PART IIA 2006
3D1: Soil mechanics
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Engineering Tripos *Datasheet: Soil Mechanics Data Book*
2006 Solutions

marks 20

1 a) rockfill $\gamma_d = 15\,kN/m^3 = G_s\gamma_w/(1+e)$

$\therefore e = 2.7\times9.8/15 - 1 = 0.764$

$\gamma_{sat} = \gamma_w(G_s+e)/(1+e)$

$\therefore \gamma_{sat} = 9.8\times3.464/1.764 = \underline{19.2\,kN/m^3}$

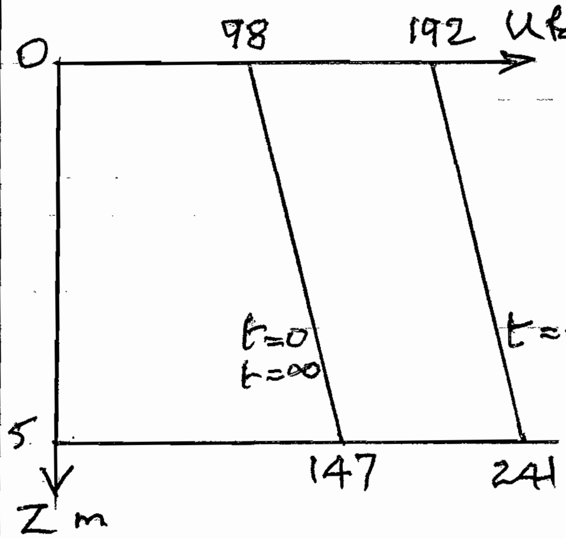
i) $\sigma_v' = \gamma'.10 = (19.2-9.8).10 = \underline{94\,kPa}$

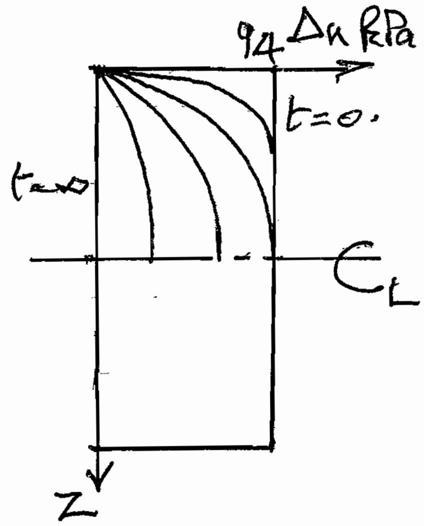
ii) except that the fill will be dry for 5m on top

$\sigma_v' = 94 + 5.15 = 94+75 = \underline{169\,kPa}$ 4

b) – treat the compression of the clay as 1D, and ignore friction at the sides.

– treat the clay as homogeneous even though it would be stiffer and less permeable at depth





4

c) – note that clay at mid-depth has $\sigma_v' = 25$ to 119 kPa and therefore remains overconsolidated

$\rho_{ult} = 5m\times94/3300 = \underline{0.142\,m}$

1 cont.

c) cont. $T_v = c_v t / d^2$

double drainage gives $d = 2.5$ m

so at 0.5 years, $T_v = 5 \times 0.5 / 2.5^2 = 0.4$

so Data Book table P.6 gives $R_v = 0.7$

$\therefore \rho_{6\,months} = 0.7 \times 0.142 = \underline{0.099\text{ m}}$

considering assumptions:

* E_o is variable with depth due to $\sigma'_{v,o}$ and the shallow layer will be proportionately <u>less</u> stiff than the deeper layer is <u>more</u> stiff.

so the E_o variation makes the previous estimate <u>low</u>

Furthermore the test range of 100 kPa is a little higher than required, so E_o will be a little higher, and the previous estimate of ρ_{ult} will be a little <u>low</u>

* the side friction in the field will reduce $\Delta\sigma'$ in the lower part of the clay, and this will tend to make our previous estimate of ρ_{ult} <u>high</u>.

* these effects may or may not cancel out. 4

d) we need to extrapolate on a κ-line for E_o

in the test, $E_o = \dfrac{\Delta\sigma'_v}{(\Delta V/V)} = 3300$ kPa

and we have $\Delta\sigma'_v = 100$ kPa, $\Delta V = \kappa \ln(125/25)$

and we can deduce V from γ and G_s:

$\gamma = (G_s + e)\gamma_w / (1+e)$

$2.041 + 2.041e = 2.7 + e$

$\therefore e = 0.633$, $\therefore V = 1.633$

so $\kappa = 0.0495/\ln 5 = \underline{0.0307}$

1 cont

d) cont. So for the σ_v' from 25 to 194 kPa

$\Delta V = 0.0307 \ln(194/25) = 0.063$

$\therefore E_o = 169/(0.063/1.633) = \underline{4380 \text{ kPa}}$

For overconsolidated clay, k does not change very much. So $C_v = E_o k/\gamma_w \propto E_o$

$\therefore$ take $C_v = 5 \times 4380/3300 = \underline{6.64 \text{ m}^2/\text{y}}$

In this case $\rho_{ult} = 5 \times 169/4380 = 0.193$ m

We desire $\rho = 0.142$ m

$\therefore R_v = 0.74$

Deduce $T_v = 0.46$

$\therefore t = 2.5^2 \times 0.46/6.64 = \underline{0.43 \text{ y}}$

So the additional surcharge should be left in place for about 5 months. 8

2 a) "Friction" emerges from shear resistance expressed in terms of effective stress, seen most readily when pore water can flow during shear to permit grain re-packing – "drained" shear.

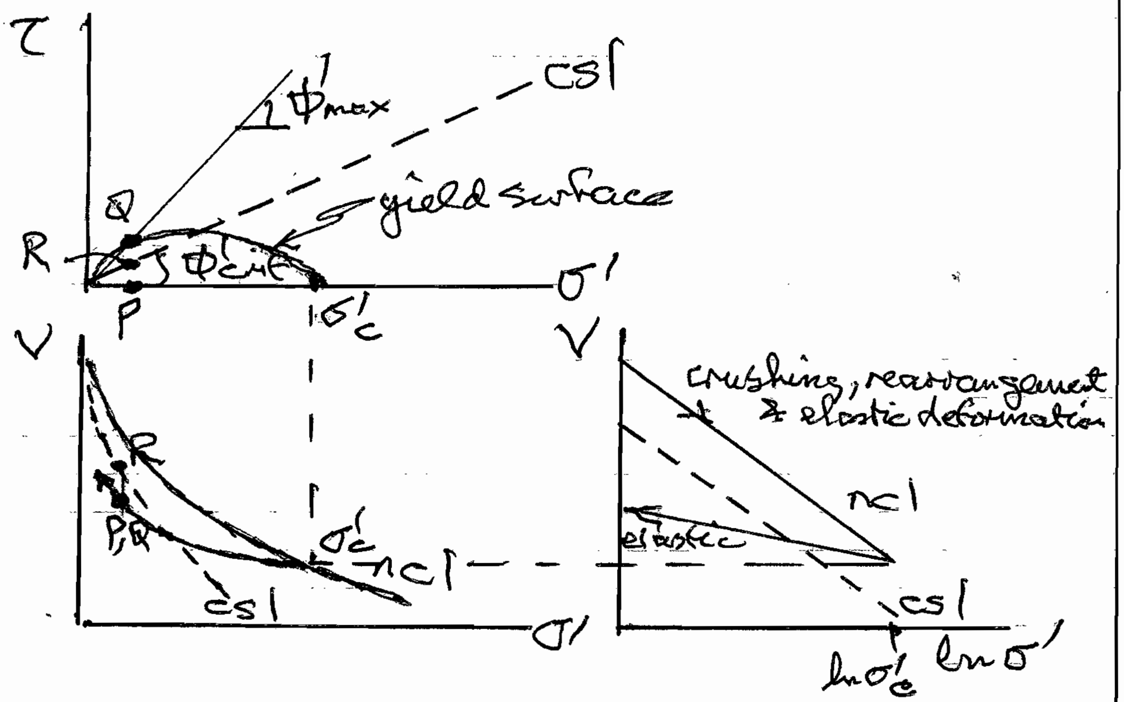
Then $\tau_{max} = \sigma' \tan\phi_{max}$ where $\phi_{max} = f(\sigma')$

$$\tau_{ult} = \sigma' \tan\phi_{crit}$$

for (τ, σ') acting on a macro failure plane.

"Cohesion" is expressed as "undrained strength" when soil shears at constant volume.

Then $\tau_{max} = c_u$ where $c_u = f(V)$



when sheared from high overconsolidation ratio (σ'_c/σ') at P, "drained" peak strength at Q falls to critical state strength at R as the soil dilates. So "friction angle" $\phi_{max} \approx \phi_{crit} + \psi$ where ψ is the angle of dilation (which varies).

2 cont. a) cont.

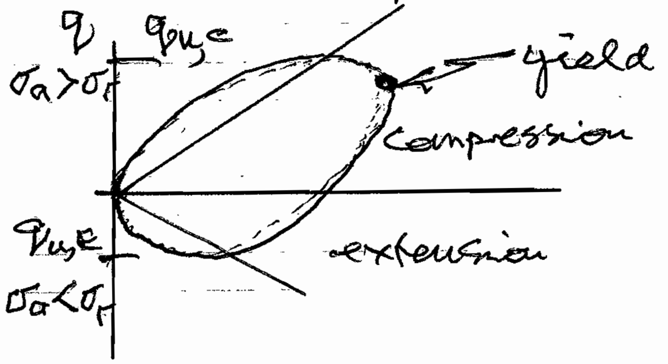
On the other hand, soil tested "undrained" is supposed to shear at constant volume to a critical state. Cam Clay use csl equations

$$\tau_{crit} = \sigma' \tan \phi_{crit}$$

$$v_{crit} = \Gamma - \lambda \ln \sigma'_{crit}$$

so fixing v fixes σ' which fixes $\tau = c_u$. This is the "apparent cohesion" of clay. The current specific volume, and the current yield surface, are functions of the maximum previous stress σ'_c permanently crushing & rearranging the fabric, and also the current stress σ'. So we need to know the effective stress history of the soil to predict its drained/undrained shear strength. 10

b) (i) Bedding planes and anisotropic fabric. Vertical cores are stronger than horizontal cores. Triaxial compression is stronger than extension.

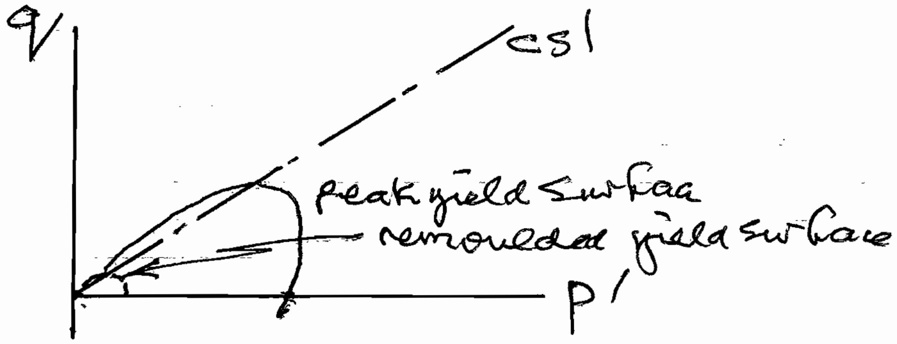


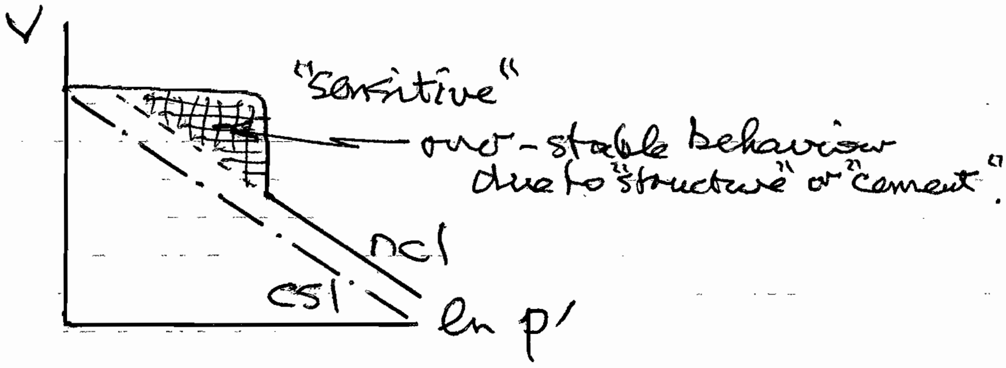
$q_{u,c} > q_{u,E}$
(eg factor 2)

Impact: dangerous just to do compression tests on vertical cores if there are "passive" zones – as with foundations of embankments.

2 cont. b)(ii) Sensitivity of "quick" clays

Occurs eg with Norwegian clays where fabric can trap a lot of water in a quasi-stable microstructure that can collapse on shearing. Then very high excess pore pressures are created, reducing effective stresses to close to zero so the clay behaves as a viscous, heavy, fluid. Responsible for catastrophic landslides.





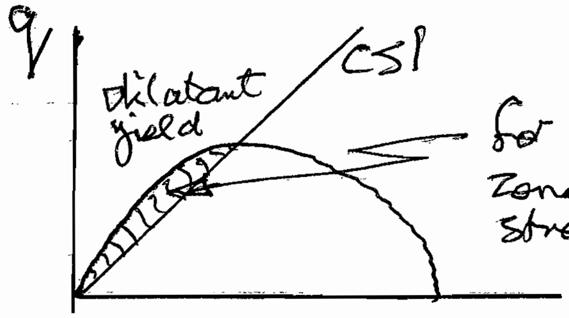
b) iii) Progressive failure of overconsolidated clay

Occurs due to non-homogeneous deformation of brittle materials where $\tau_{max} > \tau_{ult}$. Shearing occurs first at a soft spot, which dilates and gets even softer, so it continues to soften towards τ_{ult} before other elements strain harden to τ_{max}.

2 cont.

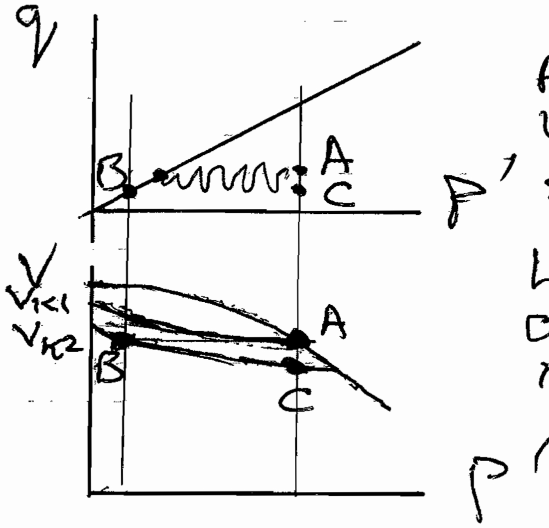
b) iii) cont.

So the mass strength is inferior to average peak strength of the constituent elements prior to being tested. More related to fracture mechanics than plasticity.



Impact: failure of trenches & cuttings e.g. in London Clay.

b) iv) Seismic liquefaction of saturated sands. Sands subject to cyclic shear at $\tau < \sigma' \tan\phi$ tend to compact, so if pore fluid can not escape excess pore pressures are created and soil effective stresses can drop to low levels. Can lead to structures sinking into sand, or collapsing, following earthquake shaking.



A → B due to shaking
$V_{K1} \rightarrow V_{K2}$ but V = const.
so p'_A drops to p'_B

Later p'_B rises back to p'_C as soil compresses on its new K-line.

Note $V = f$(strain cycles) opposes simple "cohesion" idea.

3 a) Take σ'_v = constant $\simeq$ 0, 50 or 100 kPa

Mode P: $\sigma'_{h\,max} = K_p \sigma'_v = \sigma'_v (1+\sin\phi_{max})/(1-\sin\phi_{max})$

Also, $\phi_{max} = \phi_{crit} + 3\left[I_D \ln \frac{\sigma'_c}{p'} - 1\right]$

where $p' = \sigma'_v (1+K_p)/2$

noting that factor 3 is assumed since the deformation is 3D and 3 < 5 puts ϕ_{max} on the safe side, and p' is based on plane stress reflecting the collision conditions.

At $\sigma'_v = 0$, $\underline{\sigma'_{h,max} = 0}$

At $\sigma'_v = 50$: take $p' = 50$ for 1st iteration

From Data Book p.13 select $\phi_{crit} = 36°$, $\sigma'_c = 20$ MPa

$\phi_{max} = 36° + 3[0.75 \ln 400 - 1] = 46.5°$

$K_p = 6.28$ leading to better $p' = 182$ kPa

$\phi_{max} = 43.6°$, $K_p = 5.44$, $\underline{\sigma'_{h,max} \simeq 272 \text{ kPa}}$

At $\sigma'_v = 100$: take $p' = 300$ for 1st iteration

$\phi_{max} = 42.4°$, $K_p = 5.16$, $\sigma'_{h,max} \simeq 516$ kPa

$p' \simeq 616/2 \simeq \underline{308 \text{ kPa}}$ (near enough)

Mode S: $\tau_{max} = \sigma'_v \tan\phi_{max}$

$\phi_{max} = \phi_{crit} + 5\left[I_D \ln \frac{\sigma'_c}{\sigma'_v} - 1\right]$

noting factor 5 is used to reflect plane strain

At $\sigma'_v = 0$, $\underline{\tau_{max} = 0}$

At $\sigma'_v = 50$, $\tau_{max} = 50 \tan 53.5° = \underline{67.5 \text{ kPa}}$

At $\sigma'_v = 100$, $\tau_{max} = 100 \tan 50.9° = \underline{122.9 \text{ kPa}}$

3 cont

b) $K_p = (1+\sin 36°)/(1-\sin 36°) = 3.85$

Mode P   At $\sigma_v' = 0$, $\sigma_{h,crit}' = 0$

At $\sigma_v' = 50$, $\sigma_{h,crit}' = 192.6$ kPa

At $\sigma_v' = 100$, $\sigma_{h,crit}' = \underline{385 \text{ kPa}}$

Mode S   At $\sigma_v' = 0$, $\tau_{crit} = 0$

At $\sigma_v' = 50$, $\tau_{crit} = 36.3$ kPa

At $\sigma_v' = 100$, $\tau_{crit} = \underline{72.7 \text{ kPa}}$ 2

c) ϕ_{max} drops to ϕ_{crit} as soil distorts, so the design resistance should be based on (b)

Mode P $\sigma_{h,average}' = 193$ kPa

$F = HD\,\sigma_{h,avge}' = 19.3$ MN

Mode S τ at base $= 73$ kPa

$F = \dfrac{\pi D^2}{4} \cdot \tau_{base} = 5.7$ MN

so design value of F is 5.7 MN in Mode S. 2

d) The soil is quite dilatant.

e.g at the base, $\psi_{max} = (\phi_{max} - \phi_{crit})/0.8$

so if the soil is free to dilate, $\psi_{max} = 14.9/0.8$

giving $\psi_{max} = 18.6°$, $\tan\psi_{max} = 0.34$

Ships must collide at speeds 1 m/s to 10 m/s

So the vertical component of velocity of the top surface of the sand would be 0.34 to 3.4 m/s

It is not clear how sufficient water can enter the buffer to provide this expansion of volume at the required rate. None of the boundaries can provide inflow.

3 cont

d) cont. On the other hand, constant volume shearing – at a critical state – would require an enormous p'_{crit}:

$$\ln \frac{\sigma_c'}{p'_{crit}} = \frac{1}{I_D} = 1.33$$

$$p'_{crit} = 20/3.78 = 5.3\,\text{MPa}$$

Since σ_v is limited (eg 0, 50, 100 kPa) and there is no physical restraint on the top surface, p'_{crit} would require $u \ll 0$. In reality, the sand would cavitate at $u = -100$ kPa ~ zero absolute pressure in the pore water.

Then, at the base of the buffer,

$$\sigma_v' = \sigma_v - u = 200 + 100 = 300\,\text{kPa}$$

If the soil reaches a critical state at this stress, then in mode 5:

$$\underline{\tau_{crit} = 218\,\text{kPa}}$$

and $\underline{F = 3 \times 5.7 = 17.1\,\text{MN}}$ 4

4 a) $\tan\phi_{crit} = 44/90 = 0.489$

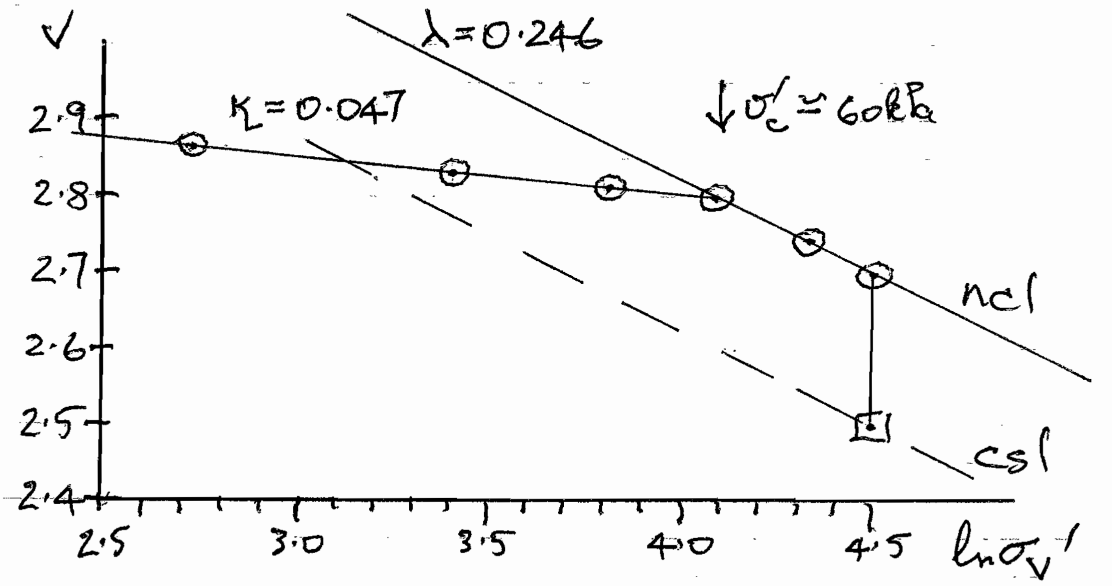
$\therefore \underline{\phi_{crit} = 26°}$

specific volume $v \propto$ height h

initially, $w = 0.69$ so $e = 2.7 \times 0.69 = 1.863$

so initial $v = 2.863$

σ_v'/kPa:	15	30	45	60	75	90	90
v:	2.863	2.832	2.811	2.798	2.742	2.697	2.495
$\ln\sigma_v'$:	2.71	3.40	3.81	4.09	4.32	4.50	4.50



For Cam Clay $\Gamma + \lambda - \kappa - \lambda \ln 90 = 2.697$

$\therefore \underline{\Gamma = 3.605}$

Note: Δv on shearing should be $\lambda - \kappa = 0.199$ and is measured as 0.202: $\underline{\text{correct.}}$ 6

b) Precompression $\sigma_c' = 60$ kPa

Water table is at ground level

$\gamma' = (G_s - 1)\gamma_w/(1+e) = 1.7 \times 9.8/2.863$

$\therefore \gamma' = 5.8\ \text{kN/m}^3$

$\therefore \sigma_{v,i}'$ at 5m depth $= 5 \times 5.8 = 29$ kPa

4 cont. b) cont. $\therefore \Delta\sigma_v' = 60 - 29 = 31$ kPa

So if $\sigma_{v,i}' = 5.8z$

and $\sigma_{v,max}' = 5.8z + 31$

then $n = 1 + 31/(5.8z) = \underline{1 + 5.3/z}$ 8

c) $v = \Gamma + \lambda - \kappa - \lambda \ln(5.8z + 31) + \kappa \ln(1 + 5.3/z)$

$= \Gamma - \lambda \ln \sigma_u'$ for undrained shear

$\therefore \ln \sigma_u' = -\frac{(\lambda - \kappa)}{\lambda} + \ln(5.8z + 31) - \frac{\kappa}{\lambda} \ln(1 + \frac{5.3}{z})$

z m	0.5	1	2	4	10
$\ln \sigma_u'$	2.246	2.445	2.672	3.023	3.598
σ_u'	9.4	11.5	14.5	20.6	36.5 kPa
c_u	4.6	5.6	7.1	10.1	17.9 kPa

So, roughly, $\underline{c_u = 4 + 1.4z \text{ kPa}}$

For normal consolidation $z \to \infty$

$\sigma_u' = 5.8z \exp(-0.809)$

and $c_u = \sigma_u' \tan\phi_{crit} = \underline{1.26z \text{ kPa}}$

So the OC clay has slightly superior strength, with an intercept at mud-line of $\underline{0.135 \Delta\sigma_v'}$ 6

3D1 2006 Answers

1 (a) 19.2 kN/m3 (i) 94 kPa (ii) 169 kPa
(b) 94 kPa
(c) 0.142 m, 0.099 m
(d) 5 months

3 (a) $\sigma'_{h,max}$ = 0, 272 kPa, and 516 kPa approximately at 0, 5 and 10 m respectively
τ_{max} = 0, 68 kPa, and 123 kPa approximately at 0, 5 and 10 m respectively
(b) σ'_{hcrit} = 0, 193 kPa, and 385 kPa approximately at 0, 5 and 10 m respectively
τ_{crit} = 0, 36 kPa, and 73 kPa approximately at 0, 5 and 10 m respectively
(c) 5.7 MN in Mode S
(d) 17.1 MN in Mode S, based on cavitation

4 (a) 26°, 3.605, 0.047, 0.246
(b) 60 kPa, 31 kPa, $n = 1 + 5.3/z$, $v = \Gamma + \lambda - \kappa - \lambda \ln(5.8z + 31) + \kappa \ln(1 + 5.3/z)$
(c) $c_u \approx 4 + 1.4z$ kPa