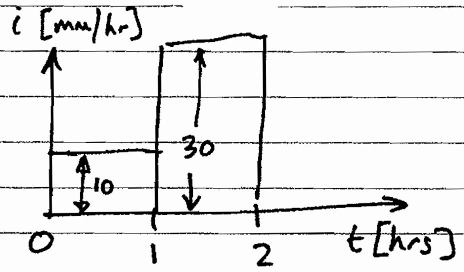
PART IIA 2006
3D5: Environment engineering I
Principal Assessor: Dr A Al-Tabbaa

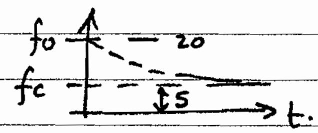
Datasheet: Environmental Engineering I Data Sheet

3D5.

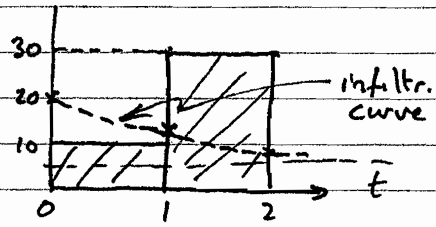
Q1 (a) Hyetograph



Horton: $f = f_c + (f_0 - f_c)e^{-K_f t}$



∴



t	f
0	$5 + 15 = 20$ mm/hr
1	$5 + 15e^{-0.7} = 12.45$ mm/hr
2	$5 + 15e^{-1.4} = 8.70$ mm/hr.

For first hour, infiltration curve is <u>above</u> rainfall
∴ All rain infiltrates in 1st hour.

For second hour, we need to start infiltration eqn at t_{equiv}, for 10mm prev. total infiltration.

$$\text{Total infiltration eqn} = \int_0^t f \, dt$$

$$= \int_0^t f_c + (f_0 - f_c)\exp(-K_f t)\, dt$$

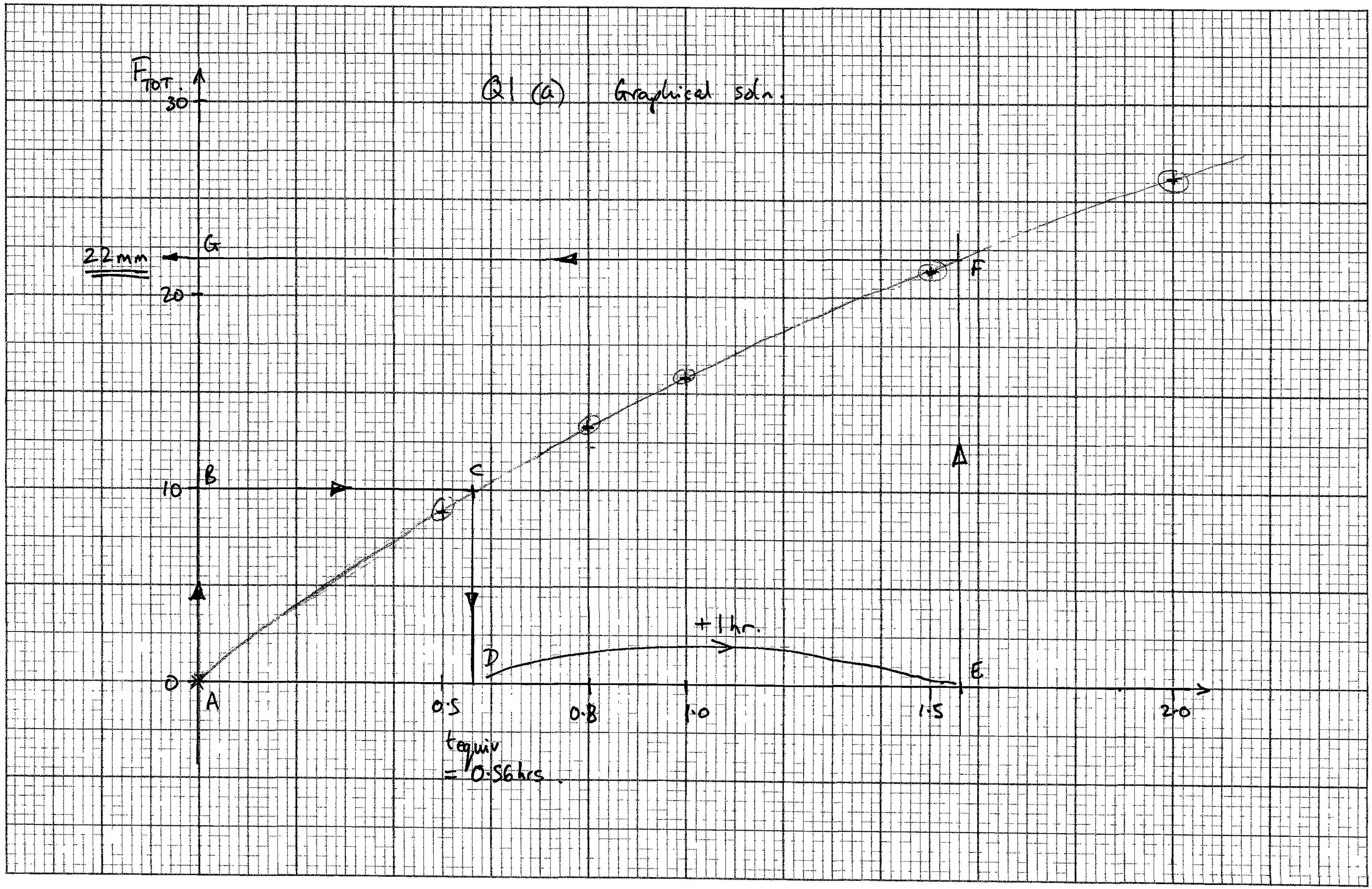
$$= f_c t + \frac{(f_0 - f_c)}{-K_f}\left[\exp(-K_f t) - \exp[0]\right]$$

$$F_{TOT} = f_c t + \frac{(f_0 - f_c)}{K_f}\left[1 - \exp[-K_f t]\right]$$

Need to find t when $F_{TOT} = 10$ mm.

Can't solve analytically – it's a transcendental eqn.

Solve graphically – (or numerically).


Q1 (a) Graphical soln.
F_{TOT}
30
20
10
0
22 mm
G
B
C
D
E
F
A
0.5
0.8
1.0
1.5
2.0
+ 1 hr.
t_{equiv} = 0.56 hrs.

Q1 (b). Distrib %'s = 4, 14, 30, 25, 16, 8, 3
Accumulate $\rightarrow$ 4, 18, 48, 73, 89, 97, 100

Draw on graph paper, at "½ time points"
i.e. 2hrs, 6hrs, 10hrs, . . .
2, 6, 10, 14, 18, 22, 26

Shift by 2 hours. and subtract : —

Then read off ~~base~~ flow at "½ time points" on a 2hr basis.

= 1, 3, 5, 7, 9 hrs etc

Dist %'s over 2hr base
= 2, 4, 7, 10·5, 14, 16, 12·5, 10·5, 8·5, 6·5, 5, 2·5, 1

Total run off = 2hrs × 10 mm/hr × 10 km²
= 20 mm × 10 km²
= 20×10^{-3} m × 10×10^{6} m²
= 200×10^{3} m³

Now 100% of this runs off in 26 hrs

$\therefore$ Average runoff = $\dfrac{200 \times 10^3 \text{ m}^3}{26 \times 3600 \text{ s}} = 2{\cdot}14$ m³/s.

Average height of hydrograph = $\Sigma \dfrac{2+4+7+\ldots}{13} = \dfrac{100}{13}$
= 7.69

$\therefore$ 7·69 $\Leftrightarrow$ 2·14 m³/s

$\therefore$ 16 $\Leftrightarrow$ $\dfrac{16}{7{\cdot}69} \times 2{\cdot}14 = 4{\cdot}45$ m³/s.

1 $\Leftrightarrow$ $\dfrac{2{\cdot}14}{7{\cdot}69}$ m³/s = 0.278 m³/s.

$\therefore$

2, 4, 7, 10·5, 14, 16, 12·5, 10·5, 8·5, 6·5, 5, 2·5, 1

$\Rightarrow$ 0·56, 1·11, 1·94, 2·92, 3·90, 4·45, 3·48, 2·92, 2·36, 1·81, 1·39, 0·70, 0·29 m³/s.

$\rightarrow$ plot. at ½ time pts
1, 3, 5, 7, etc.

Q1 (a) cont'd.

t (hrs)	F
0	0
1	$5(1) + (15/0.7)(1 - e^{-0.7}) = 15.8$ mm
2	$5(2) + (15/0.7)(1 - e^{-1.4}) = 26.1$ mm
0.5	$5(0.5) + 15/0.7\,(1 - e^{-0.35}) = 8.8$
0.8	$5(0.8) + 15/0.7\,(1 - e^{-0.56}) = 13.2$
1.5	$5(1.5) + 15/0.7\,(1 - e^{-1.05}) = 21.4$

Plot on graph (see attached).

Answer – follow the letters.
In first hour, total infiltration is 10 mm (as rainfall less than infiltration) so follow letters ABCD
– go up to 10 mm (A → B)
read off t equiv (C → D) → t equiv = 0.56 hrs.

In second hour, rainfall exceeds infiltration rate, Follow letters DEFG.
– move forward 1 hour (D → E)
read off total infiltration (F → G) → 22mm total infiltration.

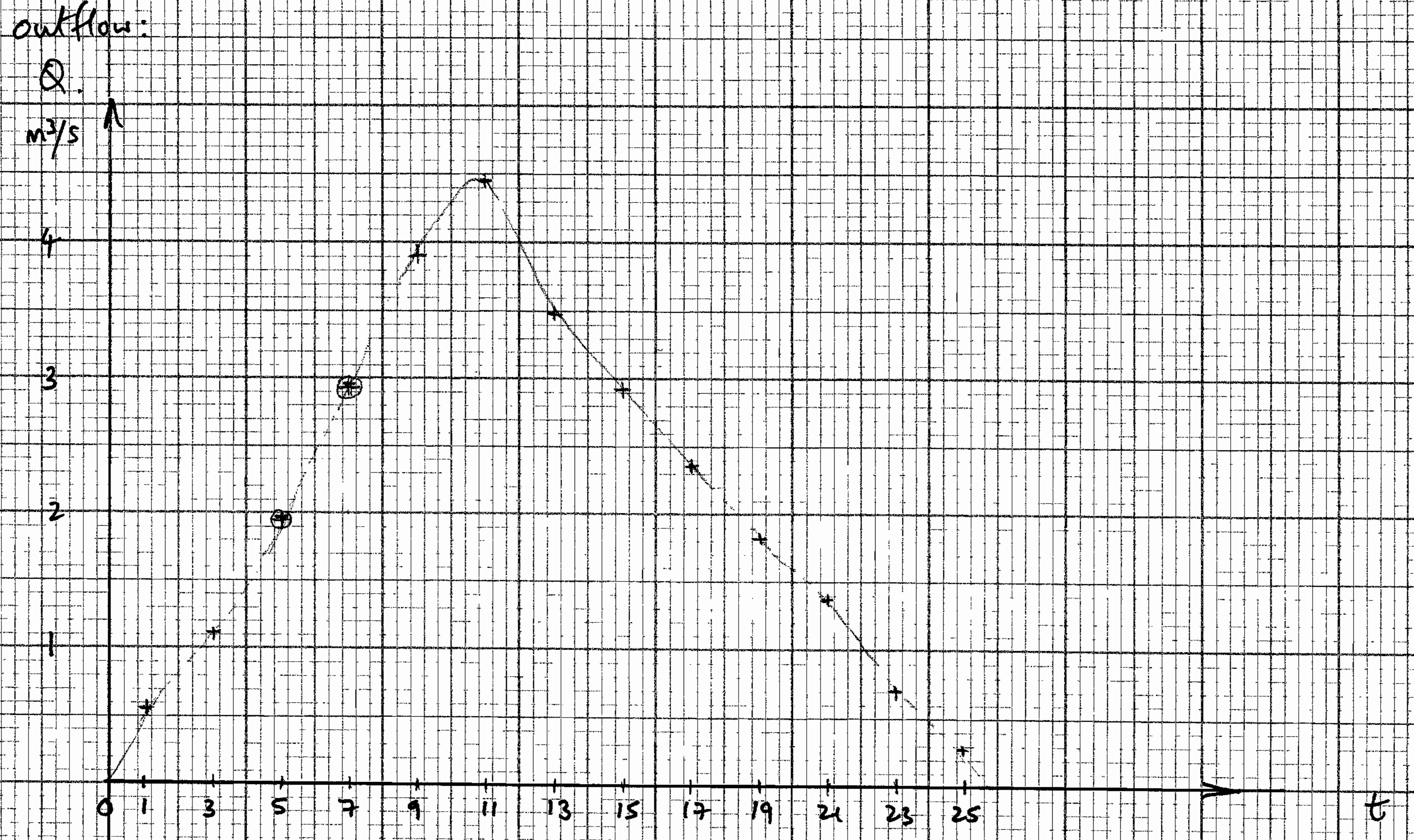
$$\text{Runoff} = (\underbrace{40\,\text{mm}}_{\text{rainfall}} - \underbrace{22\,\text{mm}}_{\text{infiltration}}) \times (\underbrace{20\,\text{km}^2}_{\text{area}})$$

$$= 18\,\text{mm} \times 20\,\text{km}^2$$

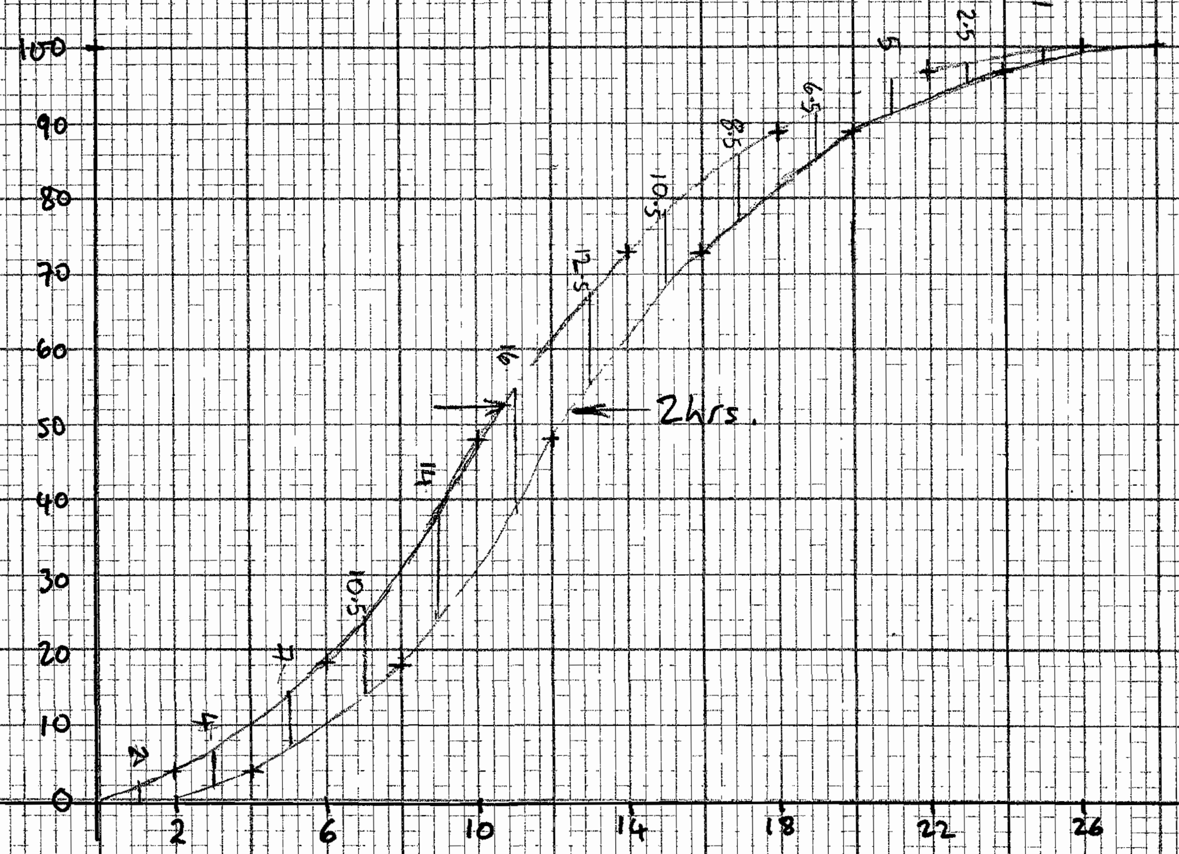
$$= 18 \times 10^{-3}\,\text{m} \times 20 \times 10^{6}\,\text{m}^2$$

$$= \underline{\underline{360 \times 10^3\,\text{m}^3}}$$

Q1 (b).



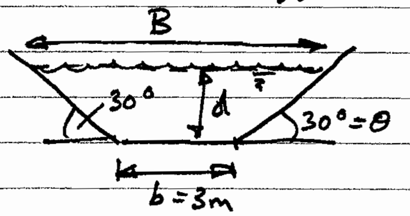
Q1 (b) S-curve.



100
90
80
70
60
50
40
30
20
10
0
2
6
10
14
18
22
26
2
4
7
10.5
14
16
12.5
10.5
8.5
6.5
5
2.5
1
2hrs.

Q2 a) Energy $E = d + \dfrac{v^2}{2g} = d + \dfrac{Q^2}{2gA^2}$

Critical energy when $\dfrac{dE}{dd} = 0 \Rightarrow 0 = \dfrac{dE}{dd} = 1 - \dfrac{Q^2}{2gA^3}\,2\,\dfrac{dA}{dd}$.



Area $A = (b + d\cos\theta)\,d$

$\dfrac{dA}{dd} = (b + 2d\cos\theta) = B$

∴ Critical condition is $0 = 1 - \dfrac{Q^2 B}{gA^3}$

or $Q^2 = \dfrac{gA^3}{B}$ $\qquad \cos\theta = \dfrac{\sqrt{3}}{2}$

∴ Critical flow from (when $d = 2m$) $Q^2 = g\dfrac{(b + \cancel{2}\sqrt{3}/\cancel{2})^3\, 2^3}{(b + 2.2.\sqrt{3}/2)}$

$= 9.81\dfrac{(3 + 1.732)^3\, 2^3}{(3 + 2(1.732))} = 9.81\dfrac{(4.732)^3 . 8}{6.464}$

$= \cancel{1608}\ 1286$

$\rightarrow \quad Q_{crit} = \sqrt{\cancel{1608}\ 1286} = \cancel{12.68}\ 35.9 \ m^3/s$.

The actual flow is $40\ m^3/s$, which is ~~less~~ greater than this,

and thus the flow is ~~sub~~ super-critical. (It must be going faster than critical velocity).

n.b. The formula $v_{crit} = \sqrt{g\, d_{crit}}$ is only valid for wide channels.

~~the~~ Incorrect use of this formula here would give

$Q_{crit} = v_{crit} A \cancel{dd}$

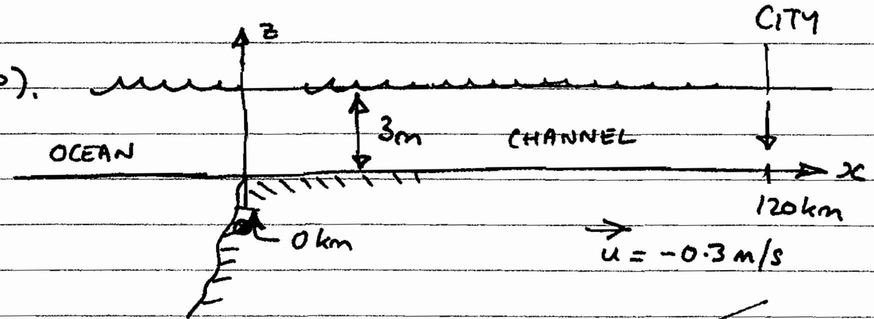
And when ~~A~~ $d = 2m$, $A = (3 + 2\sqrt{3}/2)\,2 = 9.46\ m^2$

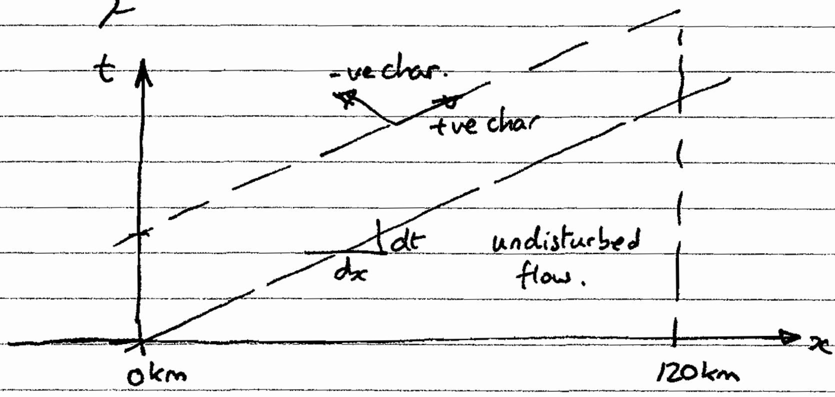
$v_{crit} = \sqrt{gd} = \sqrt{9.81 \times 2} = 4.43\ m/s$

∴ $Q_{crit} = 4.43 \times 9.46 = 41.9\ m^3/s$.

and since the actual flow is less than this, one would incorrectly infer that the flow was ~~super~~ sub critical.

Q2(b).





Prelims: From data sheet $\begin{cases} \bar{u}+2c = \text{const on +ve char}, & \frac{dx}{dt} = \bar{u}+c \\ \bar{u}-2c = \text{const on -ve char}, & \frac{dx}{dt} = \bar{u}-c \end{cases}$

and $c = \sqrt{gd}$

Now, $\bar{u} - 2c$ = const on all -ve char.s (from data sheet), and all -ve chars can be traced back to undisturbed region where d is const, thus c is const and u = const $\therefore$ $u - 2c$ = const in undist. region, and thus $u - 2c$ = const <u>everywhere</u>

$u - 2c = k_-$ say.

Along +ve char: $\frac{1}{\text{slope}} = \frac{dx}{dt} = \bar{u}+c$

Along +ve char: $\bar{u}+2c = \text{const} = k_+$ say

$\bar{u} - 2c = \text{const} = k_-$ <u>everywhere</u>

$\therefore$ Along +ve char: $\bar{u}$ = const

c = const

$\therefore$ $\underline{d = \text{const}}$ (from $c = \sqrt{gd}$) on +ve char,

also slope $= \left(\frac{dx}{dt}\right)^{-1} = (u+c)^{-1}$ = const

$\therefore$ +ve characteristics are straight lines.

[Note to marker:— students may begin with "+ve chars are straight" — this is OK — they do not need to derive it if they know it]

Q2(b) cont'd: +ve characteristics are straight lines with $(\text{slope})^{-1}$

$$= \left(\frac{dx}{dt}\right) = \bar{u} + c$$

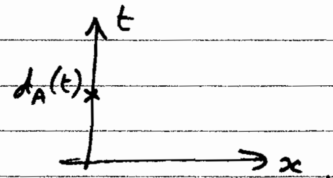
$$= (\bar{u} - 2c) + 3c$$

Now $\bar{u} - 2c$ = const everywhere

∴ use values from undisturbed region

$$\bar{u} - 2c = -0.3 - 2\sqrt{(9.81).3} = -11.15 \text{ m/s}$$

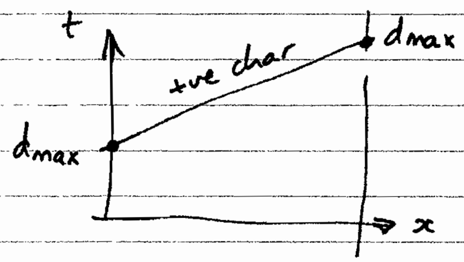
Now $+3c$ term:— use $c = \sqrt{g d_A}$ where d_A is some boundary condition at $x = 0$.



Question asks for "max level at city".

Now, since i) +ve chars are straight and
ii) d is const on +ve char

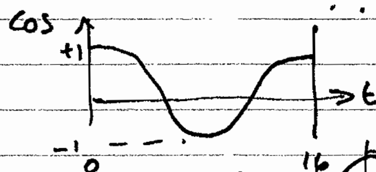
then max level at city is on +ve char from max level at ocean.

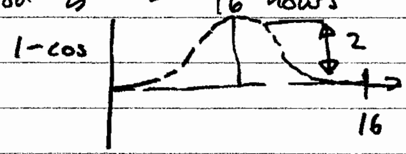


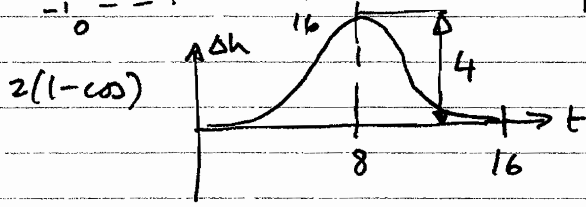
Now $d(x=0) = 3 + \Delta h$ metres

$$= 3 + 2\left(1 - \underbrace{\cos(\pi t/8)}_{\cos\frac{2\pi t}{16}}\right)$$

∴ Period is 16 hours







∴ max depth at $x=0$ when $t = 8$ (and then $d = 3 + 4 = 7$m).

Q2(b) cont'd.

$\therefore$ $+3c$ term uses $c = \sqrt{gd} = \sqrt{9.81.(7)}$

on +ve char

$\therefore$ $c = 8.29$ m/s on +ve char thru $(x,t) = (0, 8\text{hrs})$

$3c = 24.86$ m/s.

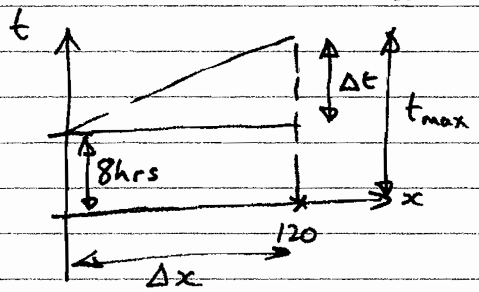
$\therefore$ $\frac{dx}{dt} = (\bar{u} - 2c) + 3c = -11.15 + 24.86$

$= +13.71$ m/s.

$\frac{3600}{1000}$

$= 13.71 \times 3.6$ km/hr.

$=$ ~~49.3~~ 49.36 km/hr.

$\therefore$ 

$t_{max} = 8\text{hrs} + \Delta t = 8\text{hrs} + \left(\frac{dt}{dx}\right)\Delta x$

$= 8\text{hrs} + \frac{\Delta x}{(dx/dt)} = 8\text{hrs} + \frac{120\text{ km}}{49.3\text{ km/hr}} = 8 + 2.43$

$\therefore$ Max level at city is when $t = 10.43$ hrs

[Note to marker: computer simulation shows there are no hydraulic jumps in the channel]

Q3. slope $S = 0.0001$ grain $D = 0.1 \times 10^{-3}$ m
depth $d = 1.0$ m $k_s = 0.01$ m

wide, $\therefore$ Hydraulic radius $R = d \simeq 1.0$ m

Bed shear stress
$$\tau_0 = \rho g R S$$
$$= \cancel{1000}(9.81)(1)(0.1 \times \cancel{10^{-3}})$$
$$= \underline{\underline{0.981 \text{ N/m}^2}}$$

shear velocity
$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{0.981}{1000}} = \underline{\underline{0.0313}} \text{ m/s.}$$

For hydraulic roughness
$$\frac{u_* k_s}{\nu} = \frac{(0.0313)(0.01)}{10^{-6}} = \underline{\underline{313}} > 70$$
$\therefore$ Hydraulically rough.

Prandtl-Karman
$$\bar{u} = 2.5\, u_* \log\left(\frac{12.1 R}{k_s}\right)$$
$$= 2.5(0.0313)\log\left(\frac{12.1\,(1)}{0.01}\right)$$
$$= \underline{\underline{0.555 \text{ m/s}}}.$$

Froude
$$\frac{\bar{u}}{\sqrt{gd}} = \frac{0.555}{\sqrt{9.81(1)}} = \underline{\underline{0.18}}$$

Fall velocity $D < 0.0005$
$\therefore$
$$W = (56 \times 10^4)\, D^2 (\rho_s - \rho)/\rho$$
$$= (56 \times 10^4)(0.0001)^2\, 1.65$$
$$= 0.0092 \text{ m/s}.$$

$$\frac{W}{K u_*} = \frac{0.0092}{(0.4)(0.0313)} = 0.74.$$

$$A = \frac{30.2}{k_s} \text{ (rough)} = \frac{30.2}{0.01} = 3020.$$

$$\frac{b}{d} = \frac{\text{bed thickness}}{\text{depth}} = \frac{0.01}{1} = 0.01$$

Q3 <u>cont'd.</u>

$$\int_b^d C u \, dy = 11.6 \, u_* C_b \, b \left[I_1 \log(Ad) + I_2 \right]$$

	$w/\kappa u_* = 0.6$	$w/\kappa u_* = 0.74$	$w/\kappa u_* = 1.0$
$b/d = 0.01$	$I_1 = 2.174$ $I_2 = 4.854$		$I_1 = 0.788$ $I_2 = 2.107$.

interpolate.

$$\overbrace{0.6 \;|\; 0.74}^{0.14} \overbrace{\;|\; 1.0}^{0.26}$$

$$\frac{0.14}{0.40} = 0.35 \qquad \frac{0.26}{0.40} = 0.65$$

$$\therefore \quad I\Big|_{0.74} = 0.65 \, I\Big|_{0.6} + 0.35 \, I\Big|_{1.0}$$

$$\therefore \quad I_1 = 0.65(2.174) + 0.35(0.788) = 1.69$$

$$I_2 = 0.65(4.254) + 0.35(2.107) = 3.50$$

$$11.6 \, u_* C_b \, b = 11.6(0.0313)(13.6)(0.01)$$

$$= 0.0494 \quad \text{kg s}^{-1}/\text{m}.$$

$$I_1 \log(Ad) + I_2 = 1.69 \log(3020(1)) + 3.50$$

$$= 17.0$$

$$\therefore \int_b^d C u \, dy = (0.0494)(17) = \underline{\underline{0.84 \ \text{kg s}^{-1}/\text{m}}}.$$

Q3 (b).

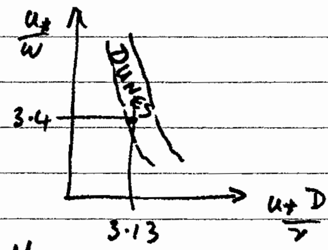
Shields. $\dfrac{\tau_0}{(\rho_s-\rho)gD} = \dfrac{0.981}{(1650)(9.81)0.1\times10^{-3}} = 0.61$

Froude. $\dfrac{\bar{u}}{\sqrt{gd}} = 0.18$ (see earlier).

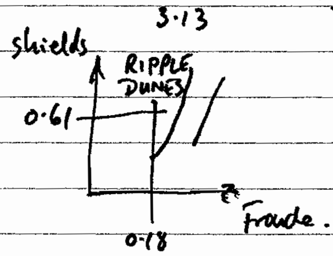
$\dfrac{u_*}{w} = \dfrac{0.0313}{0.0092} = 3.4$

$\dfrac{u_* D}{\nu} = \dfrac{(0.0313)(0.1\times10^{-3})}{10^{-6}} = 3.13.$

$\therefore$ Albertson Simons + Richardson $\Rightarrow$ DUNES (just)



Garde + Albertson $\Rightarrow$ RIPPLE-DUNES



[(Not needed for exam:)

if dunes, wavelength from $K_d \sim \dfrac{1}{F_r^2} \sim \dfrac{1}{0.18^2} \sim 31$

$K_d = \dfrac{2\pi d}{L}$ $\therefore$ $L = \dfrac{2\pi d}{K_d} = \dfrac{2\pi(1)}{31} = 0.2\,m$ wavelength.

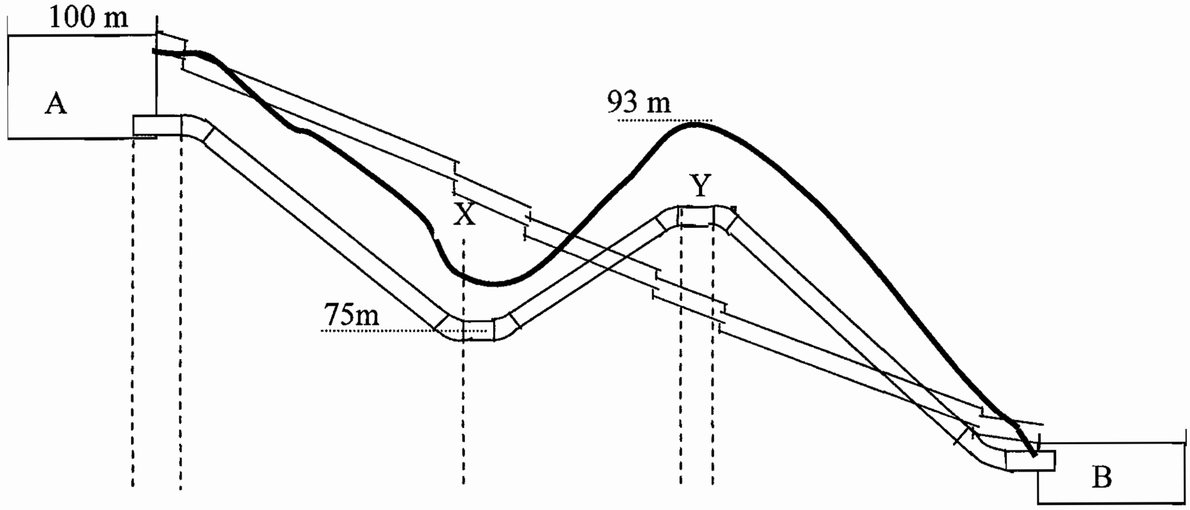
steepness $\dfrac{h_r}{L} \sim 0.3$ for $F_r \sim 0.18$, Shields ~ 0.6

$\rightarrow h_r \sim 0.3 \times 0.2 = \underline{0.06\,m}$

[Hence why justified in having surface roughness height $k_s \gg D$]

1)

a) Energy and hydraulic grade lines



b) i) Apply Bernoulli Equation between A and X to find V_x (hence Q)

$$\frac{V_A^2}{2g} + \frac{P_A}{\rho g} + z_A = \frac{V_x^2}{2g} + \frac{P_x}{\rho g} + z_x + \underset{\text{friction}}{h_f} + \underset{\text{minor losses}}{h_L}$$

P_A is atmospheric pressure at reservoir = 0 $\qquad$ V_A is negligible $\simeq 0$

Minor losses at entry and at 2 bends.

$$P_x = 120\ kN/m^2 = \frac{120000}{9806} = 12.237m$$

$$\text{Relative roughness} = k/D = \frac{0.26\ mm}{450\ mm} = 0.00058$$

From Moody diagram (assuming fully turbulent flow) $\lambda = 0.017$

Thus:

$$0 + 0 + 100 = \frac{V_x^2}{2g} + 12.237 + 75 + \Big(\underset{\text{entry}}{0.5} + \underset{\text{friction}}{0.017\,\frac{500}{0.45}} + \underset{\text{bends}}{0.25 + 0.25}\Big)\frac{V_x^2}{2g}$$

$$12.763 = 20.889\,\frac{V_x^2}{2g} \Rightarrow \underline{\underline{V_x = 3.462\ m/s}}$$

Check Reynolds Number $Re = \frac{VD}{\nu} = \frac{3.462 \times 0.45}{1.14 \times 10^{-6}} = 1.367 \times 10^6$ $\qquad$ from Moody Diagram

$\Rightarrow \lambda = 0.0175$

Recompute with revised λ:

$$12.763 = \frac{V_x^2}{2g} + \left(0.5 + 0.0175\,\frac{500}{0.45} + 0.25 + 0.25\right)\frac{V_x^2}{2g}$$

$$12.763 = 21.444\,\frac{V_x^2}{2g} \Rightarrow V_x = 3.417\ m/s$$

(Check $Re = \frac{3.417 \times 0.45}{1.14 \times 10^{-6}} = 1.349 \Rightarrow \lambda = 0.0175$ – accept)

$$Q = V_x \frac{\pi D^2}{4} = \frac{\pi \times 0.45^2}{4} \times 3.417 = \underline{\underline{0.543\ m^3/s}}$$

ii) Take distance from ground surface to pipe at Y as a.

Apply Bernoulli between X & Y:

$$\frac{V_X^2}{2g} + \frac{P_X}{\rho g} + Z_X = \frac{V_Y^2}{2g} + \frac{P_Y}{\rho g} + Z_Y + h_f + h_L$$

$V_X = V_Y$

$$12.237 + 75 = -10.1 + (93 - a) + \left(0.0175 \times \frac{800-500}{0.45} + 0.25 + 0.25\right)\frac{3.417^2}{2g}$$

$$\Rightarrow 90.094 = (93 - a) \Rightarrow \underline{\underline{a = 2.906\,m}} \text{ (below ground)}$$

(Elevation of Y = 90.094 m)

iii) Apply Bernoulli between reservoirs:

$$V_A^2/2g + \frac{P_A}{\rho g} + Z_A = \frac{V_B^2}{2g} + \frac{P_B}{\rho g} + Z_B + h_f + h_L$$

$P_A = P_B = 0 \qquad V_A = V_B \simeq 0$

$$Z_A - Z_B = \left(\lambda \frac{L}{D} + k_{entry} + k_{exit} + 6 \times k_{bend}\right) V^2/2g$$

$$100 - Z_B = \left(0.0175 \frac{1500}{0.45} + 1.0 + 0.5 + (6 \times 0.25)\right)\frac{3.417^2}{2g}$$

$$\underline{\underline{Z_B = 63.486\,m}}$$

4)

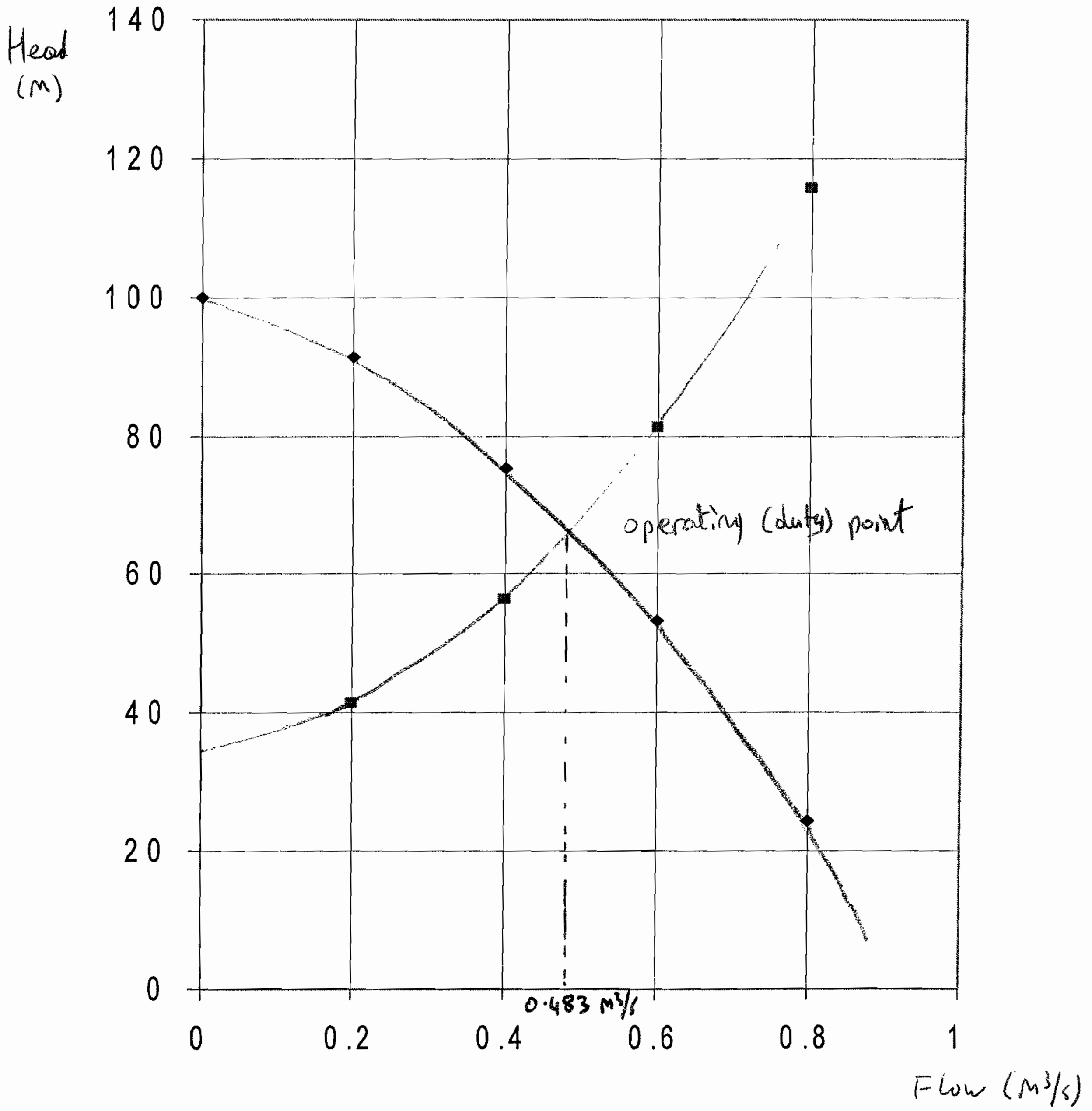
c) Develop a system curve for the pipeline

Static head = 100 − 63.486 m = $\underline{\underline{36.514\ m}}$

Q m^3/s	$V = Q/A$ m/s	$h_f + h_L = \left(0.0175\ \frac{1500}{0.45} + 1.0 + 0.5 + 1.5\right)\frac{V^2}{2g}$	Total head $36.514 + h_f + h_L$ *
0.2	1.258	4.949	41.463
0.4	2.515	19.781	56.305
0.6	3.773	44.519	81.033
0.8	5.030	79.124	115.638

(* minor losses could be ignored)

Plot pump & system characteristic:



Pump will deliver 0.483 m³/s

ENGINEERING TRIPOS PART IIA 2006

MODULE 3D5: ENVIRONMENTAL ENGINEERING I

NUMERICAL ANSWERS

1. (a) Volume of the runoff = 360×10^3 m^3.
   (b) Peak instantaneous outflow = 4.45 m^3/s

2. (a) Flow is super-critical.
   (b) Time = 10.43 hrs.

3. (a) Sediment transport rate = 0.84 kg/s per metre width of channel.
   (b) Albertson Simons and Richardson – Dunes
   Garde and Alberton – Ripple-Dunes.

4. (b) (i) Q = 0.543 m^3/s.
   (ii) Minimum acceptable depth at Y = 2.906m.
   (iii) Water level in reservoir B = 63.486m.

   (c) Flow rate that could be delivered to reservoir A = 0.483m^3/s.