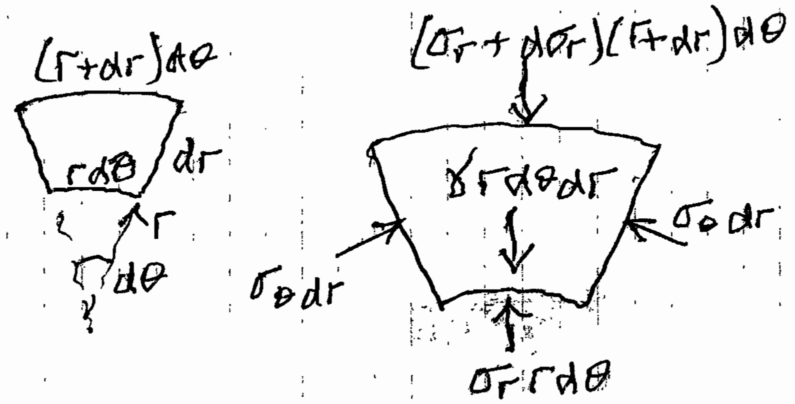
PART IIA 2006
3D2: Geotechnical engineering
Principal Assessor: Dr D J White

Datasheet: Soil Mechanics Data Book

Engineering Tripos Part II A 2006
Module 3D2 Geotechnical Engineering
Solutions

a)



Keeping only first order terms & preserving radial equilibrium on the vertical plane:

$$\left.\begin{aligned} &\sigma_r r d\theta + d\sigma_r r d\theta + \sigma_r dr d\theta \\ &- \sigma_r r d\theta - \sigma_\theta dr d\theta + \gamma r d\theta dr \end{aligned}\right\} = 0$$

$$\therefore \frac{d\sigma_r}{dr} + \frac{(\sigma_r - \sigma_\theta)}{r} = -\gamma$$

b) Given radial symmetry of displacements

$$\varepsilon_{\theta,a} = \frac{\rho_a}{a}$$

Since undrained ($\varepsilon_{vol} = 0$)

$$\varepsilon_{r,a} = -\frac{\rho_a}{a}$$

$$\therefore \varepsilon_{\gamma,a} = (\varepsilon_{r,a} - \varepsilon_{\theta,a}) = -\frac{2\rho_a}{a}$$

Again, for constant volume deformation

so $\varepsilon_\gamma \propto \frac{1}{r^2}$ (ρr = constant) and $\varepsilon_\gamma = -\frac{2\rho_a a}{r^2}$

c) Take $\sigma_r - \sigma_\theta = 2\tau = -2c_u\left(\frac{\varepsilon_\gamma}{\varepsilon_{\gamma,f}}\right)^\beta$

Then $\frac{d\sigma_r}{dr} + (-2c_u)\left(\frac{2\rho_a a}{\varepsilon_{\gamma,f}}\right)^\beta r^{-1-2\beta} = -\gamma$

So $\int_0^0 d\sigma_r - A\int_a^b r^{-1-2\beta}dr = -\gamma\int_a^b dr$

where $A = 2c_u\left(\frac{2\rho_a a}{\varepsilon_{\gamma,f}}\right)^\beta$

Then $\frac{A}{2\beta}\left(b^{-2\beta} - a^{-2\beta}\right) = -\gamma(b-a)$

c) cont.

So $2c_u \frac{2^\beta}{\varepsilon_{\gamma,f}^\beta}\frac{(\rho_a a)^\beta}{2\beta} = \frac{\gamma(b-a)}{(a^{-2\beta} - b^{-2\beta})}$

And since continuity gives $\rho_a a = \rho_b b$ then

$$(\rho_b b)^\beta = \frac{\gamma(b-a)}{c_u(a^{-2\beta} - b^{-2\beta})} \cdot \frac{\varepsilon_{\gamma,f}^\beta}{2^\beta}\beta$$

$$\text{so } \frac{2\rho_b b}{\varepsilon_{\gamma,f}} = \left[\frac{\gamma}{c_u}\beta\frac{(b-a)}{a^{-2\beta} - b^{-2\beta}}\right]^{\frac{1}{\beta}}$$

$$\text{or } \frac{2\rho_b}{b\,\varepsilon_{\gamma,f}} = \left[\frac{\gamma}{c_u}\beta\frac{(b-a)}{\left\{\left(\frac{b}{a}\right)^{2\beta} - 1\right\}}\right]^{\frac{1}{\beta}}$$

Substitute $\beta = 0.5$.

$$\frac{2\rho_b}{b\,\varepsilon_{\gamma,f}} = \left[\frac{\gamma}{c_u}\beta a\right]^2$$

So given $a = 2.5\,m$, $b = 10\,m$, $\gamma = 20\,kN/m^3$
$c_u = 100\,kPa$, $\varepsilon_{\gamma,f} = 0.02$

we get $\rho_b = \frac{10 \times 0.02}{2} \times \left[\frac{20}{100} \times 0.5 \times 2.5\right]^2$

$= 0.1 \times 0.25^2\,m$

$\rho_b = 6.25\,mm$

Question 2

Properties of London Clay from databook, page 10:

$\lambda = 0.161$, $\kappa = 0.062$, $\Gamma = 2.759$, $M = 0.89$ or 0.69.
$\phi_{crit} = 23°$, $G_s = 2.75$.

a) Earth pressure coefficients in 1D compression and swelling taken from databook, p.17:

$$\alpha = 1.2 \sin\phi_{crit} = 0.469$$

At A', $K_{o,nc} = 1 - \sin\phi_{crit} = 0.609 \Rightarrow \underline{\sigma_h' = 121.8\text{ kPa}}$

$$\underline{q} = \sigma_v'(1 - K_{o,nc}) = 200 \times 0.391 = \underline{78.2\text{kPa}}$$
$$\underline{p'} = \tfrac{1}{3}\sigma_v'(1 + 2K_{o,nc}) = 200 \times 0.739 = \underline{147.9\text{kPa}}$$

At B', $K_o = K_{o,nc} \times n^{\alpha}$ where $n = \sigma_{v,max}'/\sigma_v' = 4$

$$\Rightarrow K_o = 0.609 \times 4^{0.469} = 1.17 \Rightarrow \underline{\sigma_h' = 58.5\text{kPa}}$$

As above, $\underline{q} = 50(1 - 1.17) = \underline{-8.5\text{ kPa}}$
$\underline{p'} = 50(1 + 2 \times 1.17)/3 = \underline{55.6\text{kPa}}$

Critical state stress ratios for $\sigma_v' - \sigma_h'$ axes:

$$K_a = \frac{1 - \sin\phi_{crit}}{1 + \sin\phi_{crit}} = 0.44, \quad K_p = \frac{1}{K_a} = 2.28.$$

[SEE DIAGRAM ON FOLLOWING PAGE]

b) Path B'C' is undrained and quasi-elastic, so p' is constant

$\Rightarrow$ at C': $p' = 55.6\text{ kPa}$, $q = 65\text{ kPa}$ (as given)
hence: $\sigma_v' + 2\sigma_h' = 3p' = 166.8\text{ kPa}$, $\sigma_v' - \sigma_h' = 65\text{ kPa}$

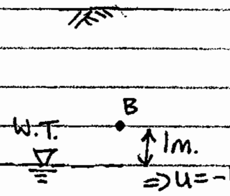
Eliminating $\sigma_v' \Rightarrow \sigma_h' = 33.9\text{kPa}$, $\Rightarrow \sigma_v' = 98.9\text{kPa}$

At D', $q = 70\text{ kPa}$ (given) and soil is on CSL $\Rightarrow p' = \frac{q}{M} = 78.9\text{kPa}$

Solving for σ_v', σ_h' as previously $\Rightarrow \sigma_h' = 55.3\text{kPa}$, $\sigma_v' = 125.3\text{kPa}$.

Question 2 continued

c) To find state B, given B':



$\Rightarrow u = -10\,kPa$.

$\sigma_v' = 50\,kPa \Rightarrow \sigma_v = 40\,kPa$

$\sigma_h' = 58.5\,kPa \quad \sigma_h = 48.5\,kPa$.

Stress path during excavation:

Effective stresses:

pre-failure $\Rightarrow p' = \text{constant}$.

$\Rightarrow \Delta\sigma_v' = -\Delta\sigma_h'$ (if it is assumed that

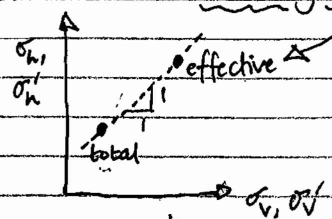
$\Rightarrow$ direction of path known $\quad \sigma_2' = \dfrac{\sigma_h' + \sigma_v'}{2}$)

Total stresses:

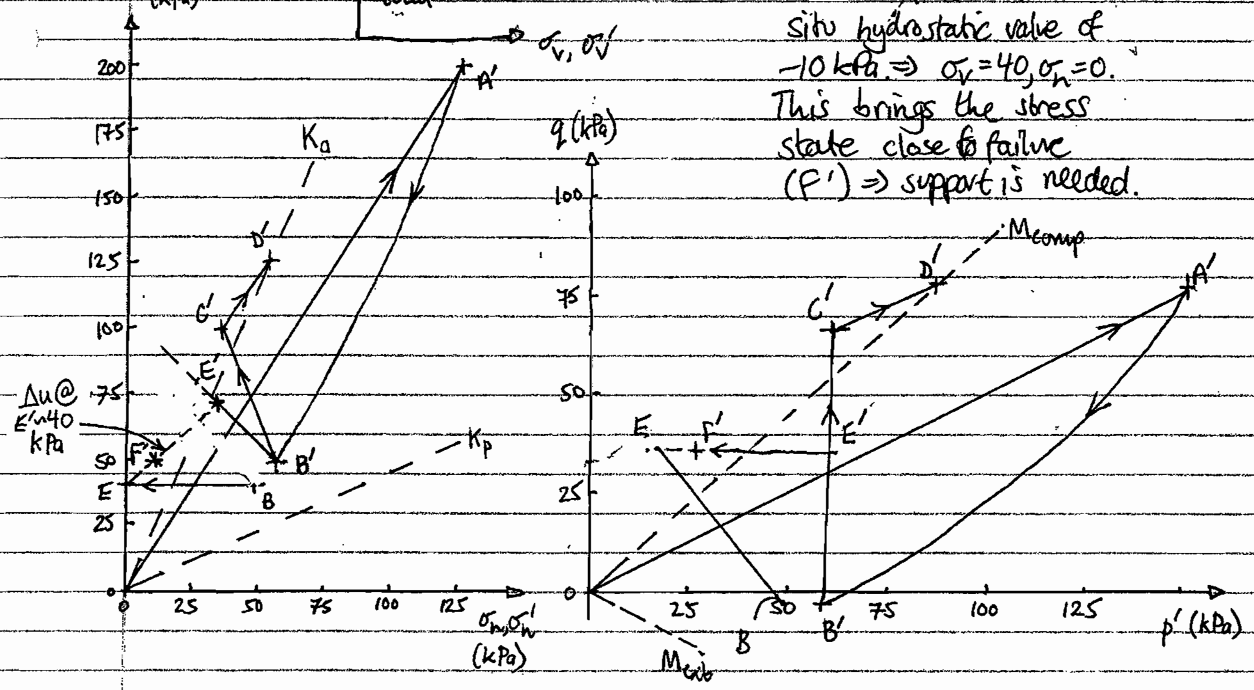
End point: $\sigma_h = 0$, $\sigma_v = 40\,kPa$ (overburden, unchanged)

To find end point in effective stress terms, note that vertical and horizontal total and effective stresses must differ by same amount, equal to pore pressure.

from intersection of these conditions, at E':
$\sigma_h' \simeq 35\,kPa$
$\sigma_v' \simeq 70\,kPa$
$\Rightarrow p' \sim 52.5\,kPa$.
$q \sim 35\,kPa$



d) In long term, dissipation of excess pore pressure will return value to in situ hydrostatic value of $-10\,kPa \Rightarrow \sigma_v = 40, \sigma_h = 0$. This brings the stress state close to failure (F') $\Rightarrow$ support is needed.



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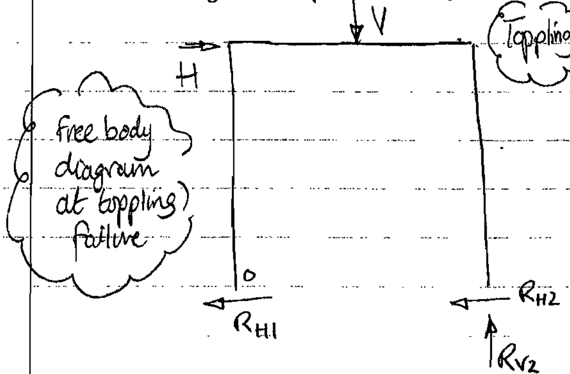
Question 3

Two-footed structure of weight V, under applied load H.

Pinned feet, so not determinate, but question say H is divided equally between feet.

Approach: ① Two failure modes are possible, as shown on interaction diagram: toppling ($R_V = 0$) or bearing/sliding ② Check each mechanism



Toppling

$$M\Big)_0 \Rightarrow \quad Ha + V\frac{a}{2} = R_{V2}a \quad -①$$

$$R\uparrow \Rightarrow \quad R_{V2} = V \quad -②$$

Since $V = 3bls_u$, combining ①, ② $\Rightarrow$

$$H = \frac{3}{2}bls_u$$

$R_{H1} < bls_u$ so sliding not critical

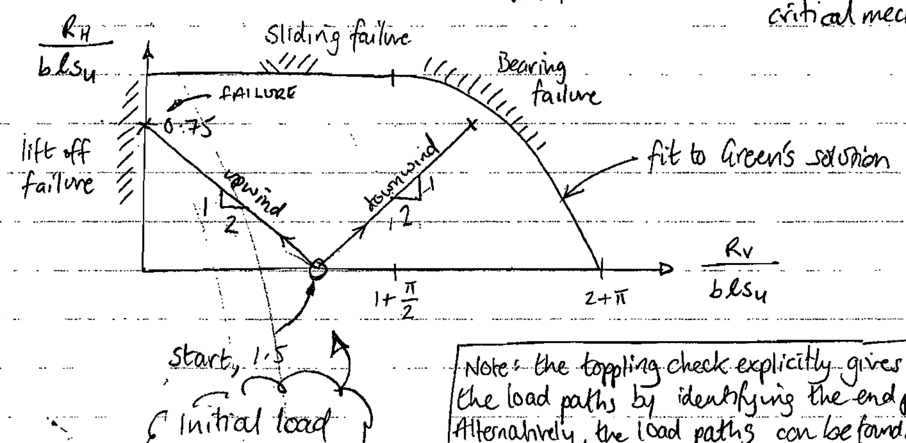
Check: does downwind footing fail first, in bearing/sliding?

$$\frac{R_{H2}}{R_{H2,ULT}} = 3/4 \Rightarrow \frac{R_{V2}}{R_{V2\,ULT}} = \frac{1}{2} + \frac{1}{2}\sqrt{1-\frac{3}{4}} = \frac{3}{4} = 0.75$$

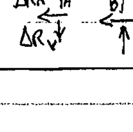
algebraic fit to Green's solution in databook.

.. at failure of downwind footing.

.. but, at lift off of upwind footing, $\frac{R_{V2}}{R_{V2\,ULT}} = \frac{3}{2+\pi} = 0.58$, so this is critical mechanism.



Note: the toppling check explicitly gives the load paths by identifying the end point. Alternatively, the load paths can be found by:



$$\vec{R} \Rightarrow \Delta R_H = H/2$$

$$M\Big)_{A,B} \Rightarrow \Delta R_V = \pm H$$

$$\Rightarrow \frac{\Delta R_H}{\Delta R_V} = 1/2$$

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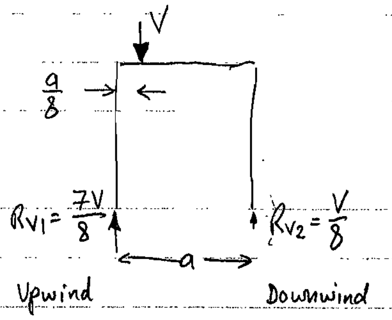
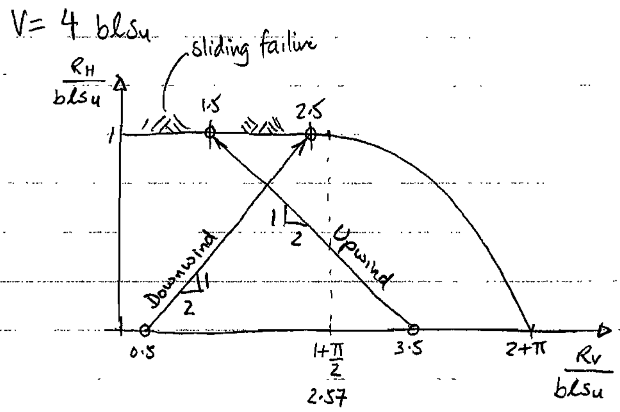
Question 3 continued.

b) Same load path as previously ($\Delta R_H / \Delta R_V = 1/2$) but different initial loads on footings.





Both footing fail simultaneously in sliding.

$$H = R_{H1} + R_{H2} = 2 b l s_u$$

Capacity is increased by 33% from $\frac{3}{2} b l s_u$ to $2 b l s_u$.

c) i) Effective depth of foundation is increased
→ hence N_c is higher, and $\gamma' \frac{b}{2}$ term is introduced to compressive bearing capacity equation
→ passive resistance (and shear) added to H (and M) capacity

ii) Tensile (uplift) capacity is provided since free water cannot reach tension interface between foundation at soil

d) $V = 2\, bl \, \underbrace{\left(N_c s_u + \gamma' \frac{b}{2} \right)}_{\text{bearing pressure}} = 2bl\left((2+\pi) \times 1.15 s_u + \gamma' \frac{b}{2}\right)$

2 footings — area — embedment factor on N_c (Skempton, databook)

$1 + \frac{1}{3}\tan^{-1}\left(\frac{b/2}{b}\right) = 1.15$

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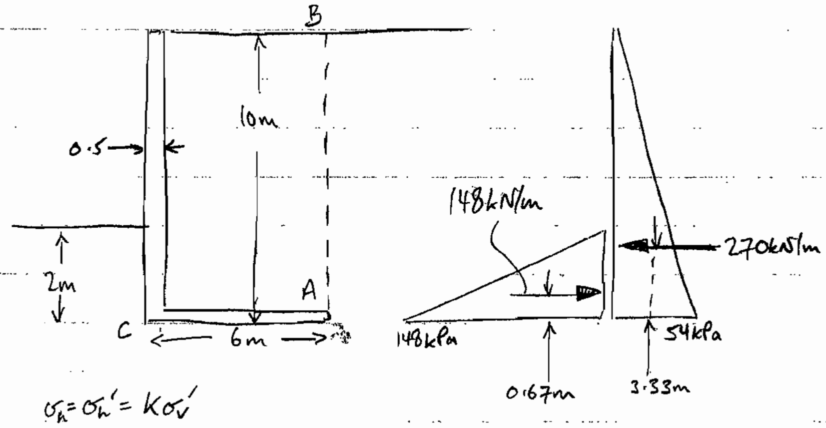
Question 4

a) Rankine's lower bound earth pressures : $\sigma_h' = K\sigma_v' = K\gamma' z$

self-weight.

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 35}{1+\sin 35} = \frac{0.43}{1.57} = 0.27$$

$K_p = 3.7$



	$\sigma_v = \sigma_v'$	$\sigma_h = \sigma_h' = K\sigma_v'$	
A	200kPa	54 kPa	} active
B	0	0	
C	40kPa	148kPa	} passive

Resultant force acts 1/3 up from base of triangle

b) Net overturning moment = $270 \times 3.33 + 148 \times 0.67 = 801$ kNm/m

Restoring moment (about toe, point C) = $\underbrace{(5.5\times 9.5\times 20)\times 3.25}_{\text{soil}} + \underbrace{(0.5\times 10\times 25)\times 0.25}_{\text{wall}} + \underbrace{(0.5\times 5.5\times 25)\times 3.25}_{\text{base}}$

$= 3651$ kNm/m

FoS on overturning $= \dfrac{3651}{801} = 4.56$

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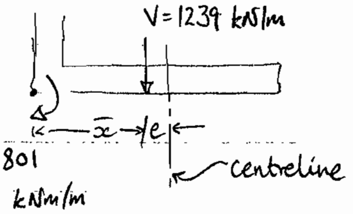
Question 4 continued

c) Vertical load $= (5.5 \times 9.5 \times 20) + (0.5 \times 10 \times 25) + (0.5 \times 5.5 \times 25)$

$V = 1239$ kN/m

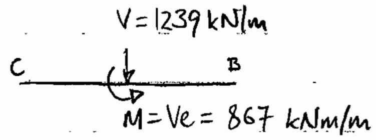
From part b), net moment at toe is:



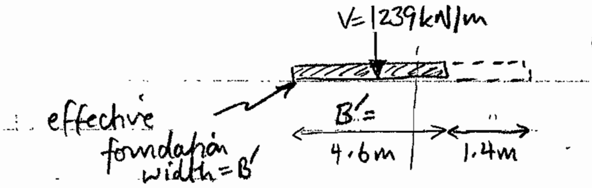
To find bearing pressure across base, find moment about centre line:

$$\bar{x} = \frac{M}{V} = \frac{2850}{1239} = 2.30\text{m} \Rightarrow e = 0.70\text{m}$$

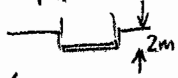
Hence system can be idealised as:



or, following Meyerhof's effective area method.



Conservative check on V_{ult} would ignore backfill on retained side:

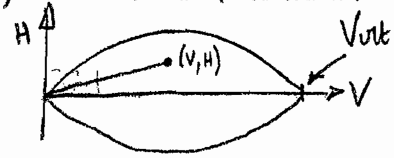


$$V_{ult} = N_\gamma \frac{\gamma B'}{2} + N_q \sigma'_{v0}$$

Also, consider horizontal load, to check against bearing/sliding failure along AC.

$H = 270 - 148 = 122$ kN/m
(from previous page)

To check stability, use V–H slice of Butterfield/Gottardi failure surface, check load is within surface



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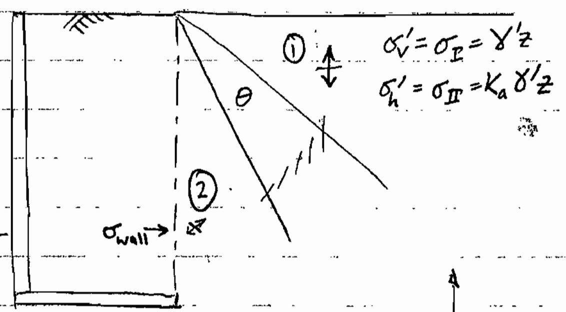
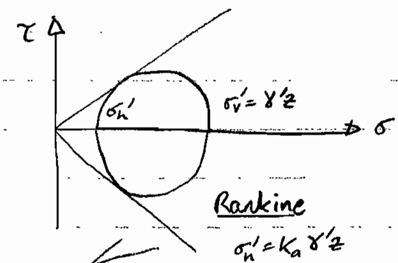
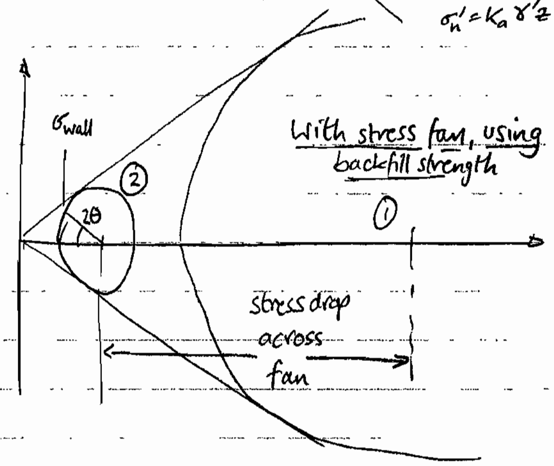
Question 4 continued

d) Strength of backfill has been ignored, leading to the assumption of hydrostatic vertical stress behind the wall and vertical and horizontal principal stresses.

If the strength of the backfill is considered, then rotation of the principal stresses can lead to a reduction in the horizontal and vertical stresses behind the wall.







3D2 Geotechnical Engineering
Exam answers 2006

Question 1: Cylindrical cavity collapse analysis of tunneling-induced settlement

(a) Proof

(b) $\varepsilon_{\gamma,a} = -\frac{2\rho_a}{a}$, $\varepsilon_{\gamma} = -\frac{2\rho_a a}{r^2} = -2\varepsilon_{\gamma,a}\frac{a^2}{r^2}$

(c) $$\frac{2\rho_b}{b\varepsilon_{\gamma,f}} = \left[\frac{\gamma}{c_u}\beta\frac{b-a}{\left\{\left(\frac{b}{a}\right)^{2\beta}-1\right\}}\right]^{1/\beta}$$

(d) $\rho_b = 6.25$ mm

Question 2: Stress paths in clay, collapse of a vertical cut

(a) A′: $\sigma'_h = 121.8$ kPa, $p' = 147.9$ kPa, $q = 78.2$ kPa
B′: $\sigma'_h = 58.5$ kPa, $p' = 55.6$ kPa, $q = -8.5$ kPa

$K_a = 0.44$, $K_p = 2.28$

(b) C′: $q = 65$ kPa, $p' = 55.6$ kPa, $\sigma'_h = 33.9$ kPa, $\sigma'_v = 98.9$ kPa
D′: $q = 70$ kPa, $p' = 78.9$ kPa, $\sigma'_h = 55.3$ kPa, $\sigma'_v = 125.3$ kPa

(c) B: $q = -8.5$ kPa, $p = 45.6$ kPa, $\sigma_h = 48.5$ kPa, $\sigma'_v = 40$ kPa
E: $q = 40$ kPa, $p' = 20$ kPa, $\sigma_h = 0$ kPa, $\sigma_v = 40$ kPa

Question 3: Combined V-H loading of a two-footing structure

(a) $H = 3blc_u/2$

(b) $H = 2blc_u$

(c) $V = 2bl\,(2+\pi)\,d_c c_u + \gamma' b/2$

Question 4: Stability of a cantilever gravity wall in dry sand

(a) $M_{overturning} = 801$ kNm/m

(b) $M_{restoring} = 3651$ kNm/m , FoS = 4.56

(c) $B' = B - 2e = B - 2(M/V) = 4.6$ m

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