1. a) ISI occurs when the system response $h(t)$ exceeds one symbol interval.

ISI reduces the eye openings consequently worsening BER performance.

[2]

b) Sample $h(t)$ with a train of $\delta$ pulses at times $kT$,

$$h_s(t) = h(t) \sum_{k=-\infty}^{\infty} \delta(t - kT) \quad ,$$

Consequently the spectrum of $h_s(t)$ is

$$H_s(\omega) = \frac{1}{T} \sum_k H\left(\omega - k\frac{2\pi}{T}\right)$$

Remember for zero ISI

$$h(nT) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

Consequently,
$h_s(t) = \delta(t)$

The spectrum of $\delta(t) = 1$, therfore

$$H_s(\omega) = \frac{1}{T} \sum_k H\left(\omega - k\frac{2\pi}{T}\right) = 1$$

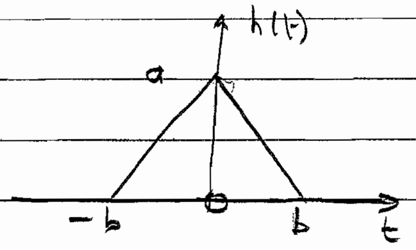
So,

$$\sum_k H\left(\omega - \frac{k2\pi}{T}\right) = T \quad .$$

(6)

A2.

c) (i) From the Electrical and Information data book we have




$$\rightleftharpoons \quad ab\,\mathrm{sinc}^2\left(\frac{\omega b}{2}\right)$$

We have,

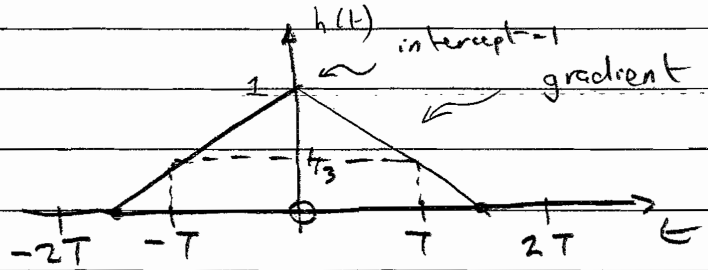
$$H(\omega) = \frac{3T}{2}\,\mathrm{sinc}^2\left(\frac{3\omega T}{4}\right)$$

Comparing with above gives

$$\frac{b}{2} = \frac{3T}{4} \quad , \quad b = \frac{3T}{2}$$

and also $a = 1$.

So the corresponding time domain pulse shape is,




therefore,

$$h(t) = \begin{cases} \frac{-2}{3T}|t| + 1 & |t| \le \frac{3T}{2} \\ 0 & \text{otherwise} \end{cases}$$

[4]

(ii) We can see from c) that $h(t)$ will introduce ISI at the sample instant. At times $+T$ and $-T$ the pulse has a value of

$t = +T$, $\quad h(T) = \frac{-2}{3T}\cdot T + 1 = 1/3$

$t = -T$, $\quad h(-T) = \frac{-2}{3T}\cdot T + 1 = 1/3$

A3.

So at the optimum sample instant the contribution of the wanted pulse is 1.

The ISI contributions (of which there are 2), have a value of 1/3 each.

So for polar line coding the worst case '1' is
$$1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}.$$

Similarly the worst case '0' is
$$-1 + \frac{1}{3} + \frac{1}{3} = -\frac{1}{3}$$

Therefore the worst case eye opening is,
$$h = \frac{1}{3} - \left(-\frac{1}{3}\right)$$
$$= \frac{2}{3}.$$

Worst case $BER = Q\left(\frac{h}{2\sigma}\right)$
$$= Q\left(\frac{2}{3 \times 2 \times 0.1}\right) = Q\left(\frac{1}{0.3}\right)$$
$$= Q(3.33)$$

Sub into formula for evaluating Q func gives

$$BER = 4.3 \times 10^{-4}$$

(6)

iii) Add an equaliser prior to the decision device.

[2]

Add forward error correction (FEC) coding

A 1.

2. a) • Block code : Data symbols are grouped into blocks, and each block is coded separately into a single codeword.

• Binary : Each data or code symbol is either '1' or '0'.

• Linear : Any two valid codewords when added together using modulo-2 arithmetic, produce a valid code word.

• Systematic: Any valid codeword contains the corresponding data word as part of the codeword (often the first part).

[4]

b) Need to determine all possible codewords and find the non-zero one with the minimum Hamming weight (i.e., min. number of '1's).

Using $c = dG$ gives,

| dataword (d) | codeword (c) |
|---|---|
| 0 0 0 | 0 0 0 0 0 0 |
| 0 0 1 | 0 0 1 1 0 1 |
| 0 1 0 | 0 1 0 0 1 1 |
| 0 1 1 | 0 1 1 1 1 0 |
| 1 0 0 | 1 0 0 1 1 0 |
| 1 0 1 | 1 0 1 0 1 1 |
| 1 1 0 | 1 1 0 1 0 1 |
| 1 1 1 | 1 1 1 0 0 0 |

[2]

see $d_{min} = 3$. [1]

so, max number of detectable errors $= d_{min} - 1$

$= 3 - 1$

$= 2$ [1]

max number of correctable errors $= \left\lfloor \dfrac{d_{min}-1}{2} \right\rfloor$

A2.

$$= \left\lfloor \frac{3-1}{2} \right\rfloor$$

$$= 1 \qquad [1]\ [5]$$

c) For a $(n, k)$ linear block code with generator matrix $G$, there is always an $(n-k) \times n$ matrix $H$ (known as the parity check matrix) such that $GH^T = 0$. This matrix has the property that $cH^T = 0$ for any valid codeword and $\hat{c} H^T \neq 0$ if $\hat{c}$ is <u>not</u> a valid codeword. 2

Now,

$$s = c_r H^T$$

is called the error syndrome of a received codeword $c_r$.

If a codeword $c$ is corrupted by the addition (bit by bit modulo-2) of an error vector $e$, then

$$c_r = c + e$$

and the syndrome is

$$s = (c+e)H^T$$

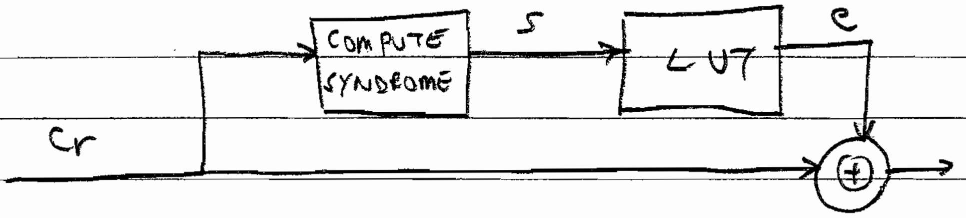
$$s = cH^T + eH^T$$

$$s = eH^T \qquad (\text{since } cH^T = 0) \qquad 2$$

Thus the syndrome depends only on the error and not on the transmitted codeword.

Thus the syndrome can be used to identify the most likely error vector. This can be performed say, using a look-up-table (LUT). The error vector is then subtracted (same as addition in modulo-2 arithmetic) from the received codeword to obtain the most likely transmitted codeword, i.e., 1

A3.




1

[6]

d) (i) Now $G = [I \mid P]$ and
$H = [-P^T \mid I]$.

In this case,

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] = [I \mid P]$$

so,

$$H = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] = [-P^T \mid I]$$

[2]

(ii) As we know from (c), $e = H^T$. Consequently to achieve a direct binary mapping, the columns of H need to form a natural binary count. Further $e_1 = [100\,000]$ is to be associated with syndrome 001, therefore the first column of the modified H, say $H_{NS}$ should be 001. So,

[1]

$$H_{NS} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

[1]

Column no. in H 6 5 2 4 3 1

Consequently

$$G_{NS} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

[1]

6

[5]

3 (a)

A BPSK phasor waveform may be defined by:

$$p(t) = e^{j\phi_0} \sum_k b_k \, g(t - kT_b)$$

where $g(t)$ is a shaping pulse, that should be limited to being non-zero only in the range 0 to $T_b$, and is usually rectangular in shape. $\phi_0$ is an constant phase shift, that is usually unknown at the receiver, due to uncertain path length and time references.

A QPSK phasor waveform may be defined by:

$$p(t) = e^{j\phi_0} \sum_k [\, b_{2k} + j\, b_{2k+1} \,] \, g(t - kT_s)$$

where $T_s = 2T_b$ is the symbol period, and $g(t)$ is now non-zero (and rectangular) in the range 0 to $T_s$. [25%]

(b)

Power Spectrum for Random Data (from lecture notes):

We observe that $p(t)$ is just a constant phasor $e^{j\phi_0}$ multiplied by a polar binary data stream, in which the data impulses have been filtered (convolved) with an impulse response $g(t)$.

Hence the discrete autocorrelation function (ACF) of the random data stream $b_k$ is:

$$R_{bb}(L) = E\{b_k \, b_{k-L}\} = \begin{cases} 1 & \text{for } L = 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Therefore the ACF of the stream of data impulses $b(t) = \sum_k b_k \delta(t - kT_b)$ is:

$$C_{bb}(\tau) = \frac{1}{T_b} \sum_L R_{bb}(L) \; \delta(\tau - LT_b) = \frac{1}{T_b} \; \delta(\tau)$$

and its power spectrum is the Fourier Transform of this (Wiener-Kintchine Theorem). Hence

$$|B(\omega)|^2 = \int_{-\infty}^{\infty} C_{bb}(\tau) \, e^{-j\omega\tau} d\tau = \frac{1}{T_b} \int_{-\infty}^{\infty} \delta(\tau) \, e^{-j\omega\tau} d\tau = \frac{1}{T_b}$$

Since the data impulses $b(t) = \sum_k b_k \delta(t - kT_b)$ are convolved with $g(t)$, the Fourier transform of $p(t)$ is given by:

$$P(\omega) = e^{j\phi_0} B(\omega) \; G(\omega)$$

Version: 2

So the power spectrum of $p(t)$ is:

$$|P(\omega)|^2 = |e^{j\phi_0}|^2\ |B(\omega)|^2\ |G(\omega)|^2 = \frac{1}{T_b}\ |G(\omega)|^2$$

If $g(t)$ is a rectangular pulse of amplitude $a_0$ and duration $T_b$:

$$|G(\omega)| = a_0 T_b \operatorname{sinc}\left(\frac{\omega T_b}{2}\right) \quad \text{and so} \quad |P(\omega)|^2 = a_0^2 T_b \operatorname{sinc}^2\left(\frac{\omega T_b}{2}\right)$$

[25%]

(c)

Power Spectrum of QPSK:

If each quadrature carrier of amplitude $a_0$ is BPSK modulated by rectangular data pulses at a symbol rate of $1/T_s$, then, using the BPSK result, the spectrum of each carrier is given by:

$$|P_I(\omega)|^2 = |P_Q(\omega)|^2 = a_0^2\ T_s \operatorname{sinc}^2(\omega T_s/2)$$

The data on the two carriers are uncorrelated, so the power spectra add to give a total spectrum of:

$$|P(\omega)|^2 = 2\ a_0^2\ T_s \operatorname{sinc}^2(\omega T_s/2) = 4\ a_0^2\ T_b \operatorname{sinc}^2(\omega T_b) \qquad \text{since } T_b = T_s/2$$

Fig 4.2 from the lecture notes (see next page) shows the BPSK and QPSK spectra on a log (dB) vertical axis. [25%]

(d)

Since the argument of the sinc function for the QPSK spectrum is twice that for BPSK, the QPSK spectrum is *half as wide* as the BPSK spectrum for a given data rate - a big advantage!

Usually, modulation schemes that are more bandwidth efficient are also more sensitive to noise, but in the case of QPSK vs BPSK, this is not the case. Both QPSK and BPSK have the same bit error rate in a given level of noise, because QPSK can just be regarded as two independent BPSK systems operating in parallel.

Hence there is no major disadvantage in QPSK to offset the advantage of its improved bandwidth efficiency. Its only slight disadvantage is increased complexity, but this is of minimal importance due to the current low cost of highly complex chips.

QPSK is currently the method of choice for Digital Audio Broadcasting (DAB), because of its high resilience to noise and interference (needed for good mobile reception) and relatively good bandwidth efficiency. [25%]

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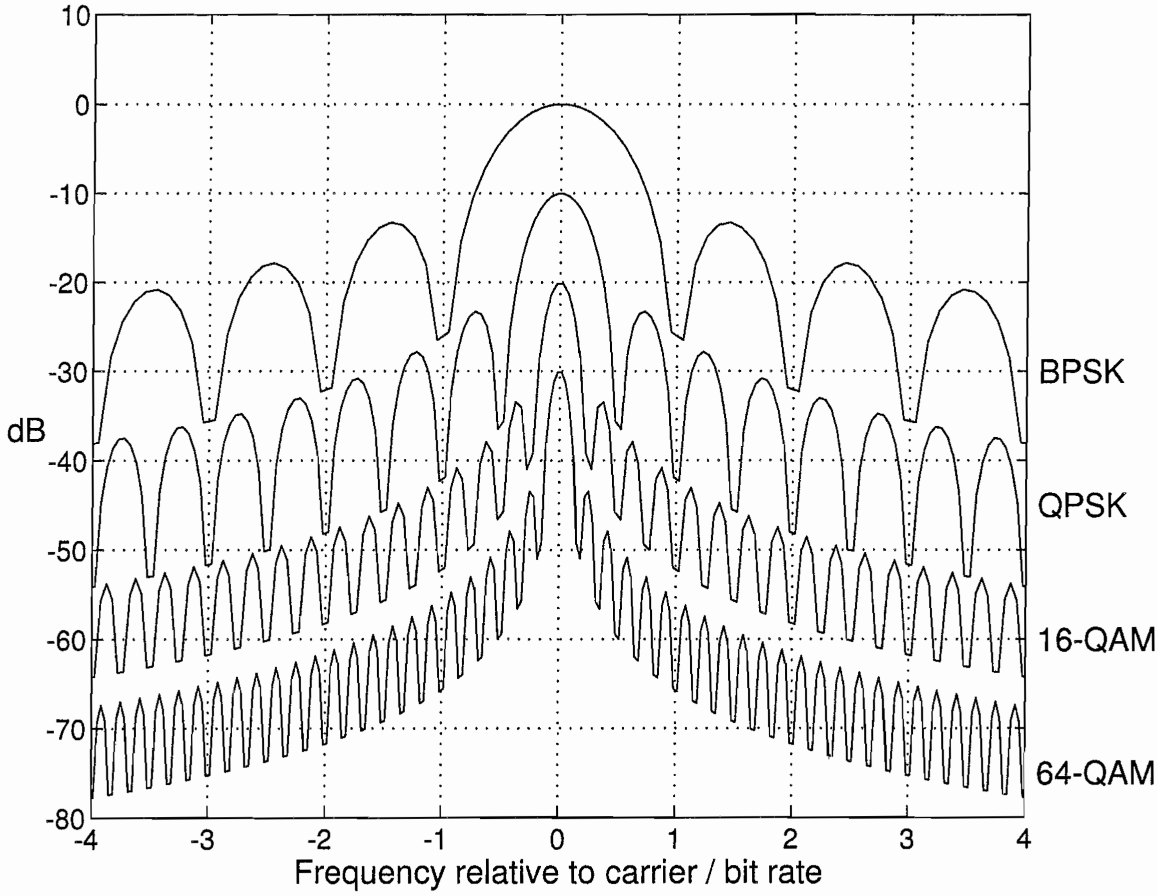


Fig 4.2: Power spectra of BPSK, QPSK, 16-QAM and 64-QAM for a given bit rate.

Version: 2

4 (a)

Energy per bit ( $E_b$ ) is the signal energy of each bit in the transmitted signal, which is equal to the signal power multiplied by the bit period. Usually we assume unit impedance, so the signal power is just the mean square of the signal voltage.

Energy per symbol ( $E_s$ ) is the signal energy of each symbol in the transmitted signal. For an $M$-level modulation scheme where $M = 2^m$, each symbol represents $m$ bits of information and so $E_s = mE_b$.

Energy of the $g(t)$ shaping pulse ( $E_g$ ) is given by

$$E_g = \int_0^{T_s} g^2(t)\, dt$$

where the amplitude of each symbol is some integer multiple $s_k$ times $g(t - kT_s)$. $E_s$ is equal to $E_g$ times the mean square value of $s_k$, averaged over all symbols $k$.

If we use $m$-bit Gray coding for the $M$-levels, each symbol error to an adjacent state will only cause a single bit error in each $m$-bit word. Other coding schemes, which did not use unit-distance codes, would result in higher output bit error rates. [25%]

(b)

To analyse QAM noise performance, we consider each carrier separately. We shall analyse the inphase component and then assume the same performance for the other component.

Starting from the formula given, the mean probability of symbol error on each component is

$$P_{SE} = 2(1 - \frac{1}{M})\, Q\left(\sqrt{\frac{2E_g}{N_0}}\right)$$

If we use $m$-bit Gray (unit distance) coding for the $M$ levels of the inphase component, each symbol error to an adjacent state will only cause a single bit error in each $m$-bit word. We ignore errors to non-adjacent states (unlikely except at poor SNR). Since there are $m$ bits for every symbol

$$\text{Mean probability of bit error, } P_{BE} = \frac{P_{SE}}{m} = \frac{2}{m}(1 - \frac{1}{M})\, Q\left(\sqrt{\frac{2E_g}{N_0}}\right)$$

We now need to express $E_g$ in terms of the energy per bit $E_b$. Using the given formula, the average symbol energy on the inphase carrier is

$$E_s = \frac{1}{M} \sum_{i=0}^{M-1} (2i + 1 - M)^2 \int_0^{T_s} g^2(t)\, dt = \frac{M^2 - 1}{3}\, E_g$$

Version: 2

Since there are $m$ bits per symbol, $E_s = mE_b$, and so

$$E_b = \frac{E_s}{m} = \frac{M^2-1}{3m} E_g \qquad \text{Hence} \quad E_g = \frac{3m\, E_b}{M^2-1}$$

Substituting this into the above expression for $P_{BE}$ gives

$$P_{BE} = \frac{2}{m}(1-\frac{1}{M})\, Q\left(\sqrt{\frac{6m}{M^2-1}\,\frac{E_b}{N_0}}\right)$$

Since both the inphase and quadrature channels of a QAM system operate in parallel and are orthogonal to each other (and hence independent), we get the same result for $P_{BE}$ on the quadrature channel, and hence the same result for the two channels combined (as $E_b$ and $N_0$ are the same for all three cases). [30%]

(c)

First calculate $E_b/N_0$ required for 64-QAM with $P_{BE} = 10^{-3}$.
$M^2 = 64$, so $M = 8$ and $m = 3$.

$$10^{-3}\frac{m}{2(1-1/M)} = Q\left(\sqrt{\frac{6m}{M^2-1}\,\frac{E_b}{N_0}}\right)$$

so

$$Q\left(\sqrt{\frac{18}{63}\,\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2}{7}\,\frac{E_b}{N_0}}\right) = \frac{3\,.\,4\,.\,10^{-3}}{7} = 0.00171$$

Using the result given:

$$Q(2.928) = 0.00171$$

Hence

$$\frac{E_b}{N_0} = \frac{7}{2}(2.928)^2 = 30.0$$

Now for 4-QAM (QPSK), $M^2 = 4$ so $M = 2$ and $m = 1$. Therefore

$$10^{-3} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Using the other result given:

$$Q(3.09) = 0.001$$

Hence

$$\frac{E_b}{N_0} = \frac{1}{2}(3.09)^2 = 4.774$$

$N_0$ is assumed to be constant, so the increase in $E_b$ required for 64-QAM over that for 4-QAM, expressed in dB is

$$10\log_{10}\left(\frac{30.0}{4.774}\right) = 7.98 \text{ dB}$$

[30%]

(d)

Digital video broadcasting (DVB) is designed for high bit-rate transmissions of video material (TV) over fixed terrestrial links. In the UK, coded OFDM is combined with 64-QAM to achieve this.

Spectrum efficiency is very important here, as greater efficiencies allow more TV channels in the finite available bandwidth. To get 24 Mbit/s of user data (typically 6 or more MPEG-coded digital channels) within approx 8MHz of bandwidth (the equivalent of one analogue TV channel), requires 64-QAM, which is 3 times more spectrally efficient than QPSK, because it codes 6 bits per symbol instead of only 2. Error correction coding is required on the OFDM signal in order to overcome problems with narrow nulls in the signal spectrum due to multi-path, and the modulation scheme also has to handle the extra bit rate caused by the code redundancy, which increases the raw bit rate from 24 to about 40 Mbit/s.

The penalty paid by 64-QAM is the approx 8 dB increase in signal-to-noise ratio needed at the receiver, compared with QPSK. However because fixed transmission paths with directional antennae are available, the extra SNR can be achieved in practise. This would be much more difficult if mobile reception in vehicles were desired. [15%]

Version: 2

I. J. WASSELL
N G KINGSBURY
MAY 2005.

# Engineering Triops Part 2A
## Module 3F4. Data Transmission, May 2006 - Comments

1. Generally well answered. Most candidates had difficulty in showing the result required in part (b). This was surprising since it is a bookwork example.

2. This question was the most popular question and was in general answered very well. A few candidates used an incorrect assumption when determining the minimum Hamming distance in part (b).

3. This question was answered quite poorly. Part (a) was generally answered quite well. The evaluation of the power spectrum in part (b) posed most problems to candidates.

4. This question was generally well answered.

# Engineering Triops Part 2A
# Module 3F4. Data Transmission, May 2006- Answers

1. Generally well answered. Most candidates had difficulty in showing the result required in part (b). This was surprising since it is a bookwork example.

a) See notes.
b) See notes.
c)
(i)
(ii) $Q(3.33) = 4.3\times10^{-4}$
(iii) Add equalisation. Add Forward Error Correction (FEC).

2. This question was the most popular question and was in general answered very well. A few candidates used an incorrect assumption when determining the minimum Hamming distance in part (b).

a) See notes.
b) $d_{min} = 3$. Max no. of detectable errors = 2. Max no. correctable errors = 1.
c) See notes.
d)
(i)
(ii) $\mathrm{H} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ $\mathrm{G} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

3. This question was answered quite poorly. Part (a) was generally answered quite well. The evaluation of the power spectrum in part (b) posed most problems to candidates.

a) See notes.
b) See notes.
c) See notes.
d) See notes.

4. This question was generally well answered.

a) See notes.
b) $P_{BE} = \frac{2}{m}\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{3m}{M^2-1}\frac{2E_b}{N_0}}\right).$
c) 7.98 dB.
d) See notes.

I. J. Wassell May 2006