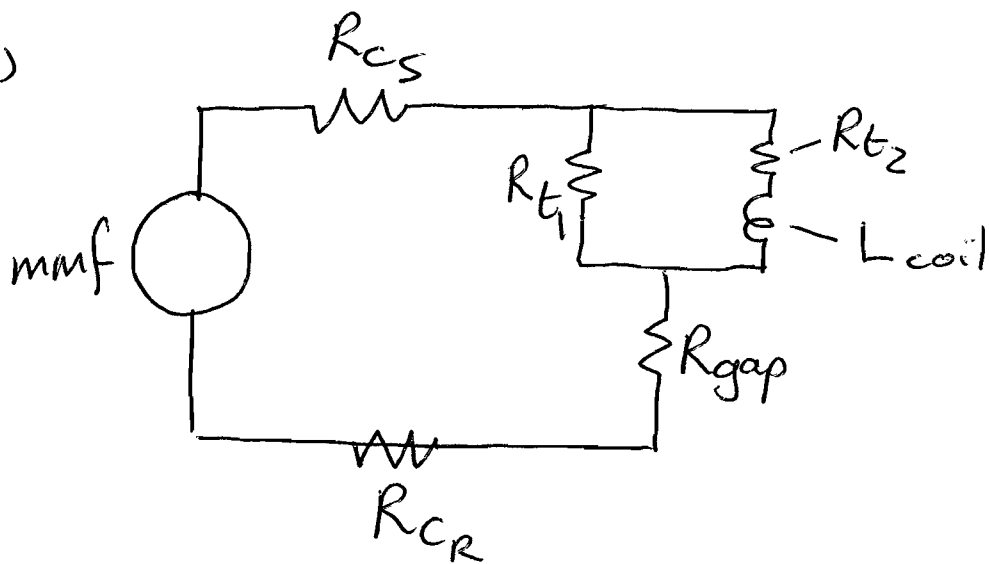
usual      usual.

$$= 259 \text{ VA.}$$

$$\text{efficiency} \times \text{powerfactor} = 0.5 \times 0.7 \quad ? \text{ low for small motors!}$$

$$P_{out} = 90 \text{ W.}$$

1 (c)



The copper coil means we get two flux components, one lagging the other. This resolves into a good rotating flux vector for good induction motor operation.

(d) 90 W is quite large, so simple PM motor drives with single ended coils would be very inefficient and not good for modern goods. Thus a full bridge is needed. 3 phases is efficient.

Ferrite is cheap, but  $B$  is low, so the motor would be large.  $NeFeB$  is much more expensive but allows for a small machine. Maybe cost effective eventually.

2 (a) Neglect  $R_1$  and  $\omega L_1$  and assume that  $\omega L_2 \ll R_2/s$ , i.e. that the slip is small.

$$T \omega_s = 3 I_2^2 \frac{R_2}{s}$$

$$I_2 = \frac{V}{R_2/s}$$

$$\begin{aligned} \text{so } T &= \frac{3}{\omega_s} V^2 \frac{R_2}{s} \left( \frac{1}{R_2/s} \right)^2 \\ &= \frac{3 V^2}{\omega_s} \cdot \frac{s}{R_2} \end{aligned}$$

$$\text{But } s = \frac{\omega_s - \omega_r}{\omega_s}$$

$$\text{so } T = \frac{3 V^2}{\omega_s^2 R_2} \cdot (\omega_s - \omega_r)$$

Note that  $\omega_s$  is the synchronous angular speed and not supply frequency so  $p$ , the number of pole-pairs, does not appear.

(b) Rated torque implies operation fully fluxed and that needs  $V/f$  control.

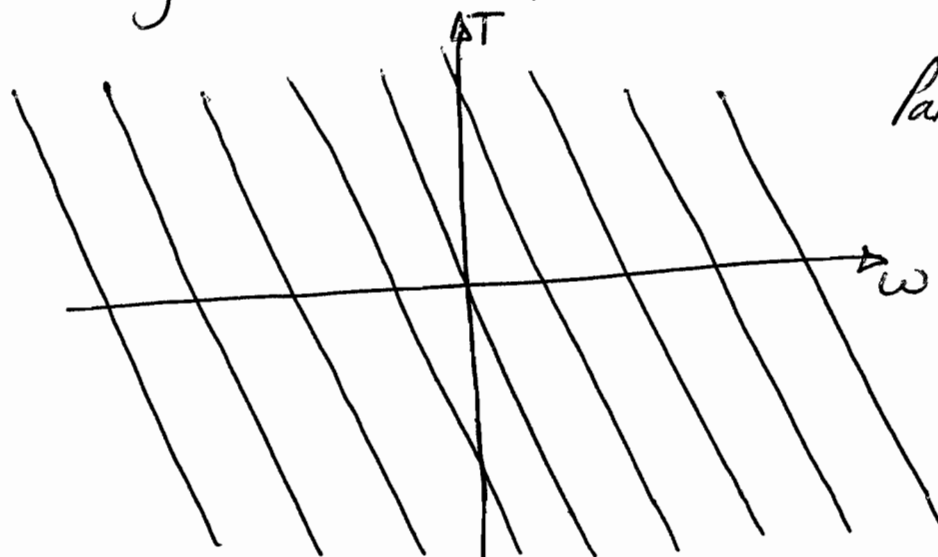
Voltage boosting made be necessary at low frequencies with  $V/f$  control.

(The phase pattern must be reversible; this is done in the modulation)

2(b) cont.

Torque - Speed in 4 quadrants:

Only want stable part of the characteristic!



Parallel lines  
with  $\frac{V}{f} = \text{const.}$

(c) Rated Torque is

$$\frac{4000}{314} = 12.7 \text{ Nm} \quad \text{assuming } s \approx 0$$

No comment so Delta

( $R_2$  referred  $< R_1$ )  
both small

(i)  $1.5 \times 12.7 \text{ Nm}$ . (Best found using similar triangles in the  $T-\omega$  curves above)

$\omega_s$  at  $33 \text{ Hz}$   $2\pi \cdot 33 = 207.3 \text{ rad/s}$

$$T = 3 \left( \frac{V}{\omega} \right)^2 \frac{(\omega_s - \omega_r)}{R_2} \quad \text{Neglect } R_1, \omega_{L2}, \omega_{Lm}$$

$$19.1 = 3 \left( \frac{415}{100\pi} \right)^2 \frac{(207.3 - \omega_r)}{2.14} \quad \omega_r = 199.5 \text{ rad/s}$$

$$= 1905 \text{ rpm}$$

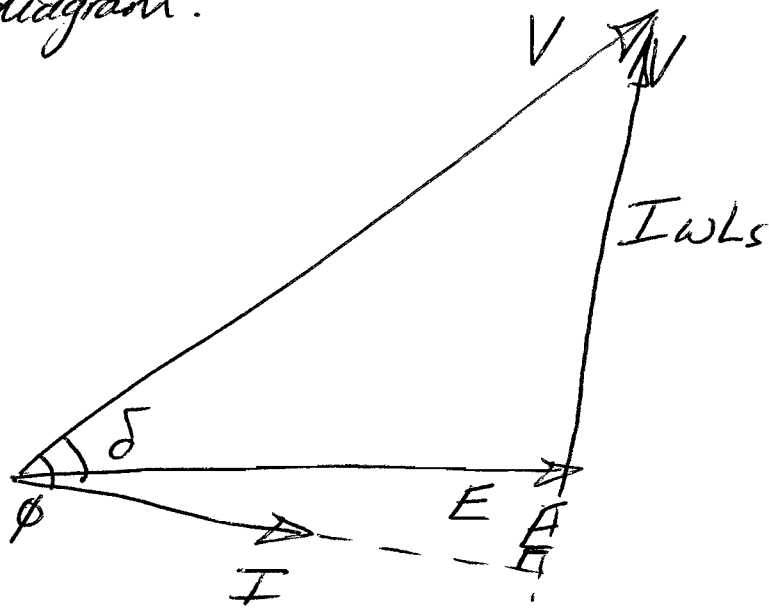
(ii)  $67 \text{ Hz} > 50 \text{ Hz}$  so field weaken (not similar triangles)

$$12.7 = 3 \left( \frac{415}{134\pi} \right)^2 \frac{(134\pi - \omega_r)}{2.14} \quad \omega_r = 411.7 \text{ rad/s}$$

$$= 3931 \text{ rpm}$$

3(a) The currents need to be sinusoidal too!  
 Thus sinusoidal references are needed for the current control loops. They must also be scalable to allow torque control. The  $\theta$  feedback maybe used to 'clock' through a sine function stored in memory, and scaled using floating point maths functions.

Phasor diagram:



$$P = 3IV \cos \phi = 3IE \sin(180 - \beta)$$

$$= 3IE \sin \beta$$

$$T = \frac{3IE}{\omega_s} \sin \beta \quad \text{For max torque per amp } \sin \beta = 1$$

ie.  $\beta = 90^\circ$

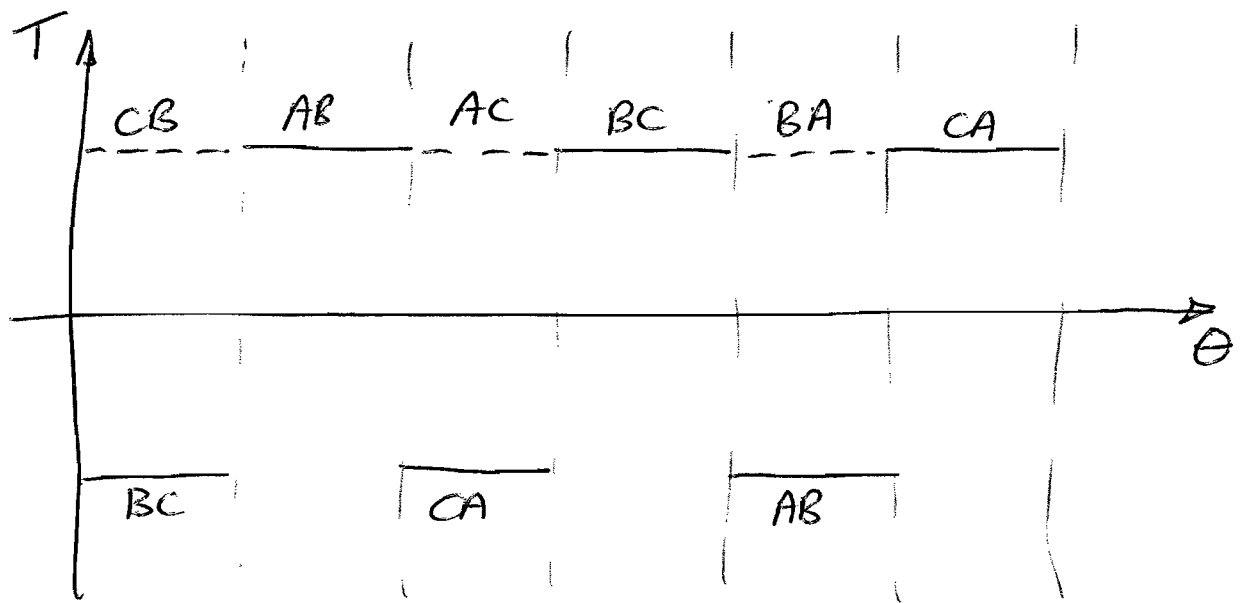
$E \propto \omega_s$  due to the permanent magnets.

$$T = KI \quad \text{like DC motors.}$$

3 (a) cont.  $T = J \frac{d^2 \theta}{dt^2}$  which is a second order

response. With a position control loop and no mechanical friction an undamped second order system will be formed. Noting that  $T = KI$ , it will be unstable. A PID controller can be added and the system may be stabilised. As  $\omega = \frac{d\theta}{dt}$ , the 'D' function necessary for damping is performed by the encoder. A 'D' function in the forward path is unattractive as the demand will be differentiated, possibly saturating the drive. Also differentiating a modern encoder signal in a microcontroller is not easy. A purpose built encoder with  $\theta$  and  $\omega$  outputs is therefore very practical.

3 (b) Star connected, for a current flowing in two phases produces the profile of Fig 3. With three phases, two on at any one time and switching between the flat sections, a constant torque can be formed:



Note that the flat part of the torque- $\theta$  characteristic must be  $\geq 60^\circ$

The difficulty is in eliminating the torque ripple at the switching instants. In a sinusoidal drive the torque ripple is small and a function of the current ripple. In trapezoidal drives there is a controlled current level, so current ripple and the finite diff at the switching  $\theta$ 's above.

4/(a) 3 Stack VR stepper:

3 Stator stacks mounted co-axially

A rotor common to the stator stacks

The rotor and stator have the same number of teeth.

Each stack has a single phase winding on all the stator poles.

Each stack of the stator is offset from the others by  $\beta$  where  $\beta = \frac{360^\circ}{3Ns}$

$$(b) \quad T_A = \frac{1}{2} I_A^2 \frac{dL_A}{d\theta} \quad \text{assuming no saturation.} \quad L_A = L_0 + L \cos m\theta$$

$$\Rightarrow T_A = -\frac{m}{2} i_A^2 L \sin(m\theta)$$

$\theta$  is taken as a displacement in tests, with a restoring torque. When working as a motor the torque is in the direction of  $\theta$

$$T_A = \frac{m}{2} i_A^2 L \sin(m\theta)$$

$$50\text{mH} - 70\text{mH} \Rightarrow 2L = 20\text{mH}, \quad L_0 = 60\text{mH}$$

$$\hat{T}_A = \frac{m}{2} i_a^2 L = \frac{8}{2} \cdot 5^2 \cdot 10\text{m} = \underline{1\text{Nm}}$$



$$H(c) \quad T_A = -\frac{m}{2} \dot{\alpha}^2 L \sin m\theta$$

$$T_B = -\frac{m}{2} \dot{\beta}^2 L \sin m(\theta - \beta)$$

$$\begin{aligned} \dot{\alpha} = \dot{\beta} \quad T_{A+B} &= -\frac{m}{2} \dot{\alpha}^2 L \left[ \sin m\theta + \sin m(\theta - \beta) \right] \\ &= -\frac{m}{2} \dot{\alpha}^2 L \cos\left(\frac{m\beta}{2}\right) \sin m\left(\theta - \frac{\beta}{2}\right) \end{aligned}$$

$$\text{For equilibrium } T_{A+B} = 0 \Rightarrow \theta = \beta/2$$

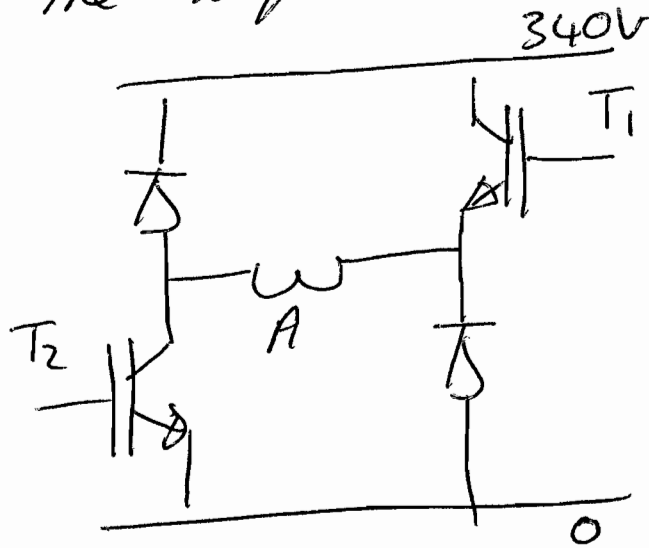
The half step position.

$$\frac{|T_{A+B}|}{|T_A|} = \frac{m \dot{\alpha}^2 L \cos\left(\frac{m\beta}{2}\right)}{\frac{m}{2} \dot{\alpha}^2 L} = 2 \cos\left(\frac{m\beta}{2}\right)$$

$$\beta = \frac{360^\circ}{3m}$$

$$= 2 \times \frac{1}{2} = 1$$

4 (d) The phase current need only be in one direction. However the power is high so current chopping is needed to control the torque.



chopping by turning  $T_1, T_2$  off and on together causes rapid changes in the current. This results in acoustic noise as a high pitched torque <sup>ripple</sup> is created. Keeping  $T_1$  on and chopping with  $T_2$  reduces the current ripple (and switching losses). Thus lowers the acoustic noise, important in a domestic appliance. (Used in some washing machines now.)  
Cheaper to manufacture. Simple electronics.