3D4 Structural Analysis and Stability - Solutions

Examiner's comments in italics

1. (a) $A = \int dA = 7640\,\text{mm}^2$; $\bar{x}.A = \int x.dA = 3056.10^3\ \text{mm}^3$ so $\bar{x} = 400\,\text{mm}$

$\bar{y}.A = \int y.dA = 4584.10^3\ \text{mm}^3$ so $\bar{y} = 600\,\text{mm}$

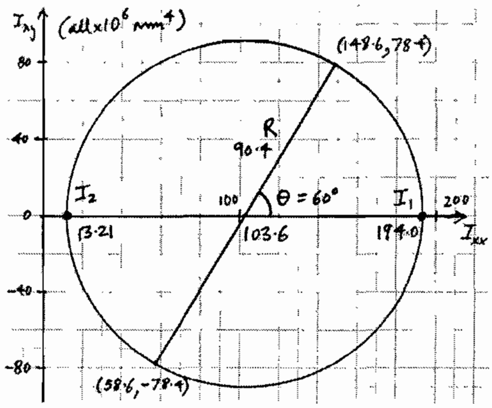
$I_{yy} + \bar{x}^2 A = \int x^2 dA = 1281.10^6\,\text{mm}^4$ so $I_{yy} = 1281.10^6 - 400^2.7640 = 58.60.10^6\,\text{mm}^4$

$I_{xx} + \bar{y}^2 A = \int y^2 dA = 2899.10^6\,\text{mm}^4$ so $I_{xx} = 2891.10^6 - 600^2.7640 = 148.6.10^6\,\text{mm}^4$

$I_{xy} + \bar{x}.\bar{y}.A = \int xy dA = 1912.10^6\,\text{mm}^4$ so $I_{xy} = 1912.10^6 - 400.600.7640 = 78.40.10^6\,\text{mm}^4$

(*Many got sign errors applying parallel-axis theorem here*)

(b) Plot Mohr's Circle



(*A significant proportion had not brought a compass to the exam so could not draw a circle to scale.*)

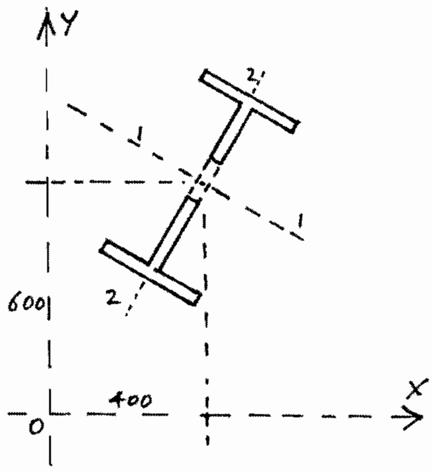
$I_1 = 194.0.10^6\ \text{mm}^4$; $I_2 = 13.21.10^6\ \text{mm}^4$

(c) Look in Data Book for suitable section; $I_{xx} = 19400\ \text{cm}^4$; $I_{yy} = 1321\ \text{cm}^4$.

Nearest section is 356×171×67 Universal Beam ($I_{xx} = 19640\ \text{cm}^4$; $I_{yy} = 1362\ \text{cm}^4$), but area is 85.5 cm^2 whereas our section has 76.40 cm^2 (so 9.1 cm^2 missing).

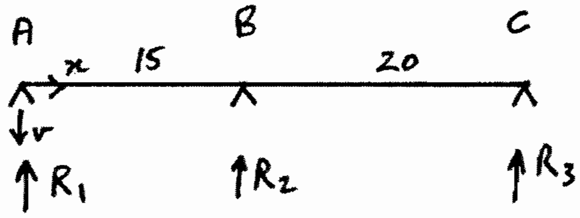
How can material be removed from section without affecting *I* significantly? Material must be removed from very close to the centroid.

Assume hole cut in web, which is 9.1 mm thick. So modification likely to be a 100 mm hole drilled through web at this section.



Most popular question and on the whole done well. The most common fault was failure to follow the sign convention of Mohr's circle so they got the mirror-image of the correct answer. About 30% got the sign wrong, which probably implies that 60% were treating it as a random process of whom half got it right by chance. The other common error was not to pay any attention to the area of the cross-section when determining which section should be selected from the data book. Bonus marks were awarded if students actually checked that this gives the measured values of I.

2. (a)



(i) $-EI\dfrac{d^2v}{dx^2} = R_1 x + R_2\{x-15\}$ Integrate twice $-EIv = R_1\dfrac{x^3}{6} + R_2\dfrac{\{x-15\}^3}{6} + Ax + B$

$x = 0$, $v = 0$, so $B = 0$

$x = 35$, $v = 0$, so $0 = R_1\dfrac{35^3}{6} + R_2\dfrac{20^3}{6} + 35A$ whence $A = -204.2R_1 - 38.1R_2$

$x = 10$, $v = \delta$, so $-EI\delta = R_1\dfrac{15^3}{6} + 15A = -2500R_1 - 571.5R_2$

Moments about C $35R_1 = -20R_2$ so $R_2 = -1.75R_1$

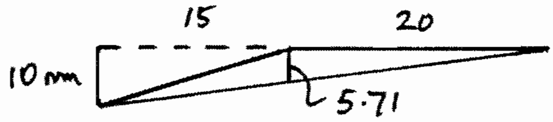
Combining these results, with $\delta = 10$ mm;

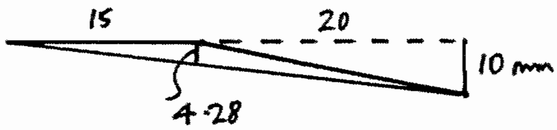
$R_1 = 66.7$ kN; $R_2 = -116.7$ kN; $R_3 = 50$ kN; $A = -9174$ kNm^2

Moment at B = $15.R_1$ = 1000 kNm (sagging)

At centre of longer span, $x = 25$, $-EIv = 66.7\dfrac{25^3}{6} - 116.7\dfrac{10^3}{6} - 25\cdot 9174$ so $v = 7.50$ mm

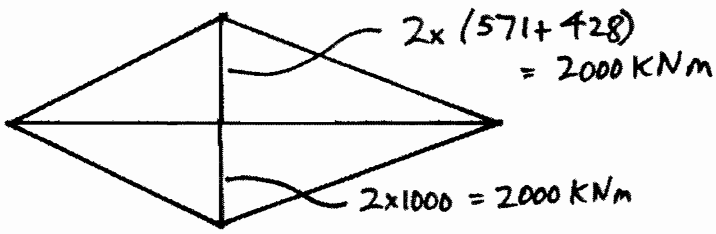
(ii) If A deflects by 10 mm relative to B and C, the beam deflection can be regarded as a rigid body rotation plus a deflection of 5.71 mm at B, so the moment induced at B will be 571 kNm (hogging).



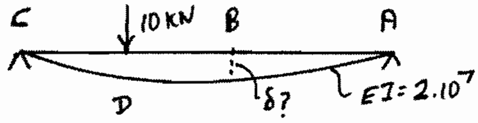


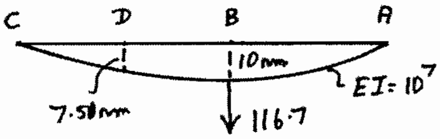
(iii) Similarly, if C deflects by 10 mm relative to A and B, a moment of 428 kNm (hogging) will be induced at B.

Combining these results, and noting that the possible settlement is 20 mm, the beam must be designed for a bending moment of ±2000 kNm at B, in addition to bending moments due to the loads.



(b) Note that the beam specified in (b) is the mirror image of the beam in (a), and also that the stiffness is now twice the stiffness in (a).





From (a)(i) we know that a load of 116.7 kN applied at B gives a deflection of 10 mm at B and 7.50 mm at D in a beam of stiffness 10^7 kNm^2.

So a load of 10 kN at B, on a beam of stiffness 2.10^7 kNm^2 would have caused a deflection at D of

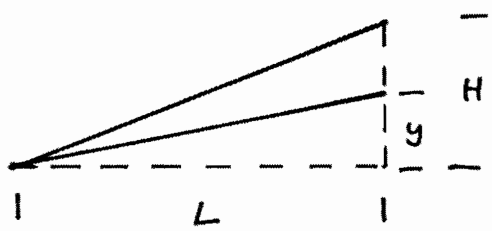
$$\frac{1}{2} \cdot \frac{7.50}{11.67} = 0.321 \text{ mm}$$

The reciprocal theorem then says that a load of 10 kN at D would have caused a deflection of 0.321 mm at B.

This question was very badly done. It had been intended, and they were told, to perform one analysis, and then to work out the rest using simple logic. The first part – a simple Macaulay analysis – was done quite well, apart from the normal range of numerical errors, but very few made use of rigid body rotations to simplify the remaining calculations, instead going back to do further analyses. Quite a few tried to invoke influence lines, which was not relevant in any way while others introduced hinges. Very few reached the part dealing with the reciprocal theorem but those who did got it right.

3. (a) Bifurcation buckling occurs in a perfect Euler strut. Limit point buckling occurs in a shallow arch.

(b) (i)



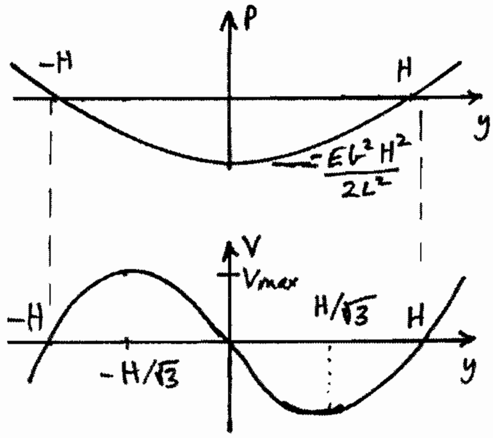
Initial length $= \sqrt{H^2 + L^2}$; Current length $= \sqrt{y^2 + L^2}$

Strain in current configuration $$= \frac{\sqrt{y^2 + L^2} - \sqrt{H^2 + L^2}}{\sqrt{H^2 + L^2}} = \frac{\sqrt{1 + \left(\frac{y}{L}\right)^2} - \sqrt{1 + \left(\frac{H}{L}\right)^2}}{\sqrt{1 + \left(\frac{H}{L}\right)^2}}$$

Applying Binomial Expansion gives strain $\approx \dfrac{y^2 - H^2}{2L^2}$ (negative because compressive)

Axial force in current configuration $P(y) = Eb^2 \varepsilon = \dfrac{Eb^2}{2L^2}\left(y^2 - H^2\right)$

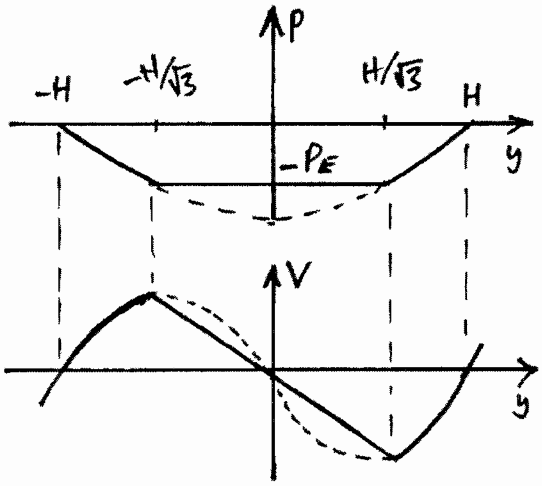
Applied load: $V(y) = 2P\frac{y}{L} = \frac{Eb^2}{L^3}\left(H^2 y - y^3\right)$



V_{max} will occur when $\frac{dV}{dy} = H^2 - 3y^2 = 0$ so $y = \pm\frac{H}{\sqrt{3}}$ and $V_{\max} = V\left(\frac{H}{\sqrt{3}}\right) = \frac{2Eb^2H^3}{3\sqrt{3}L^3}$

(ii) Note here that V_{max} and P_{max} do not occur at the same value of y. Seek to make $P\left(\frac{H}{\sqrt{3}}\right)$ equal to Euler buckling load. $P\left(\frac{H}{\sqrt{3}}\right) = \left|\frac{Eb^2}{2L^2}\left(\frac{H^2}{3} - H^2\right)\right| = \frac{\pi^2 EI}{\approx L^2} = \frac{\pi^2 Eb^2}{12L^2}$ whence $\frac{H}{b} = \frac{\pi}{2}$

Revised sketches to $P(y)$ and $V(y)$ when $P = P_E$ at V_{max}.



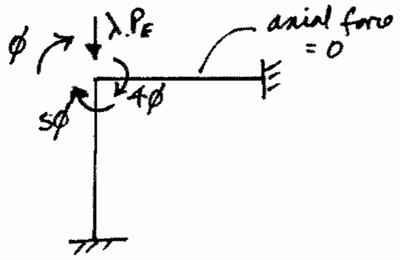
The main problem was failure to follow what was asked for in the question. There is an examples sheet question on this topic so many tried to answer the question they wish they had been asked. Many who wrote sense ran into problems because of inconsistent assumptions about small deflections, the low rise of the arch and the use of the binomial theory.

4. (a) The idea of this method is to determine the distribution of axial forces in the initial configuration and to assume that they will increase proportionally up to point of buckling. Key stages:-

- Consider as variables joint rotations (and if necessary, translations) associated with buckling
- Set up a stiffness matrix relating to these variables to corresponding couples/forces; values will depend on axial forces through the stability functions).
- Set couples/forces=0 and look for non-trivial solution of the stiffness equations, when determinant of stiffness matrix approaches zero.

For non-sway frames all variables are rotations. If the frames can sway additional displacement variables have to be considered and these will normally cause a lowering of the buckling loads.

(b)



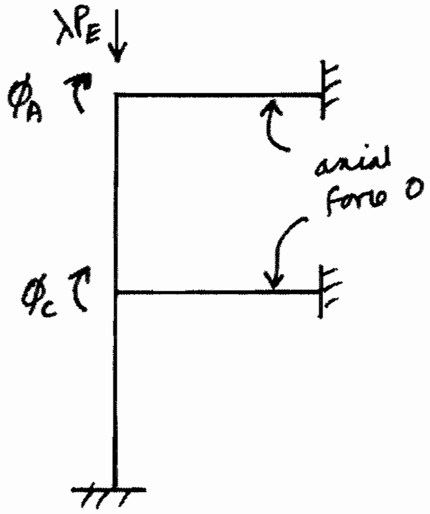
Hence, couples required for rotation ϕ

$$\begin{bmatrix} M_a \\ M_c \end{bmatrix} = \begin{bmatrix} (4+s) & cs \\ cs & s \end{bmatrix} \begin{bmatrix} \phi \\ 0 \end{bmatrix}$$

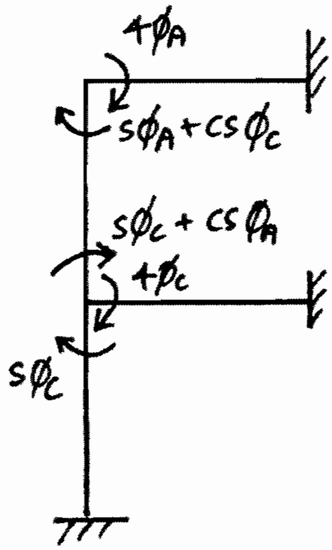
N.B. factor of *EI/L* omitted

$M_A = 4\phi + s\phi = 0$ for buckling, so $s = -4$, when $P/P_E \approx 2.86$ from table on data sheet.
(*Many set determinant of this matrix to zero, which is wrong because it takes no account of the boundary condition at the foot.)*

(c)



Key difference from case (b) is that member AC has non-zero rotations at both ends.



So couples required to impose rotations ϕ_A and ϕ_C :-

$$\begin{bmatrix} M_A \\ M_C \end{bmatrix} = \begin{bmatrix} (4+s) & cs \\ cs & (2s+4) \end{bmatrix} \begin{bmatrix} \phi_A \\ \phi_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Determinant Δ = zero when $(4 + s)(2s + 4) - (cs)^2 = 0$

Solve by trial and error starting from value of λ in case (b).

λ	s	c	Δ
2.8	-3.44	-1.71	-36.2
2.7	-2.81	-1.93	-31.3
2.6	-2.25	-2.23	-26.0
2.5	-1.75	-2.67	-20.7
2.4	-1.30	-3.34	-15.4
2.3	-0.89	-4.62	-10.0
2.2	-0.52	-7.51	-4.95
2.1	-0.18	-21.07	-0.48
2.0	0.14	24.68	5.78

So solution $\lambda \approx 2.1$

On the whole done well. There were two common errors – in part (b) many candidates did not take account of the clamped support at the foot and looked for indeterminacy in the whole matrix. In part (c) they had s+8 in place of 2s+4, implicitly ignoring the axial force in the lower column.

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1st June 2006

3D4 Structural Analysis and Stability – Examination 2006

Numerical answers

1. (a) $\bar{x} = 400$ mm, $\bar{y} = 600$ mm $I_{yy} = 58.60.10^6\,\text{mm}^4$, $I_{xx} = 148.6.10^6\,\text{mm}^4$, $I_{xy} = 78.40.10^6\,\text{mm}^4$ (b) $194.0.10^6$ mm^4, $13.21.10^6$ mm^4 (c) 356×171×67 UB

2. (a)(i) 1000 kNm (sagging); 7.50 mm (ii) 571 kNm (hogging)
(iii) ±2000 kNm at B (b) 0.321 mm

3. (b)(i) $P(y) = \dfrac{Eb^2}{2L^2}\left(y^2 - H^2\right)$; $V(y) = \dfrac{Eb^2}{L^3}\left(H^2 y - y^3\right)$; $\dfrac{Eb^2H^2}{2L^2}$; $\dfrac{2Eb^2H^3}{3\sqrt{3}L^3}$
(ii) $\dfrac{H}{b} = \dfrac{\pi}{2}$

4. (b)(i) 2.86; 2.1

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