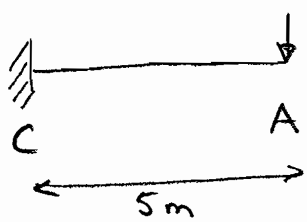
PART IIA 2006
3D3: Structural materials and design
Principal Assessor: Dr J Lees

PAPER 3D3, STRUCTURAL

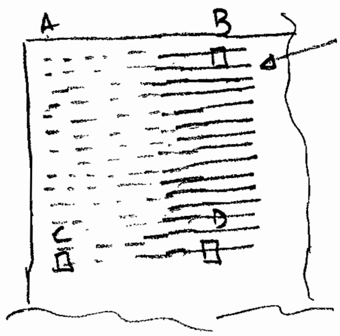
1a) bookwork - three examples required e.g.
L/d - initial design (span/depth)
b/t - buckling of compression flanges in I-beams
L/r - column buckling
L/t - for masonry arches
$\phi_e = \frac{12I}{A^2}$ - shape efficiency factor for bending
etc.
Derive formula for limiting value of one of the above e.g.
limiting L/d for simply-supported beam with uniform load

1b) i)



$M_C = 32\,kN \times 5m = 160\,kNm$

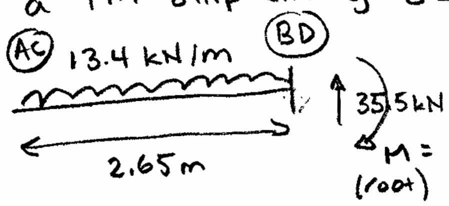
ii)



top steel resistance = 70 kNm/m

applied uniform load on slab
$w = 25\frac{kN}{m^3} \times 0.2m \times 1.4$ (dead load)
$+ 4\frac{kN}{m^2} \times 1.6$ (live load)
$w = 13.4\, kN/m^2$

for a 1m strip along BD



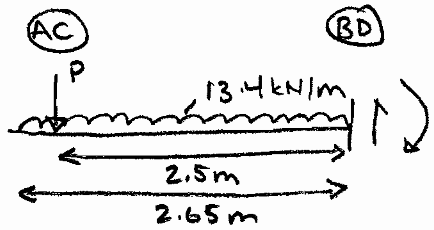
$M_{(root)} = 13.4\frac{kN}{m} \times \frac{(2.65m)^2}{2} = 47.05\, kN.m$

$M_{provided} = 70\,kN\frac{m}{m} > M_{required \text{ for uniform load}} = 47.05\frac{kNm}{m}$

∴ residual capacity to help support beam AC

1b) ii) continued

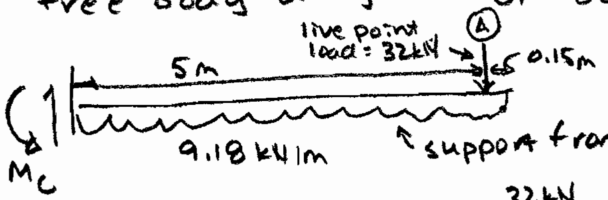
for 1m strip along BD, additional load that can be carried = P

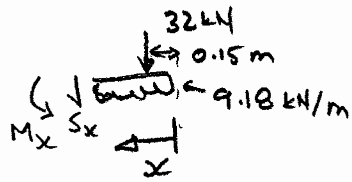


M_{root} = 70 kN m
↑ maximum resistance

$P \times 2.5m = (70 - 47.05)$ kNm   ∴ P = 9.18 kN (1m strip)

free body diagram of beam AC





$x < 0.15m$

$M_x = -9.18 \frac{x^2}{2}$, $M_A = -9.18 \frac{(0.15)^2}{2} = -0.10$ kNm

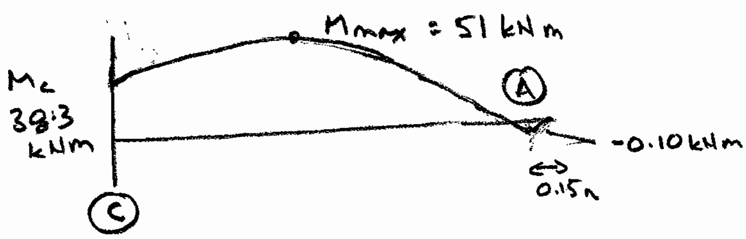
$x > 0.15m$

$M_x = 32(x - 0.15) - 9.18\, x^2/2$

$\frac{dM_x}{dx} = 32 - 9.18x = 0$ ∴ $x = 3.49$ m

$M_{max} = 32(3.49 - 0.15) - 9.18 \frac{(3.49)^2}{2}$
= 51 kNm

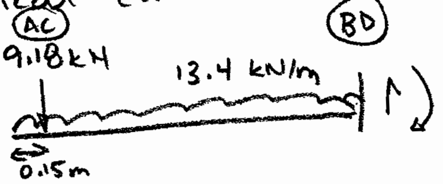
$M_c = 38.3$ kNm

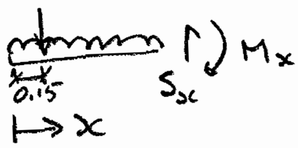


Beam AC

1b)ii) continued

typical cantilever strip



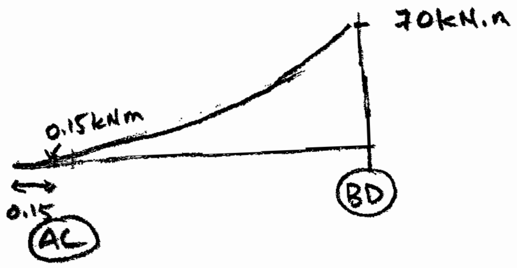


$x < 0.15$ m

$M_x = 13.4\,x^2/2 \quad \rightarrow \quad M_{0.15} = 0.15$ kNm

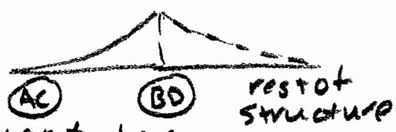
$x > 0.15$m

$M_x = \dfrac{13.4x^2}{2} + 9.18\,(x - 0.15)$



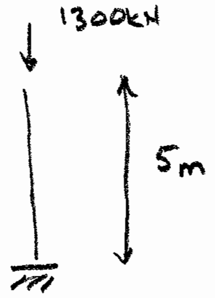
b) iii)

Need to carry the bending moment diagram (for slab at BD and the beam at C) through the rest of the structure to check that there is a feasible equilibrium system throughout. For example if the moment along BD is balanced by a moment from the rest of the structure there is no need to design for torque in BD e.g

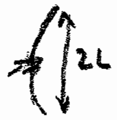


Thus the equilibrium system can be chosen so there is no resultant torque. Any load path will do and it need not be the actual one. This is an application of the lower bound theorem of plasticity theory so the assumptions of ductility, no other failure modes e.g. buckling, small deformations all apply

2a)



for a cantilever $L_{ef} = 2L = 2 \times 5m$
$= 10m$



choose a 305 × 305 section
S355 steel, $\gamma_m = 1.05$

$$\lambda_0 = \pi\left(\frac{E}{\sigma_y}\right)^{1/2} = \pi\left(\frac{210000}{355/1.05}\right)^{1/2} = 78.3$$

in 305 × 305 series r_{yy} varies between 7.69 & 8.27 cm
∴ initially select $r_{yy} = 7.69$ cm (lowest value)

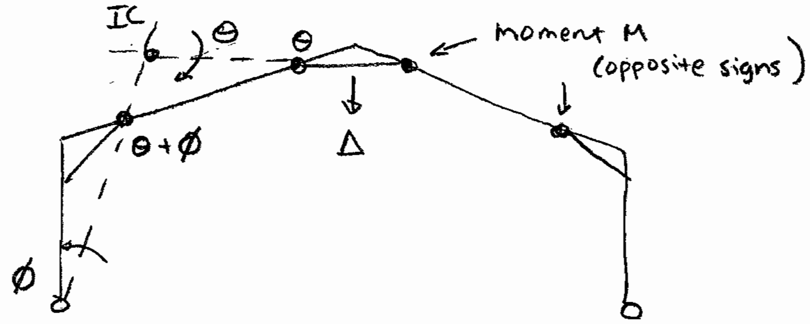
$$\lambda = \frac{L_{ef}}{r} = \frac{10000}{76.9} = 130$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_0} = \frac{130}{78.3} = 1.66 \rightarrow \chi \sim 0.3$$

so $A_{req} \approx \dfrac{1300000}{(355/1.05) \times 0.3} = 12817 \text{ mm}^2$

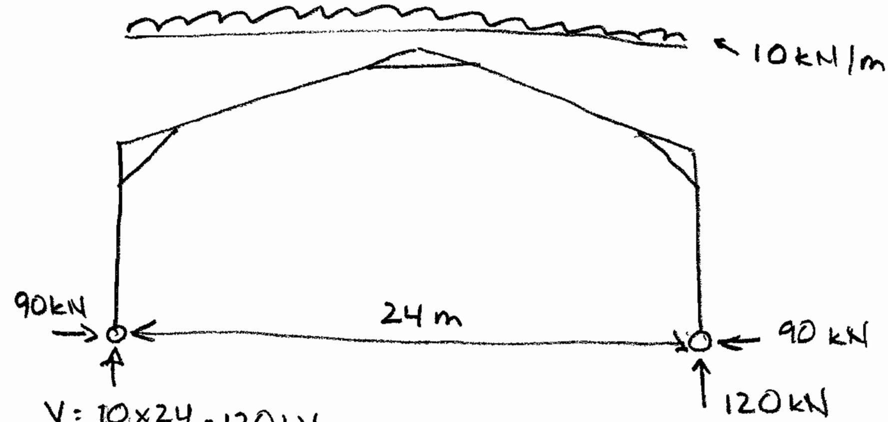
select 305 × 305 × 118 UC
$\left\{ \begin{array}{l} A = 150 \text{ cm}^2, \; r_{yy} = 7.77\text{cm} \\ \bar{\lambda} = 1.64, \quad \chi \approx 0.29 \\ P_{provided} = 0.29 \times \frac{355}{1.05} \times 15000 \\ \quad = 1471 \text{ kN} \end{array} \right\}$

bi) consider mechanism

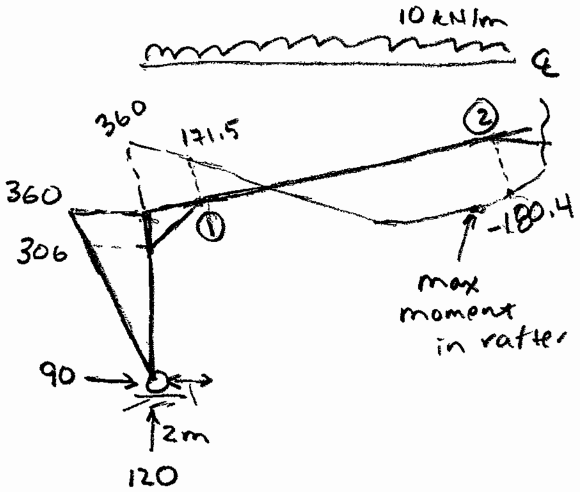


- postulate hinge locations
- find instantaneous centre
- consider a downwards displacement at A & find compatible rotations θ & φ
- use virtual work where the work done by the applied loads = energy dissipated (M(θ + θ + φ))
- find M

2b)ii)



$V = \frac{10 \times 24}{2} = 120\text{kN}$



RAFTER

M_{haunch} ① $= 90 \times 4.35 - 120 \times 2\text{m} + 10 \times 2^2/2$
$= 171.5 \text{ kN.m}$

M_{haunch} ② $= 90 \times 5.94 - 120 \times 11 + 10 \times 11^2/2$
$= -180.4 \text{ kN m}$

Choose stanchion with $M_p = 306$ kNm

$$Z_p = \frac{M_p}{\sigma_y} = \frac{306 \times 10^6}{275/1.05} = 1.168 \times 10^6 \text{ mm}^3 = 1168 \text{ cm}^3$$

say $356 \times 171 \times 67$ (1211 cm³, some allowance for axial force 120 kN)

also need to check axial interaction with bending

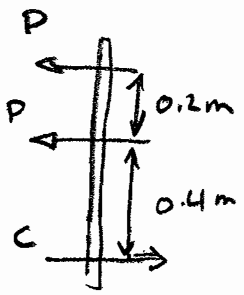
iii) Need a stiffener across stanchion at A and lateral restraint at corner A (3 compression forces meeting at a point). This could be achieved by tieing to longitudinal eaves member. Compression flange restraint required e.g. using purlins or roof cladding. Need to check overall frame is stable by providing bracing.

2b) iii) continued

3 layers of bolts, each layer takes say $\frac{120}{3} = 40\,kN$ in shear

<u>but</u> also a moment applied of 360 kNm

assume top bolts all yielded : neglect bottom bolts and that the compression force acts at say 100 mm above A



$P \times 0.6 + P \times 0.4 = 360\,kNm$

$P = 360\,kN$

one bolt on either side ∴

$P_{bolt} = 360/2 = 180\,kN$ → quite high, may need more bolts

NOTE

There were a surprising number of small errors in part (a) such as the use of the wrong effective length and/or radius of gyration.

Many students had trouble drawing the bending moment diagram in part b (ii)

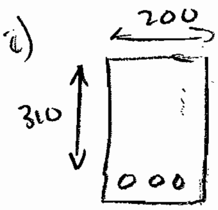
3a) - the plots indicate the properties of balanced symmetric laminates made up from lamina with particular properties

- the values used in the plots are in effect homogenized properties of the laminate structure

- the plots will have been determined by using laminate theory summating the contributions of lamina in various 0, 90 ± 45° combinations

- if the required performance attributes of a laminate are known, the carpet plots represent an easy way to get an indication of the % plies required in each direction

b) i)



$f_{cu} = 45$ MPa , $f_y = 460$ MPa
$\gamma_c = 1.5$, $\gamma_s = 1.15$
$\varepsilon_{cu} = 0.0035$, $\varepsilon_y = 0.002$
3 No 20mm Ø bars ⇒ $A_s = 942\,mm^2$

under-reinforced

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} b d} = \frac{1.5 \times 942 \times 460}{1.15 \times 0.6 \times 45 \times 200 \times 310} = 0.34$$

$$M_u = \frac{A_s f_y d(1 - 0.5x/d)}{\gamma_s} = \frac{942 \times 460 \times 310(1 - 0.5 \times 0.34)}{1.15}$$
$$= 97.7 \text{ kN.m}$$

Check span/d ratio

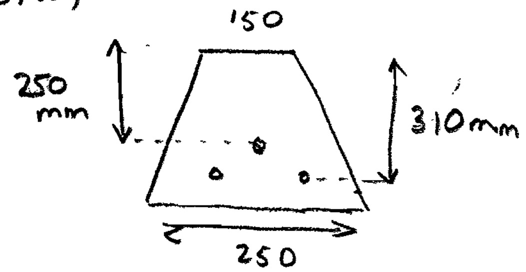
$9000/310 \approx 29$

for a one-way spanning beam (simply-supported) the IStructE guidelines suggest an L/d of 15-21 & EC2 18-25 in order to ensure serviceability requirements likely to be satisfied

in this case L/d = 29 so serviceability limit state may be an issue

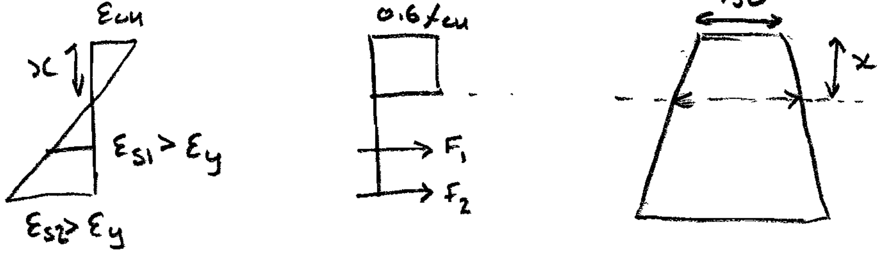
NOTE
A number of students incorrectly stated that because the beam was under-reinforced (a ULS condition), the SLS condition would be satisfied.

3b) ii)



$\varepsilon_{cu} = 0.0035$
$\varepsilon_y = 0.002$

assume steel has yielded



$b(x) = 150 + \frac{100}{350} x$

$F_c = 0.6 f_{cu}\, x\, (150 + b(x))/2 \times \gamma_m$

$F_T = F_1 + F_2 = 3 \times \frac{\pi \times 20^2}{4} \times \frac{460}{1.15} = 377$ kN

$F_c = F_T$

$\therefore \frac{0.6 f_{cu}}{1.5} \frac{x}{2} \left(150 + (150 + 100/350\, x)\right) = 377000$ N

$2.57x^2 + 2700x - 377000 = 0$

$x = \frac{-1050.6 \pm \sqrt{(1050.6)^2 + 4(1)(146611)}}{2}$

$x = (+1050.6 \pm 1300.1)/2 = 124.7$ mm

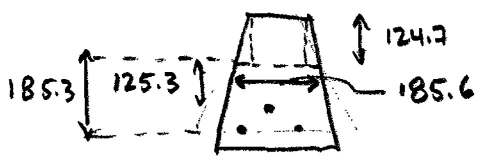
check steel has yielded (top layer will be critical)

$\varepsilon_{s1} = \frac{0.0035}{124.7} \times (250 - 124.7) = 0.0035 > \varepsilon_y \therefore$ yielded

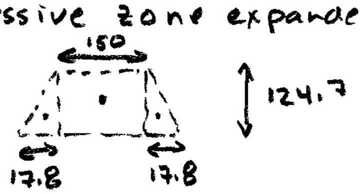
$M_u = \frac{0.6 f_{cu}}{1.5} \left[\frac{124.7^2 \times 150}{2} + \cancel{2} \times \frac{1}{3} \times \frac{35.6}{\cancel{2}} \times 124.7^2\right]$

$+ \frac{460}{1.15} \times 314 \times [125.3 + 2 \times 185.3]$

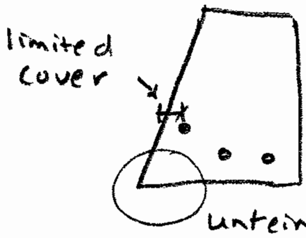
$= 24.3 \times 10^6 + 62.3 \times 10^6 = 86.6 \times 10^6$ N.mm $= 86.6$ kN m



compressive zone expanded view



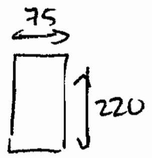
3 b) iii)



unreinforced region → susceptible to spalling

- limited cover - durability issues, fire, corrosion issues
- reinforcement no longer symmetric so need to think about y-y direction as principal axis no longer horizontal
- moment capacity about x-x axis similar to beam (b) but will now have y-y interaction in compressive zone
- bond may be a further concern (due to limited cover)

4 (a)



C18 - $E_{0.05} = 6\,GPa$
service class 3 (outside)
long-term loading
$k_{mod} = 0.55$

$L_{ef} = 5\,m$

$$M_{crit} = \frac{\pi E_{0.05} b^3 h}{L_{ef}\, 24} = \frac{\pi \times 6000 \times 75^3 \times 220}{5000 \times 24} = 14.6 \times 10^6\ N.mm$$

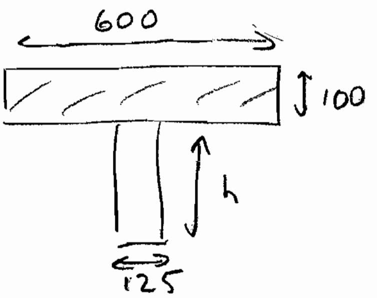
$$\sigma_{crit} = \frac{M_{crit}\, y}{I} = \frac{14.6 \times 10^6 \times 110}{75 \times 220^3/12} = 24.1\ MPa$$

$$\lambda_{rel,m} = \sqrt{f_{m,k}/\sigma_{m,crit}} = \sqrt{18/24.1} = 0.864$$

$$k_{crit} = 1.56 - 0.75 \times 0.864 = 0.91$$

$$f_{m,d} = k_{mod}\, k_h\, k_{crit}\, k_{ls}\, f_{m,k} / \gamma_m$$
$$= 0.55 \times 1 \times 0.91 \times 1 \times 18 / 1.3 = 6.94\ MPa$$

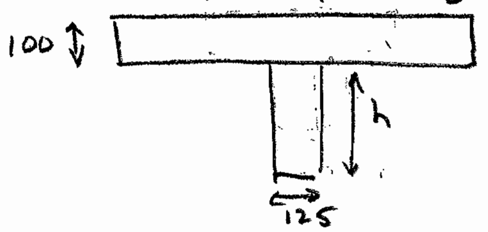
b)



$E_{conc} = 20\,GPa$
$E_{timber} = 9\,GPa$

i) transform ~~to timber~~ concrete to timber (also fine to transform timber to concrete)

$600 \times 20/9 = 1333$



$$\frac{100^2}{2} \times 1333 = 125 \times \frac{h^2}{2}$$
$$h = 327\,mm$$

if the neutral axis is at the interface the concrete is always in compression

Note
Many mistakes in transforming materials. A further concern was the number of students who stated the shear stress was zero at the neutral axis which is incorrect. The shear stress is a maximum at the neutral axis.

b) ii)

timber/concrete

$$I = \frac{125 \times 327^3}{12} + \left(\frac{327}{2}\right)^2 \times 125 \times 327 + \frac{100^3 \times 1333}{12} + 50^2 \times 1333 \times 100$$

$$= 364.2 \times 10^6 + 1.0926 \times 10^6 + 111.1 \times 10^6 + 333.3 \times 10^6$$

$$= 1.901 \times 10^9 \text{ mm}^4$$

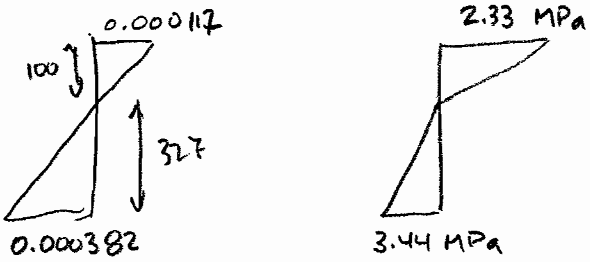
$$k = \frac{M}{EI} = \frac{20 \times 10^6}{9000 \times 1.901 \times 10^9} = 1.169 \times 10^{-6} \ \frac{1}{\text{mm}}$$

timber

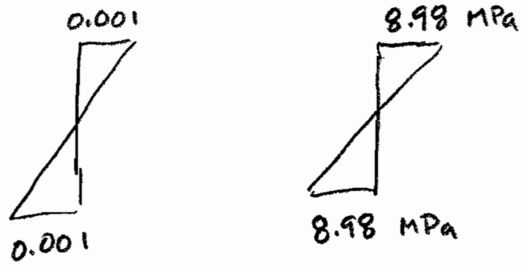
$$I = \frac{125 \times 327^3}{12} = 364 \times 10^6$$

$$k = \frac{M}{EI} = \frac{20 \times 10^6}{9000 \times 364 \times 10^6} = 6.105 \times 10^{-6} \ \frac{1}{\text{mm}} \quad \text{much higher}$$

timber/conc



timber



b) iii) adv + disadv

- composite action will result in smaller deflections, stiffer response and reduced vibrations when compared to non-composite timber system - however, will not be as stiff as a steel-concrete system
- good thermal mass (concrete slab) & timber is sustainable (if sourced from sustainable forests)
- need to consider long-term integrity between timber & concrete and thermal/moisture movement in timber
- concrete provides lateral stability to compression flange

The appropriate concrete strength will depend on the grade of timber. However even with a high grade timber (e.g. C40 might have $f_{m,d} \approx 20$MPa with typical modification factors say), for the beam shown here the resulting concrete stress (assuming still elastic) would be $\frac{20}{3.44} \times 2.33 \approx 13.5$MPa so unlikely to need very high concrete strength. Also need to think about deflection limits which may control in which case again a lower strength concrete would probably suffice.

Engineering Tripos Part IIA, 2006

Paper 3D3 Structural Materials and Design

Answers

1. (b) (i) $M_c = 160$ kNm
   (ii) $M_{max} = 51.1$ kNm

2. (a) 305 × 305 × 118 UC ($\chi \sim 0.29$, $P_{provided} = 1471$ kN)
   (b) (ii) Z_p (required) = 1168 cm^3, possible section 356 × 171 × 67 UB

3. (b) (i) $M_U = 97.1$ kNm , L/d ~ 29 $\therefore$ SLS unlikely to be satisfied
   (ii) $M_U = 86.6$ kNm

4. (a) (i) $f_{m,d} = 6.94$ MPa
   (b) (i) h = 327 mm, if neutral axis is at the interface, the concrete remains in compression
   (ii) $\kappa = 1.17 \times 10^{-6}$ mm^{-1} (timber/concrete) < $\kappa = 6.11 \times 10^{-6}$ mm^{-1} (timber)
   $\sigma_{comp} = 2.33$ MPa (timber/concrete), $\sigma_{comp} = 8.98$ MPa (timber)

J.L. June, 2006