PART IIA 2006
3C4: Machine design – transmission
Principal Assessor: Dr D J Cole

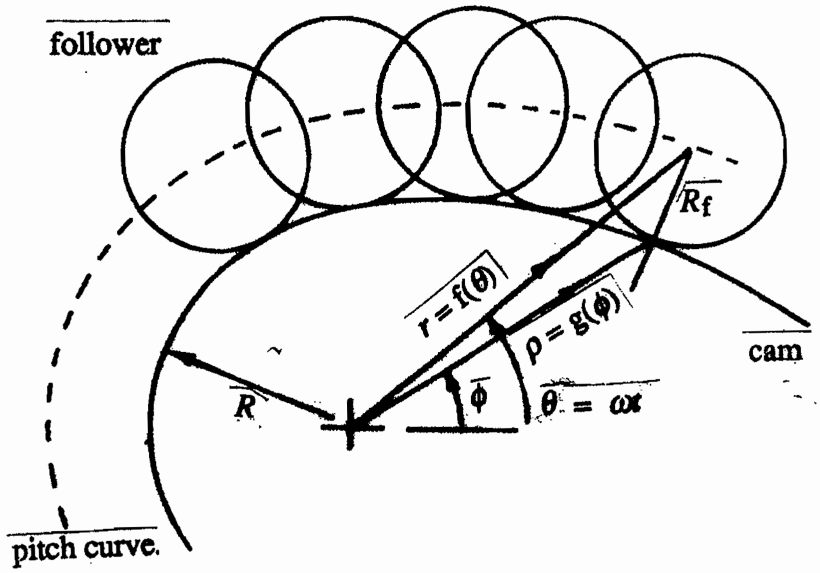
MODULE 3C4
MACHINE DESIGN

Datasheet: 3C3/3C4 Data Sheet

1/ a)

The problem is that the contact between the cam and the follower does not necessarily lie on the line between the centres of the cam and the roller. The lift cannot therefore be directly related to the radius of the cam.

In designing the shape of a cam it is convenient to imagine the cam to be stationary and the follower to roll around it rather than to allow the cam to rotate and the follower to reciprocate:



When viewed in this way, with the cam at rest, it is easy to see that the polar equation $r = f(\theta)$ of the curve followed by the centre of the follower (known as the 'pitch curve') corresponds to the lift curve $y = f(\theta) = f(\omega t)$ of the follower when the system operates normally. Just as the pitch curve is found from the cam by rolling a circle (with the radius of the follower R_f) around the cam, so the cam shape can be found from the pitch curve by rolling the same circle around the *inside* of the pitch curve.

Thus, a procedure for designing a cam to perform a given function is first to transform the required lift curve $y = f(\theta)$ into the pitch curve $r = f(\theta)$ (or $r = R + R_f + f(\theta)$, if we define $f(\theta) = 0$ when contact is on the base circle which is of radius R). Then we can choose a follower radius R_f and by rolling this around the pitch curve deduce the cam shape.

b) i) from mechanics data book:

$$R = \frac{\left\{ r^2 + \left(\frac{dr}{d\phi}\right)^2 \right\}^{\frac{3}{2}}}{r^2 + 2\left(\frac{dr}{d\phi}\right)^2 - r\left(\frac{d^2r}{d\phi^2}\right)}$$

where $r = \frac{15}{4}a + \frac{5}{4}a\sin\left(2\phi - \frac{\pi}{2}\right)$

$\phi = \frac{\pi}{2}$: $r = 5a$

$r^2 = 25a^2$

$$\frac{dr}{d\phi} = \frac{5}{4}a \cdot 2\cos\left(2\phi - \frac{\pi}{2}\right)$$

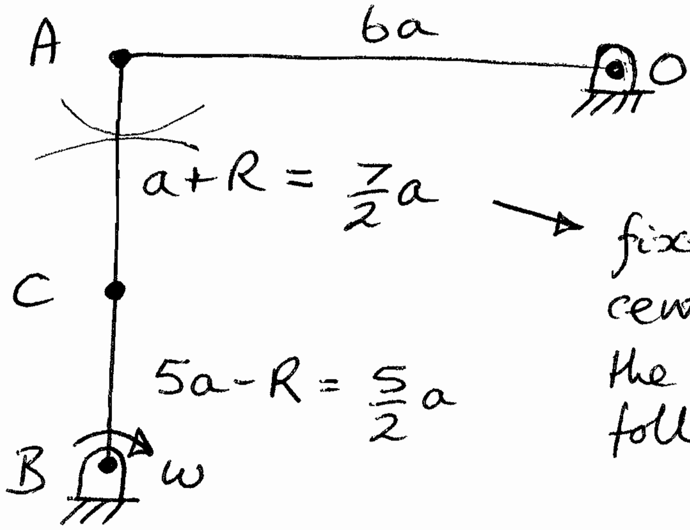
$\phi = \frac{\pi}{2}$: $= 0$

$$\frac{d^2r}{d\phi^2} = -\frac{10}{4}a \cdot 2\sin\left(2\phi - \frac{\pi}{2}\right)$$

$\phi = \frac{\pi}{2}$: $= -5a$

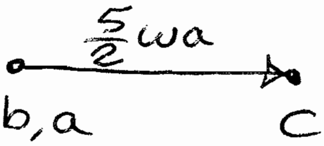
hence $R = \dfrac{\{25a^2\}^{\frac{3}{2}}}{25a^2 + 5a.5a} = \dfrac{25^{\frac{3}{2}}}{50}a = \underline{\underline{\dfrac{5}{2}a}}$

ii)

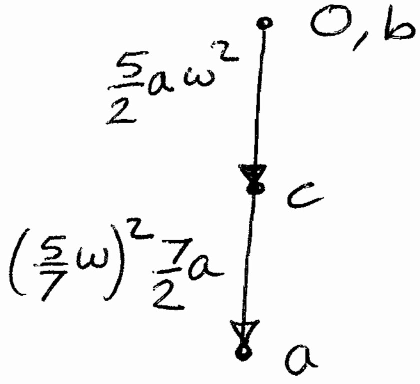


fixed distance between centres of curvature of the cam and the follower

iii) velocity diagram:



acceleration diagram:



acceleration of A is

$$\omega^2 a\left(\frac{5}{2} + \left(\frac{5}{7}\right)^2 \frac{7}{2}\right)$$

$$= \frac{30}{7}\omega^2 a \downarrow$$

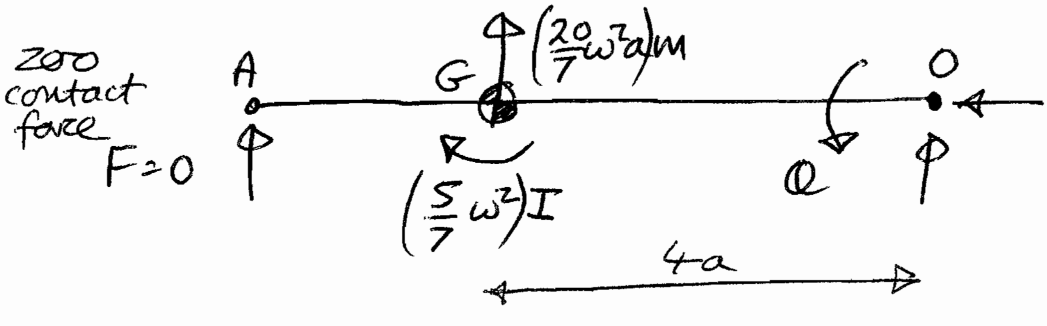
Hence acceleration at G is $\frac{4}{6}\left(\frac{30}{7}\omega^2 a\right)$

$$= \frac{20}{7}\omega^2 a \downarrow$$

angular acceleration of OA is $\frac{1}{6a} \cdot \frac{30}{7}\omega^2 a$

$$= \frac{5}{7}\omega^2 \circlearrowright$$

Free-body diagram of follower:



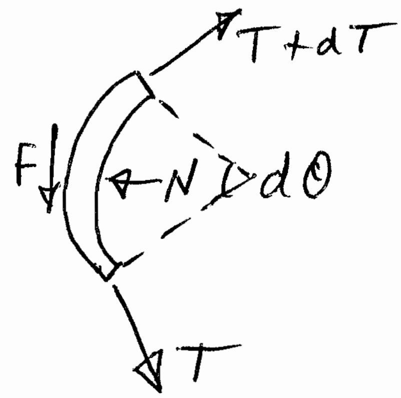
Moments about pivot sum to zero $\circlearrowleft +$

$$\frac{20}{7}\omega^2 am \cdot 4a + \frac{5}{7}\omega^2 ma^2 - Q = 0$$

$$\therefore Q = \frac{85}{7}\omega^2 a^2 m$$

2 a) There is a smaller arc of contact on the driveshaft, hence slip will occur here first. Grooves in the pulley and belt increase the effective friction coefficient and thus reduce the tendency to slip.

b) consider small element of belt:



horizontal equilibrium

$$2T\frac{d\theta}{2} = N \Rightarrow T d\theta = N$$

vertical equilibrium

$$dT = F$$

friction

$$F = \mu' N$$

(μ' = effective friction coeff) hence $T d\theta = \dfrac{dT}{\mu'}$

Integrating around the arc of contact θ_c:

$$\int_0^{\theta_c} d\theta = \frac{1}{\mu'}\int_{T_2}^{T_1} \frac{dT}{T}$$

$$\mu'\theta_c = \ln T_1 - \ln T_2$$

$$\frac{T_1}{T_2} = e^{\mu'\theta_c}$$

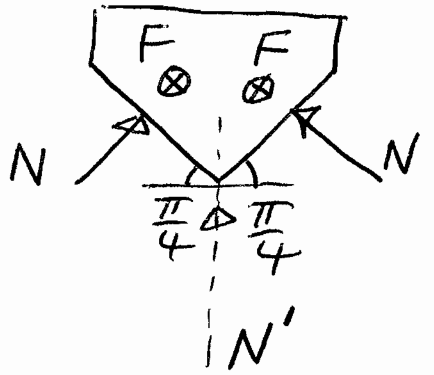
c)

Drive shaft torque $Q = (T_1 - T_2)R$

eliminating T_2 $Q = T_1 R\left(1 - \dfrac{T_2}{T_1}\right)$

$$Q = T_1 R\left(1 - e^{\mu'\theta_c}\right)$$

Now find expressions for μ':

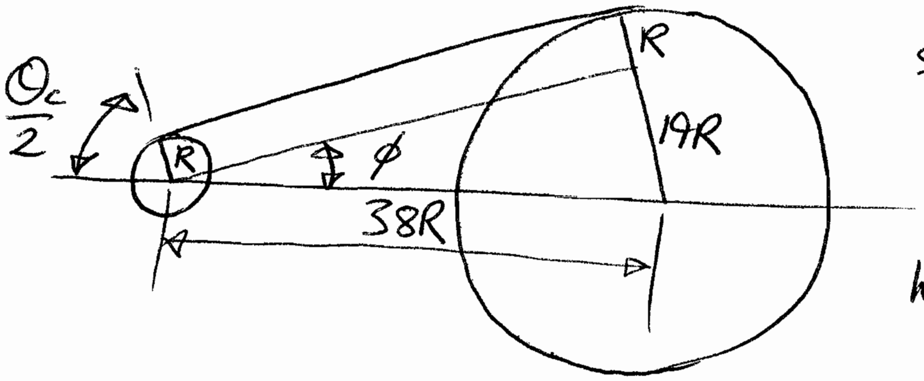


$$N' = \frac{2N}{\sqrt{2}}$$

$F = \mu N$ at the slipping condition

$$\mu' = \frac{2F}{N'} = \frac{2\mu N}{2N/\sqrt{2}} = \sqrt{2}\,\mu$$

arc of contact θ_c:



$$\sin\phi = \frac{19R}{38R} = \frac{1}{2}$$

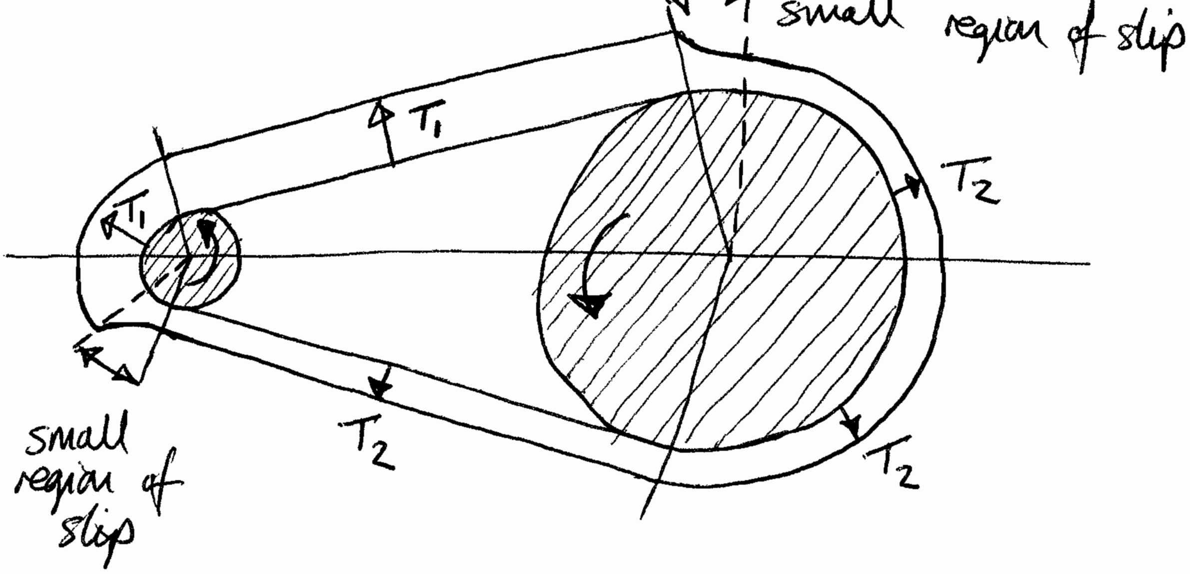
$$\phi = \frac{\pi}{6}$$

hence $\frac{\theta_c}{2} = \frac{\pi}{3}$

$$\theta_c = \frac{2\pi}{3}$$

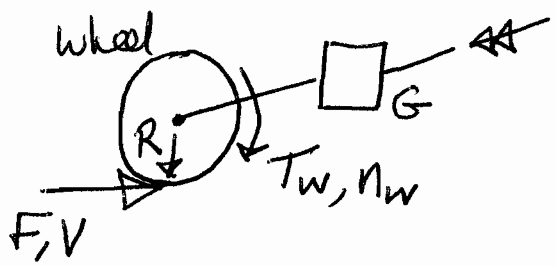
Finally $$Q = T_1 R\left(1 - e^{-\frac{2\sqrt{2}}{3}\mu\pi}\right)$$

d)



3 a) An IC engine generates maximum power over a limited speed range. Various ratios are needed to ensure that maximum power can be transmitted for a wide range of vehicle speeds, for example, travelling up a steep incline at slow speed, or travelling at high speed on a level road. The speed range over which good efficiency is achieved for a given power output is also limited. Hence a range of ratios allows efficiency to be maximised over a range of vehicle speeds.

b) i) Construct the load characteristic on top of the motor output characteristic.



$$G = \frac{n_m}{n_W} = \frac{T_W}{T_m} = 10$$

assuming no losses.

$$F = 500 + 100V$$

need to get into terms of T_m and n_m

$$F = \frac{T_W}{R} = \frac{T_m G}{R}$$

$$V = n_W \frac{2\pi}{60} R = \frac{n_m}{G} \frac{2\pi}{60} R$$

↑ converts rpm to rad/s

thus: $\frac{G T_m}{R} = 500 + 100 \frac{n_M}{G} \frac{2\pi}{60} R$

putting $G=10$ and $R=0.5$ gives:

$$T_m = 25 + 0.026\, n_m$$

at $n_m = 0$ rpm, $T_m = 25$ Nm
at $n_m = 3000$ rpm, $T_m = 103$ Nm

sketch this straightline characteristic on fig 3(a).

Maximum efficiency is 92% at about 750 rpm

Corresponding vehicle speed is $V = \frac{750}{10} \cdot \frac{2\pi}{60} \cdot 0.5 = 9.2$ m/s

ii) $V = 5$ m/s $\Rightarrow$ $n_m = V \cdot \frac{60}{2\pi} \cdot \frac{G}{R} = 5 \cdot \frac{60}{2\pi} \cdot \frac{10}{0.5}$
$= 955$ rpm

from fig 3a, corresponding oil pressure = 5 MPa
flow rate = 60 l/min
motor efficiency = 86 %

from fig 3b, pump efficiency at same pressure and flow rate = 88%

hence overall efficiency = $0.86 \times 0.88 = 76\%$

iii) From the 5m/s point on the load characteristic ($G=10$, ~50 Nm, ~1000 rpm), sketch a constant power contour, noting that the point (100 Nm, 500 rpm) will be on the contour. The point of greatest efficiency is at about (1500 rpm, 35 Nm), so to achieve 5m/s at ~1500 rpm instead of ~1000 rpm, the gear ratio G must increase from 10 to $10 \cdot \frac{1500}{1000} = 15$

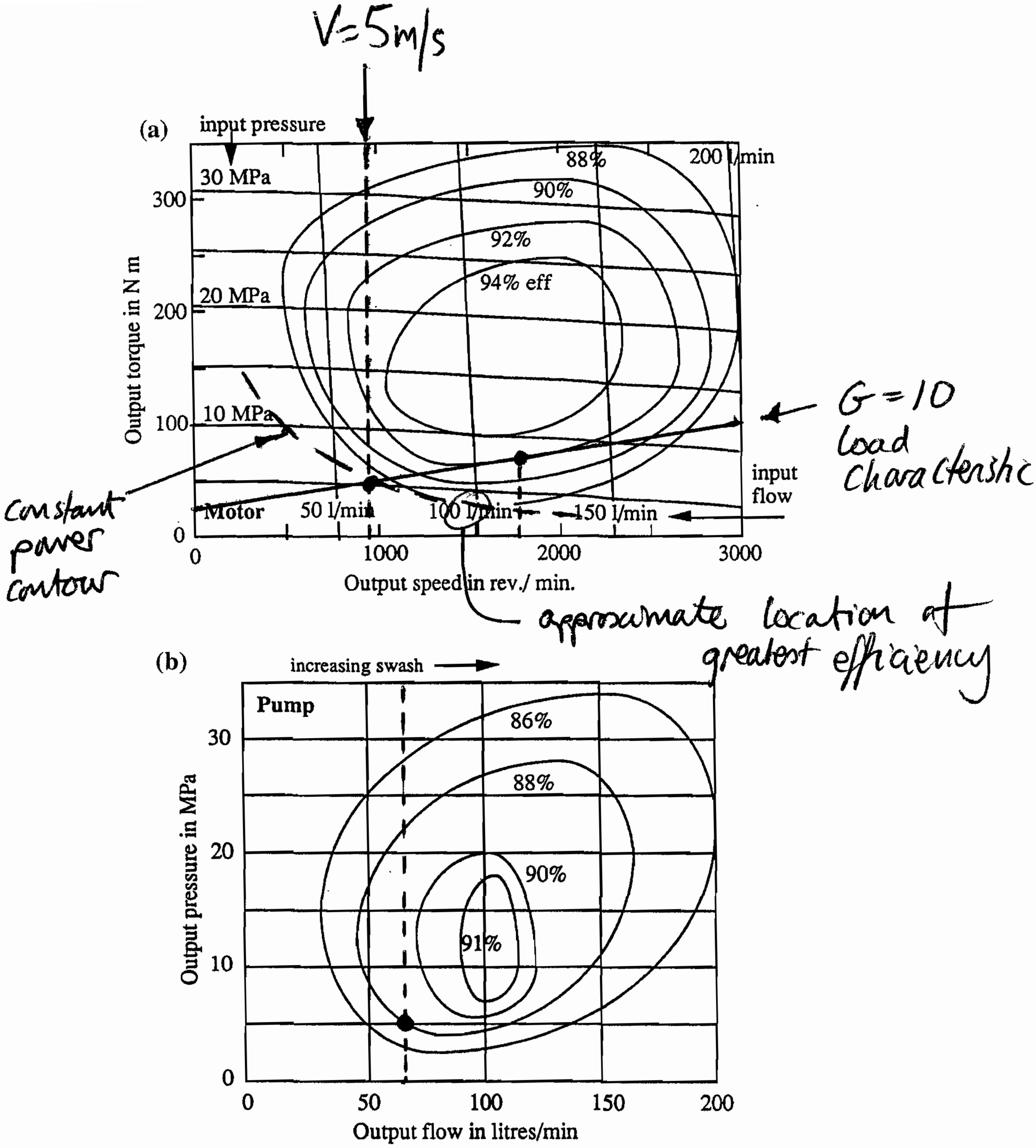

V=5m/s
(a)
input pressure
30 MPa
20 MPa
10 MPa
Motor
50 l/min
100 l/min
150 l/min
200 l/min
88%
90%
92%
94% eff
input
flow
Output torque in N m
Output speed in rev./ min.
0
100
200
300
1000
2000
3000
G=10
load
characteristic
constant
power
contour
approximate location of
greatest efficiency
(b)
increasing swash
Pump
86%
88%
90%
91%
Output pressure in MPa
Output flow in litres/min
0
10
20
30
50
100
150
200

Fig. 3

4. a)

	radial load	axial load	misalignment	axial displacement
deep groove ball	+	+	—	—
cylindrical roller	++	– –	—	+++
taper roller	++	+++ (one direction)	—	– –
spherical roller	+++	+	+++	– –

b) $L = a_1 a_{23} \left(\frac{C}{P}\right)^p$

$a_1 = 0.62$ (95% reliability)

$p = \frac{10}{3}$ (roller bearing)

$P = 10\,kN$ (radial force)

$C = 117\,kN$ (data sheet)

a_{23}: $d = 60\,mm$ $D = 100\,mm$

$$d_m = \frac{d+D}{2} = \frac{60+100}{2} = 80\,mm$$

from diagram 1 on data sheet, $\nu_1 = 15\,mm^2/s$

hence $\kappa = \frac{\nu}{\nu_1} = \frac{8}{15} = 0.53$

from diagram 3, $a_{23} = 0.32$ (lower boundary)

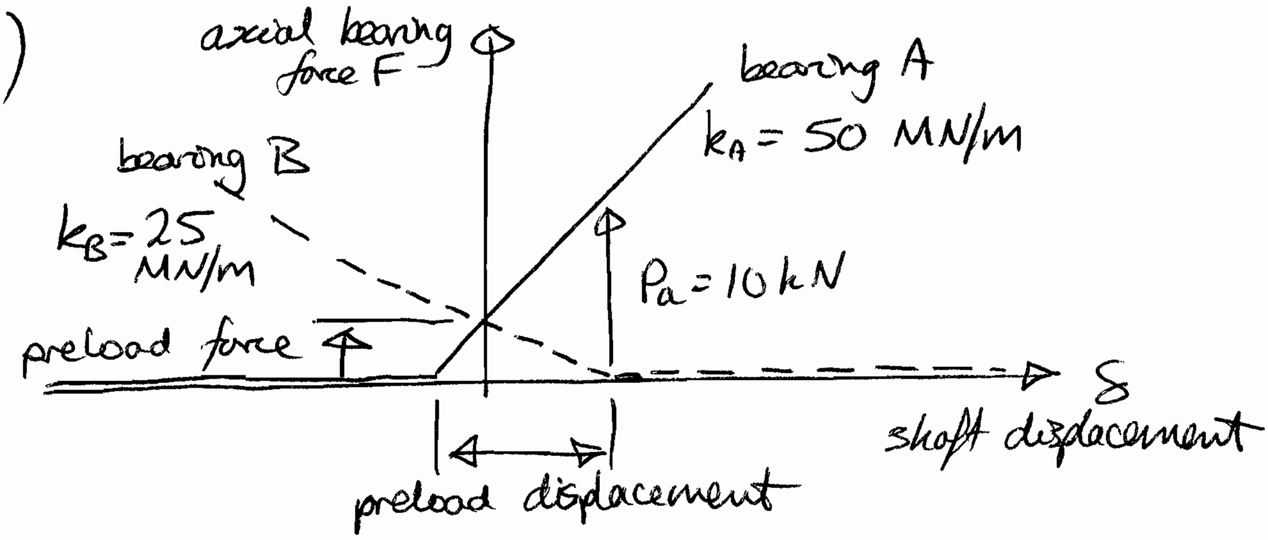
Hence $L = 0.62 \cdot 0.32 \left(\frac{117}{10}\right)^{\frac{10}{3}} = 721.4$ million revolutions

BUT from bearing data sheet, $P_u = 19{,}600\,N$

Hence bearing life is infinite.

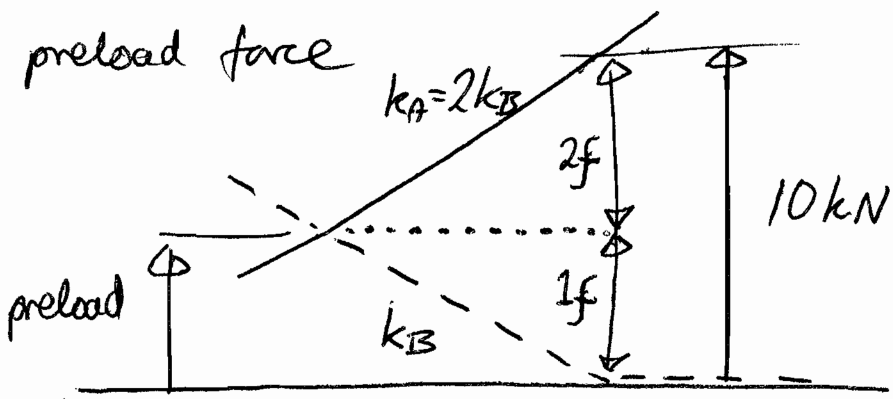
c) i) Application of axial force causes axial clearance in one of the bearings, leading to poor shaft alignment accuracy and low axial stiffness. Radial force on a bearing with axial clearance leads to premature failure, because the force is carried by only the small number of rollers that are in contact.

ii)



from the diagram:

$$\text{preload displacement} = \frac{P_a}{k_A} = \frac{10.10^3}{50.10^6} = 0{\cdot}2.10^{-3}\,m = \underline{\underline{0.2\,mm}}$$



$$\text{preload force } f = \frac{1}{3} \cdot 10\,kN = \underline{\underline{3{\cdot}3\,kN}}$$

Engineering Tripos Part IIA 2006

Module 3C4

Machine Design – Transmissions

1	(b)	(iii)	$Q = \frac{85}{7}\omega^2 a^2 m$	
2	(c)	$Q = T_1 R\left(1 - \exp\left(-\frac{2\sqrt{2}}{3}\mu\pi\right)\right)$		
3	(b)	(i)	9.2 m/s	92%
		(ii)	76%	
		(iii)	15	
4	(b)	infinite		
	(c)	(ii)	0.2 mm	3.3 kN