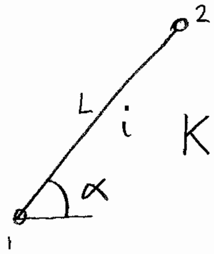
1) The overall stiffness matrix for this structure can be derived from the stiffness matrix for a pin-jointed

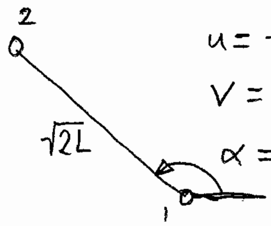
(a) bar, see the data sheet: $Kd = p$



$$K_i = \frac{1}{a}\left[\begin{array}{cc|cc} u^2 & uv & -u^2 & -uv \\ & v^2 & -uv & -v^2 \\ \hline & & u^2 & uv \\ \text{symm.} & & & v^2 \end{array}\right] \qquad a = \frac{L}{AE}, \quad u = \cos\alpha, \quad v = \sin\alpha$$

For bars I & II only a 2×2 sub-matrix of K_i is needed (one node is fully constrained)

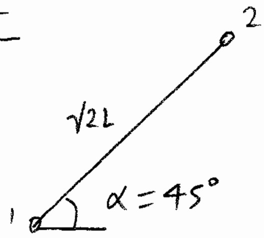
Bar I



$u = -1/\sqrt{2}$
$v = 1/\sqrt{2}$
$\alpha = 135°$

$$K_I = \frac{AE}{\sqrt{2}L}\left[\begin{array}{cc|cc} & & & \\ \hline & & 1/2 & -1/2 \\ & & -1/2 & 1/2 \end{array}\right]$$

Bar II

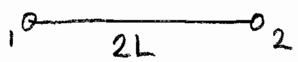


$u = 1/\sqrt{2}$
$v = 1/\sqrt{2}$

$$K_{II} = \frac{AE}{\sqrt{2}L}\left[\begin{array}{cc|cc} & & & \\ \hline & & 1/2 & 1/2 \\ & & 1/2 & 1/2 \end{array}\right]$$

Bar III

$\alpha = 0°$



$u = 1$
$v = 0$

$$K_{III} = \frac{AE}{2L}\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

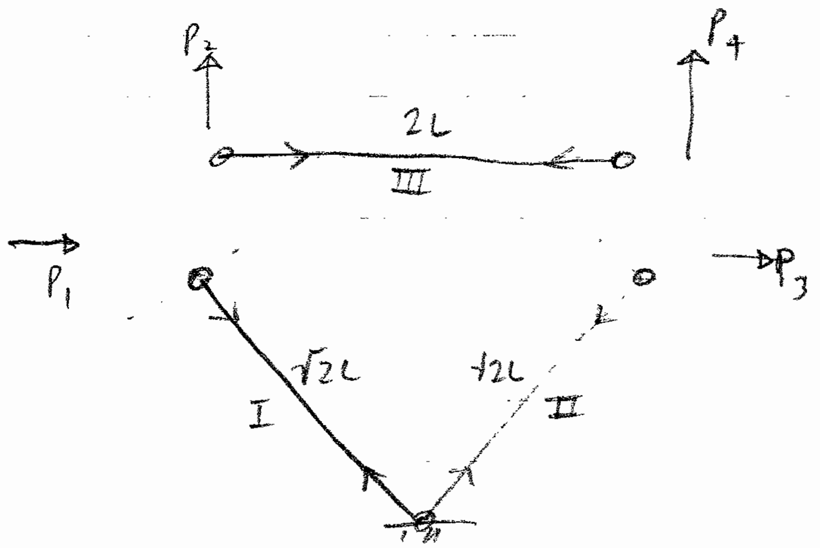
The overall stiffness matrix is obtained from the above sub-matrices

$$K = \frac{AE}{L}\left[\begin{array}{cc|cc} \frac{1}{2\sqrt{2}} + \frac{1}{2} & -\frac{1}{2\sqrt{2}} & -1/2 & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ \hline -1/2 & 0 & \frac{1}{2\sqrt{2}} + 1/2 & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{array}\right]$$

1) (a)

Alternative method to obtain K

$e = \frac{TL}{EA}$



$Kd = P$

$e_{III} = d_3 - d_1$ $\qquad$ $T_{III} = \frac{EA}{2L}(d_3 - d_1)$

$e_{II} = \frac{1}{\sqrt{2}}(d_3 + d_4)$ $\qquad$ $T_{II} = \frac{EA}{\sqrt{2}L}\frac{1}{\sqrt{2}}(d_3 + d_4) = \frac{EA}{2L}(d_3 + d_4)$

$e_{I} = \frac{1}{\sqrt{2}}(-d_1 + d_2)$ $\qquad$ $T_{I} = \frac{EA}{\sqrt{2}L}\frac{1}{\sqrt{2}}(-d_1 + d_2) = \frac{EA}{2L}(-d_1 + d_2)$

$-\frac{T_I}{\sqrt{2}} - T_{III} = P_1$ $\qquad$ $\frac{EA}{L}\left(\frac{d_1}{2\sqrt{2}} - \frac{d_2}{2\sqrt{2}} - \frac{d_3}{2} + \frac{d_1}{2}\right) = P_1$

$\frac{T_I}{\sqrt{2}} = P_2$ $\qquad$ $\therefore \frac{EA}{L}\left(-\frac{d_1}{2\sqrt{2}} + \frac{d_2}{2\sqrt{2}}\right) = P_2$

$\frac{T_{II}}{\sqrt{2}} + T_{III} = P_3$ $\qquad$ $\frac{EA}{L}\left(\frac{d_3}{2\sqrt{2}} + \frac{d_4}{2\sqrt{2}} + \frac{d_3}{2} - \frac{d_1}{2}\right) = P_3$

$\frac{T_{II}}{\sqrt{2}} = P_4$ $\qquad$ $\frac{EA}{L}\left(\frac{d_3}{2\sqrt{2}} + \frac{d_4}{2\sqrt{2}}\right) = P_4$

$\therefore \quad K d = P$

$$\frac{EA}{L}\begin{bmatrix} \frac{1}{2\sqrt{2}}+\frac{1}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2\sqrt{2}}+\frac{1}{2} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

1)

$$K = \frac{AE}{2\sqrt{2}L}\left[\begin{array}{cc|cc} 1+\sqrt{2} & -1 & -\sqrt{2} & 0 \\ -1 & 1 & 0 & 0 \\ \hline -\sqrt{2} & 0 & 1+\sqrt{2} & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

(b) For the first load condition the stiffness equations are:

$$\frac{AE}{2\sqrt{2}L}\begin{bmatrix} 2.414 & -1 & -1.414 & 0 \\ -1 & 1 & 0 & 0 \\ -1.414 & 0 & 2.414 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

(part (c))

We try to solve this by Gaussian elimination

$$K^* \,|\, p^* = \frac{AE}{2\sqrt{2}L}\left[\begin{array}{cccc|c} 2.414 & -1 & -1.414 & 0 & 1 \\ 0 & 0.5858 & -0.5858 & 0 & 0.4142 \\ 0 & -0.5858 & 1.5858 & 1 & 0.5858 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{c} 1 \\ 0.4142 \text{ or} \\ -0.4142 \\ 0 \end{array}$$

$$\downarrow$$

$$\frac{AE}{2\sqrt{2}L}\left[\begin{array}{cccc|c} 2.414 & -1 & -1.414 & 0 & 1 \\ 0 & 0.5858 & -0.5858 & 0 & 0.4142 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right] \begin{array}{c} 1 \\ 0.4142 \text{ or} \\ 0 \\ 0 \end{array}$$

There is no solution to the last equation and ∴ the load cannot be equilibrated in the configuration shown

If we attempt to take moments about the pivot point it is obvious that this load cannot be equilibrated

1) (c) Continuing from (b) we simply modify $K^*|p^*$ for the load $\underset{\sim}{p} = [1\ 0\ -1\ 0]^T$:

$$K^*|p^* = \frac{AE}{2\sqrt{2}L}\left[\begin{array}{cccc|c} 2.414 & -1 & -1.414 & 0 & 1 \\ 0 & 0.5858 & -0.5858 & 0 & 0.4142 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

And now proceed with a full Gauss-Jordan elimination:

$$K^*|p^* = \frac{AE}{2\sqrt{2}L}\left[\begin{array}{cccc|c} 2.414 & 0 & -2.414 & 0 & 1.707 \\ 0 & 0.5858 & -0.5858 & 0 & 0.4142 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\downarrow$$

$$K^*|p^* = \frac{AE}{2\sqrt{2}L}\left[\begin{array}{cccc|c} 2.414 & 0 & 0 & 2.414 & 1.707 \\ 0 & 0.5858 & 0 & 0.5858 & 0.4142 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

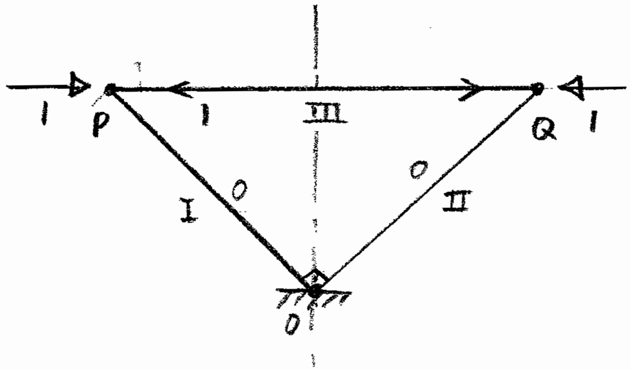
$$\downarrow$$

$$\frac{AE}{2\sqrt{2}L}\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0.7071 \\ 0 & 1 & 0 & 1 & 0.7071 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The last equation is now always satisfied. d_4 is the free variable, set to 0 for particular solution, -1 for null space vector.
So general solution is:

$$\underset{\sim}{d} = \underset{\sim}{d_0} + \underset{\sim}{M}x = \frac{2\sqrt{2}L}{EA}\begin{bmatrix} 0.7071 \\ 0.7071 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}x = \frac{L}{EA}\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}x$$

1) (c) Alternative method:

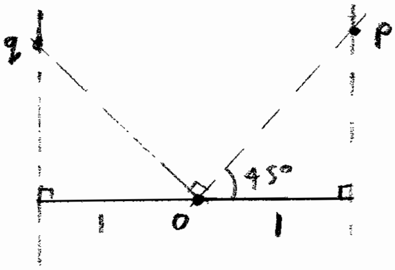


Assume symmetrical deformation:
i.e. $d_1 = -d_3 \quad d_2 = d_4$

$$e_{I} = e_{II} = 0 \qquad e_{III} = \frac{1 . 2L}{EA} = \frac{2L}{EA}$$

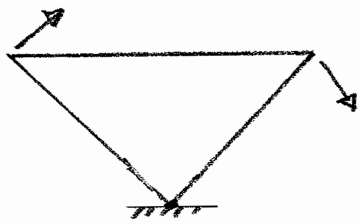
$$\therefore \quad d_1 = -d_3 = \frac{L}{EA} \qquad \text{and} \therefore \; d_2 = d_4 = \frac{L}{EA}$$

Displacement diagram:
(units of $\frac{L}{EA}$)



$$\underset{\sim}{d}_0 = \frac{L}{EA}\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

Mechanism:



$$\underset{\sim}{M} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore$

$$\underset{\sim}{d} = \underset{\sim}{d}_0 + \underset{\sim}{M}\, x' = \frac{L}{EA}\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} x'$$

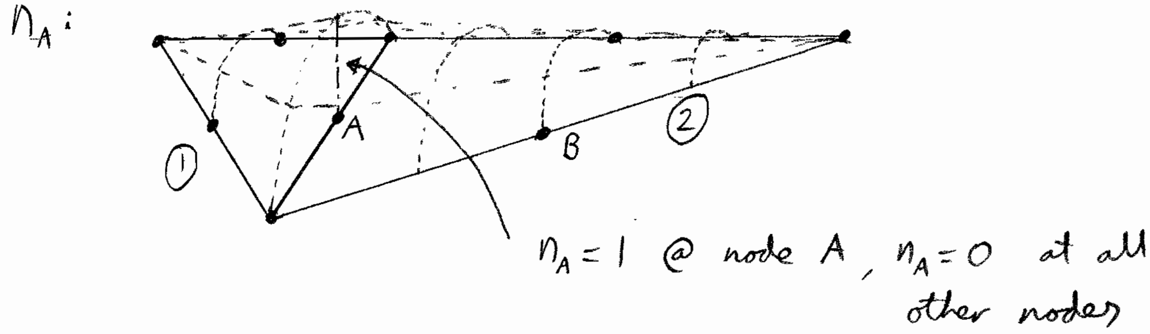
OR (by inspection) $= \frac{L}{EA}\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} x$ as before

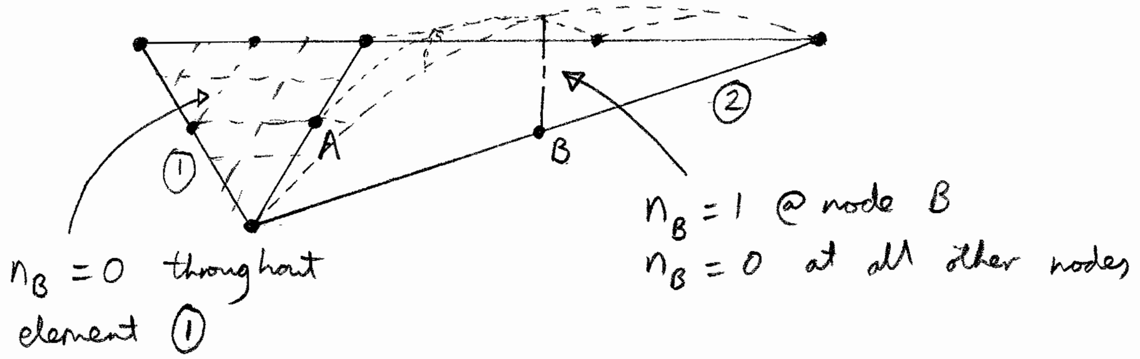
2) (a)

6 node triangular elements are linear strain elements. There will be no discontinuities in the displacement field between elements but there may be discontinuities in strain (and hence stress) fields.

(b) Shape functions (quadratic)

n_A:





(c) Shape functions (sketched above)
From data book, or by inspection

$A = (0,1)$
$B = (2,1)$

$$n_{A1} = (2-y)(x+y)$$

$$n_{A2} = \frac{1}{2}(2-y)(2y-x)$$

$$n_{B1} = 0$$

$$n_{B2} = \frac{1}{2}(2-y)x$$

2) (c) (continued)

Displacements $\quad \underset{\sim}{u} = \underset{\sim}{N}\,\underset{\sim}{d} = \underset{\sim}{N}\begin{bmatrix} d_{Ax} \\ d_{Ay} \\ d_{Bx} \\ d_{By} \end{bmatrix} = \underset{\sim}{N}\begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} \times 10^{-4}$ mm

$$u_1 = n_{A1}\, d_{Ax} = (2-y)(x+y) \times 10^{-4}$$

$$v_1 = n_{A1}\, d_{Ay} = 0$$

$$u_2 = n_{A2}\, d_{Ax} + n_{B2}\, d_{Bx} = \left[\tfrac{1}{2}(2-y)(2y-x) + \tfrac{1}{2}(2-y)x\right] \times 10^{-4}$$

$$= (2-y)y \times 10^{-4}$$

$$v_2 = n_{A2}\, d_{Ay} + n_{B2}\, d_{By} = \left[0 - \tfrac{1}{2}(2-y)x\right] \times 10^{-4}$$

$$= \tfrac{1}{2}(y-2)x \times 10^{-4}$$

(d) $\underset{\sim}{\varepsilon} = \underset{\sim}{B}\,\underset{\sim}{d} \qquad \varepsilon_x = \dfrac{\partial u}{\partial x} \qquad \varepsilon_y = \dfrac{\partial v}{\partial y} \qquad \gamma_{xy} = \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}$

@ $P = (0, 1.5)$
In element 1

$$\varepsilon_{x1} = \frac{\partial u_1}{\partial x} = (2-y) \times 10^{-4} = 0.5 \times 10^{-4}$$

$$\varepsilon_{y1} = \frac{\partial v_1}{\partial y} = 0$$

$$\gamma_{xy1} = \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0 + (2 - x - 2y) \times 10^{-4}$$

$$= -1 \times 10^{-4}$$

@ $P = (0, 1.5)$
In element 2

$$\varepsilon_{x2} = \frac{\partial u_2}{\partial x} = 0$$

$$\varepsilon_{y2} = \frac{\partial v_2}{\partial y} = \tfrac{1}{2}x \times 10^{-4} = 0$$

$$\gamma_{xy2} = \frac{\partial v_2}{\partial x} + \frac{\partial u_2}{\partial y} = \left[\tfrac{1}{2}(y-2) + 2 - 2y\right] \times 10^{-4}$$

$$= \left[1 - \tfrac{3}{2}y\right] \times 10^{-4}$$

$$= -1.25 \times 10^{-4}$$

2) (continued)

(e) Stresses @ $P = (0, 1.5)$ (only σ_{xx} asked for)

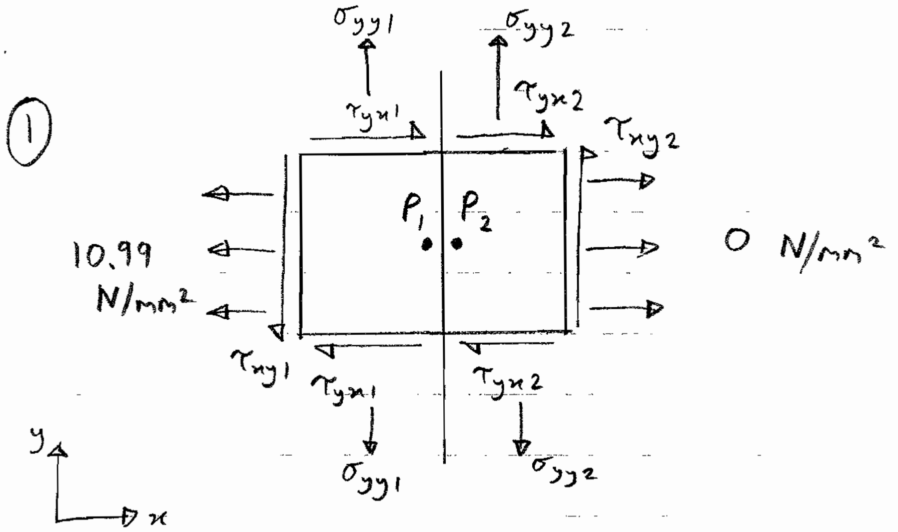
In element ① $\sigma_{x1} = \dfrac{E}{1-\nu^2}(\varepsilon_{x1} + \nu\,\varepsilon_{y1})$

$= \dfrac{200\times10^3}{1-0.3^2}(0.5 + 0.3\times0)\times10^{-4}$

$= 10.99 \ N/mm^2$

In element ② $\sigma_{x2} = \dfrac{E}{1-\nu^2}(\varepsilon_{x2} + \nu\,\varepsilon_{y2})$

$= 0$



In the x-direction we need only consider σ_{xx} to see that this section is not in equilibrium because of the discontinuity in stress between elements.

3

(a) Considering an element of (dx, dy and dz), the continuity condition gives that

$$(q_x + dq_x)dydz - q_x dydz + (q_y + dq_y)dxdz - q_y dxdz + (q_z + dq_z)dxdy - q_z dxdy = Qdxdydz$$

$$\frac{dq_x}{dx} + \frac{dq_y}{dy} + \frac{dq_z}{dz} = Q$$

Substituting the constitutive equation,

$$\frac{d\left(-k_x \partial h / \partial x\right)}{dx} + \frac{d\left(-k_y \partial h / \partial y\right)}{dy} + \frac{d\left(-k_z \partial h / \partial z\right)}{dz} = Q$$

Hence,

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} + Q = 0$$

(b) For the one-dimensional condition, the governing equation becomes

$$k_x \frac{d^2 h}{dx^2} + Q = 0$$

Applying a weight function v to the governing equation and integrate between x = a and x = b.

$$v\left[\frac{d}{dx}(k_x \frac{dh}{dx}) + Q\right] = 0$$

$$\int_b^a v\left[\frac{d}{dx}(k_x \frac{dh}{dx}) + Q\right]dx = 0$$

Integrate by parts to derive the weak form.

$$\left[v k_x \frac{dh}{dx}\right]_b^a - \int_b^a \frac{dv}{dx} k_x \frac{dh}{dx} dx + \int_b^a vQdx = 0$$

Water flow is defined as $q = -k_x (dh/dx)$

$$\int_b^a \frac{dv}{dx} k_x \frac{dh}{dx} dx = (vq)_{x=b} - (vq)_{x=a} + \int_b^a vQdx$$

$$\int_b^a \frac{dv}{dx} k_x \frac{dh}{dx} dx = (v)_{x=b} q_b - (v)_{x=a} q_a + \int_b^a vQdx$$

(c) Substituting the shape functions given and noting that a is not a function of x,

$$\left(\int_b^a \mathbf{Bc} k_x \mathbf{B} dx\right)\mathbf{a} = (\mathbf{Nc})_{x=b} q_b - (\mathbf{Nc})_{x=a} q_a + \int_b^a \mathbf{Nc} Q dx$$

Using vector product manipulation, $\mathbf{Bc} = \mathbf{c}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}$, $\mathbf{Nc} = \mathbf{c}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}$

$$\left(\int_b^a \mathbf{c}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}k_x\mathbf{B}dx\right)\mathbf{a} = \left(\mathbf{c}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}\right)_{x=b} q_b - \left(\mathbf{c}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}\right)_{x=a} q_a + \int_b^a \mathbf{c}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}Qdx$$

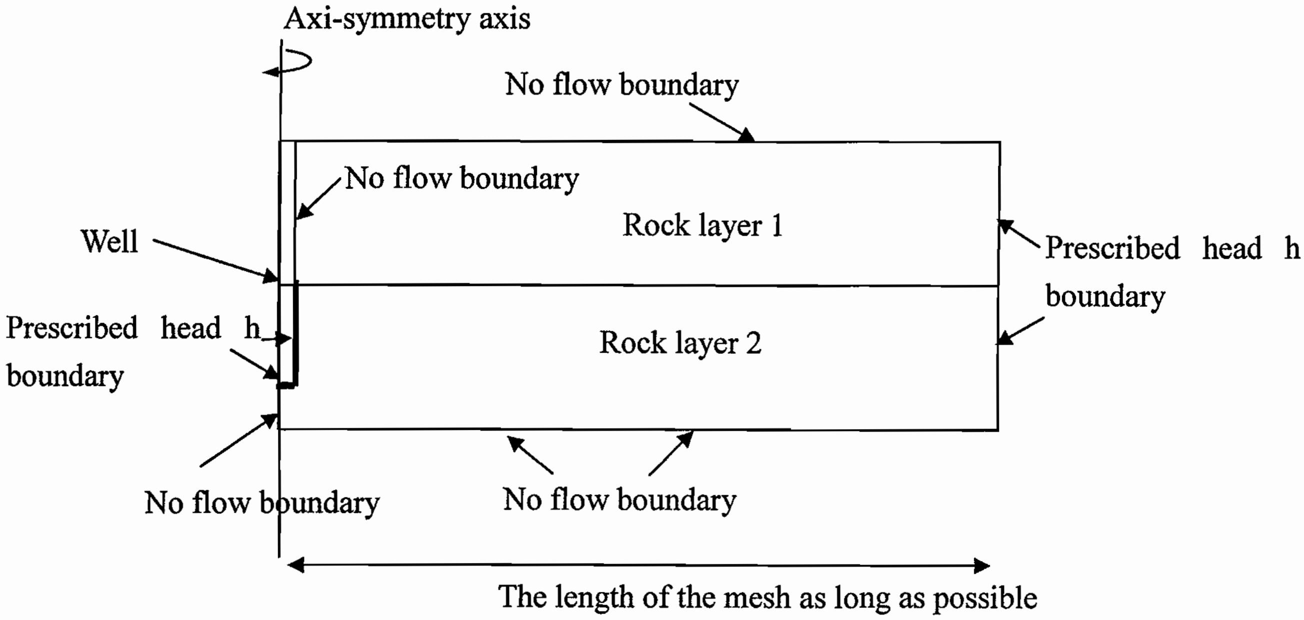
Since $\mathbf{c}$ is not a function of x,

$$\mathbf{c}^{\mathrm{T}}\left(\int_b^a \mathbf{B}^{\mathrm{T}}k_x\mathbf{B}dx\right)\mathbf{a} = \mathbf{c}^{\mathrm{T}}\left\{\left(\mathbf{N}^{\mathrm{T}}\right)_{x=b} q_b - \left(\mathbf{N}^{\mathrm{T}}\right)_{x=a} q_a + \int_b^a \mathbf{N}^{\mathrm{T}}Qdx\right\}$$

Cancelling $\mathbf{c}^{\mathrm{T}}$, the following finite element formulation can be obtained.

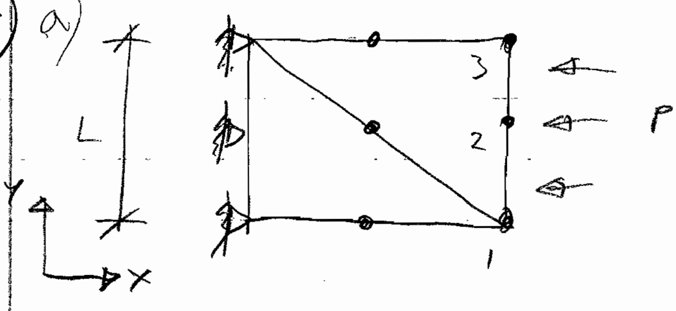
$$\left(\int_b^a \mathbf{B}^{\mathrm{T}}k_x\mathbf{B}dx\right)\mathbf{a} = \left(\mathbf{N}^{\mathrm{T}}\right)_{x=b} q_b - \left(\mathbf{N}^{\mathrm{T}}\right)_{x=a} q_a + \int_b^a \mathbf{N}^{\mathrm{T}}Qdx$$

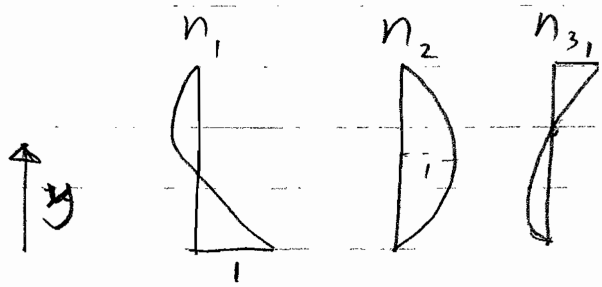
(d)&(e) Axi-symmetric condition can be used. Hence, it becomes a two dimensional problem.



The pumping rate at the well is computed by integrating the flow rate evaluated at the well boundary. The flow rate can be computed from the constitutive equation q = k(dh/dx), but the spatial gradient of the head at the well boundary needs to be evaluated. The spatial gradients are most accurate at the integration points. Hence, the spatial gradient at the element boundary need to be extrapolated from the integration values for accurate estimation of spatial gradient at the element boundary.

4) a)





$$n_1 = (y-L)\left(y-\frac{L}{2}\right)\frac{2}{L^2} = \left(y^2 - Ly - \frac{L}{2}y + \frac{L^2}{2}\right)\frac{2}{L^2}$$

$$= 2\left(\frac{y}{L}\right)^2 - 3\left(\frac{y}{L}\right) + 1$$

$$n_2 = y(L-y)\frac{4}{L^2} = (yL - y^2)\frac{4}{L^2} = 4\left(\frac{y}{L}\right) - 4\left(\frac{y}{L}\right)^2$$

Equivalent nodal loads $\quad P_{y1} = P_{y2} = P_{y3} = 0$

$$P_{x1} = \int_0^L n_1 \cdot -p \, dA = -tp\int_0^L 2\left(\frac{y}{L}\right)^2 - 3\left(\frac{y}{L}\right) + 1 \, dy$$

$$= -tp\left[\frac{2}{3}\frac{y^3}{L^2} - \frac{3}{2}\frac{y^2}{L} + y\right]_0^L$$

$$= -tp\left\{\frac{2}{3} - \frac{3}{2} + 1\right\}L$$

$$= -\frac{1}{6}tpL$$

$$P_{x2} = -tp\int_0^L n_2 \, dy = -tp\int_0^L 4\frac{y}{L} - 4\left(\frac{y}{L}\right)^2 dy$$

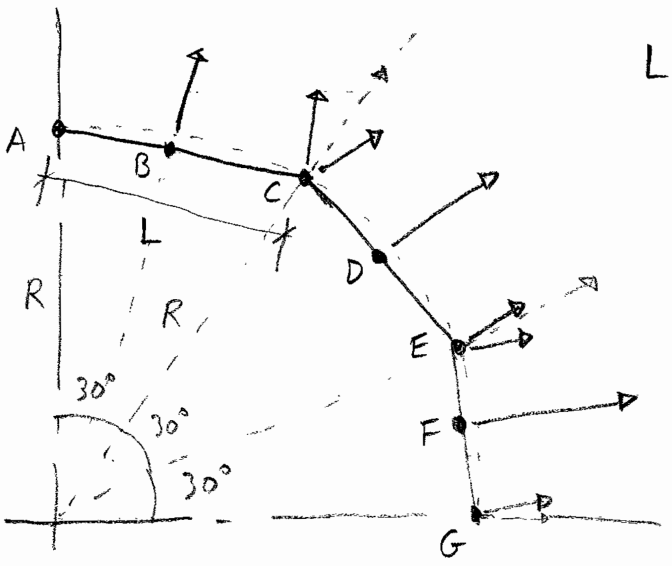
$$= -4tp\left[\frac{y^2}{2L} - \frac{y^3}{3L^2}\right]_0^L$$

$$= -4tp\left\{\frac{1}{2} - \frac{1}{3}\right\}L$$

$$= -\frac{4}{6}tpL$$

$$P_{x3} = P_{x1} = -\frac{1}{6}tpL$$

b)



$$L = 2R \sin 15^\circ$$

$$P_B = \frac{4}{6} t p L = \frac{4}{6} t p \cdot 2R \sin 15^\circ = 0.345\, t p R$$

$$P_C = 2 \cdot \frac{1}{6} t p L \cdot \cos 15^\circ = \frac{1}{3} t p \cdot 2R \sin 15^\circ \cdot \cos 15^\circ = 0.167\, t p R$$

Node	Equiv. Nodal Load	Direction (from X axis)
A	$0.0833\, tpR$	90°
B	$0.345\, tpR$	75°
C	$0.167\, tpR$	60°
D	$0.345\, tpR$	45°
E	$0.167\, tpR$	30°
F	$0.345\, tpR$	15°
G	$0.0833\, tpR$	0°

3D7 Answers

1. (a) $\dfrac{AE}{2\sqrt{2}L}\begin{bmatrix} 1+\sqrt{2} & -1 & -\sqrt{2} & 0 \\ -1 & 1 & 0 & 0 \\ -\sqrt{2} & 0 & 1+\sqrt{2} & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(b) –

(c) $\dfrac{L}{EA}\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} x$ or $\dfrac{L}{EA}\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} x$

2. (a) –

(b) –

(c) Element 1: $u_1 = (2-x)(x+y) \times 10^{-4}$, $v_1 = 0$,

Element 2: $u_2 = (2-y)y \times 10^{-4}$, $v_2 = (1/2)x(y-2) \times 10^{-4}$

(d) Element 1: $\varepsilon_x = 0.5 \times 10^{-4}$, $\varepsilon_y = 0$, $\gamma_{xy} = -1 \times 10^{-4}$

Element 2: $\varepsilon_x = 0$, $\varepsilon_y = 0$, $\gamma_{xy} = -1.25 \times 10^{-4}$

(e) Element 1: $\sigma_x = 10.99$ N/mm^2

Element 2: $\sigma_x = 0$

3. (a)&(b) –

(c) $\left(\int_b^a \mathbf{B}^T k_x \mathbf{B} dx\right)\mathbf{a} = \left(\mathbf{N}^T\right)_{x=b} q_b - \left(\mathbf{N}^T\right)_{x=a} q_a + \int_b^a \mathbf{N}^T Q dx$

(d) & (f) –

4. (a) $p_{x1} = -(1/6)tpL$, $p_{x2} = -(4/6)tpL$, $p_{x3} = -(1/6)tpL$, $p_{y1} = p_{y2} = p_{y3} = 0$

(b) Node A, 0.0833tpR, 90° from x axis

Node B, 0.345tpR, 75° from x axis

Node C, 0.167tpR, 60° from x axis

Node D, 0.345tpR, 45° from x axis

Node E, 0.167tpR, 30° from x axis

Node F, 0.345tpR, 15° from x axis

Node G, 0.0833tpR, 0° from x axis