ENGINEERING TRIPOS PART IIA

Thursday 27 April 2006 9 - 12

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (4 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 A cylinder of radius a is spinning with an angular velocity of Ω about its longitudinal axis in a uniform flow of velocity U as shown in Fig. 1. This flow is to be modelled as a doublet and a vortex located at the origin in a uniform flow.

(a) What is the appropriate strength of the doublet and the appropriate circulation of the vortex to model this flow correctly? [30%]

(b) Write down the complex potential $F(z)$ for this flow $(z = x + iy)$. [30%]

(c) Find the position of the stagnation points for the particular cases:

(i) $\Omega a/(2U) < 1$;
(ii) $\Omega a/(2U) = 1$;
(iii) $\Omega a/(2U) > 1$.

and sketch the streamline pattern for the flow *outside* the cylinder for each case. Indicate regions on the surface of the cylinder where separation might occur in the real flow. [40%]

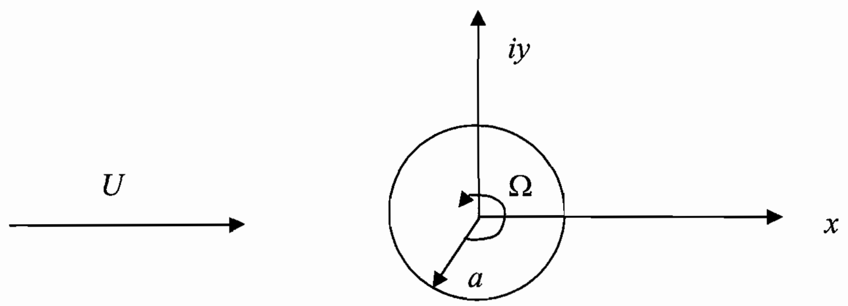

Fig. 1.

2 A vortex of strength Γ is located a distance b above a plane wall in a still fluid of density ρ as shown in Fig. 2.

(a) What is the velocity U_v of the vortex relative to the stationary wall? [20%]

(b) By moving in a frame of reference in which the vortex is stationary (i.e. "sitting on the vortex"), so that the flow is steady, find the complex potential $F(z)$ for this flow $(z = x + iy)$. [20%]

(c) Find an expression for the velocity along the wall as a function of the distance along the wall x in this moving frame. [20%]

(d) Using your result from (b), or otherwise, find an expression for the change of gauge pressure with time that would be measured by a pressure transducer located on the wall at the origin (as shown in Fig. 2) as the vortex moves over the transducer, from far left in the diagram to far right.

Sketch a plot of the non dimensional pressure $P = p\pi^2 b^2 / \rho\Gamma^2$ versus non-dimensional time $T = t\Gamma / 4\pi b^2$, where p is the pressure and t is time. [40%]

Hint: You will need to change back to the frame of reference in which the wall is stationary and use the fact that the pattern moves past at the constant speed of the vortex. In this way distance in the moving frame of reference is related to time for the transducer by $x = U_v t$ where U_v is the speed of the vortex.

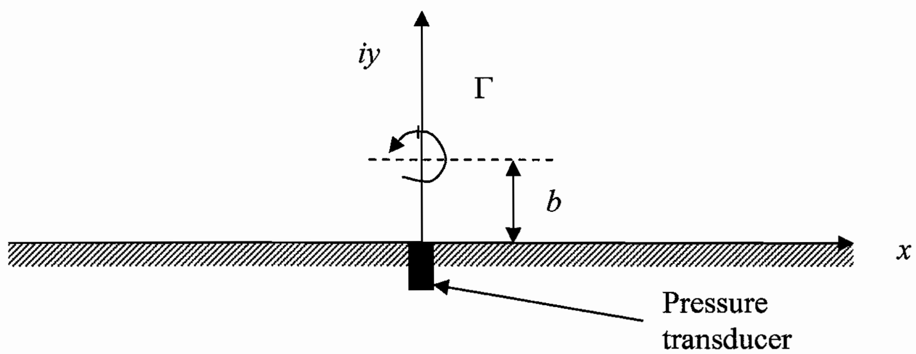

Fig. 2

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3 Two sources of strength m are separated by a distance $2d$ in a uniform flow of velocity U as shown in Fig. 3.

(a) Write down the complex potential $F(z)$ for this flow $(z = x + iy)$. [20%]

(b) Find an expression for the position of the stagnation points in this flow. [40%]

(c) Dye is released along the centreline $(y = 0)$ between the sources. Find d_{crit}, the critical value of d, in terms of m and U such that when $d < d_{crit}$ the dye from far upstream no longer passes between the sources and instead is forced to pass around the pair. [20%]

(d) When $d = d_{crit}$ how far upstream is the single stagnation point from the sources in terms of m and U? [20%]

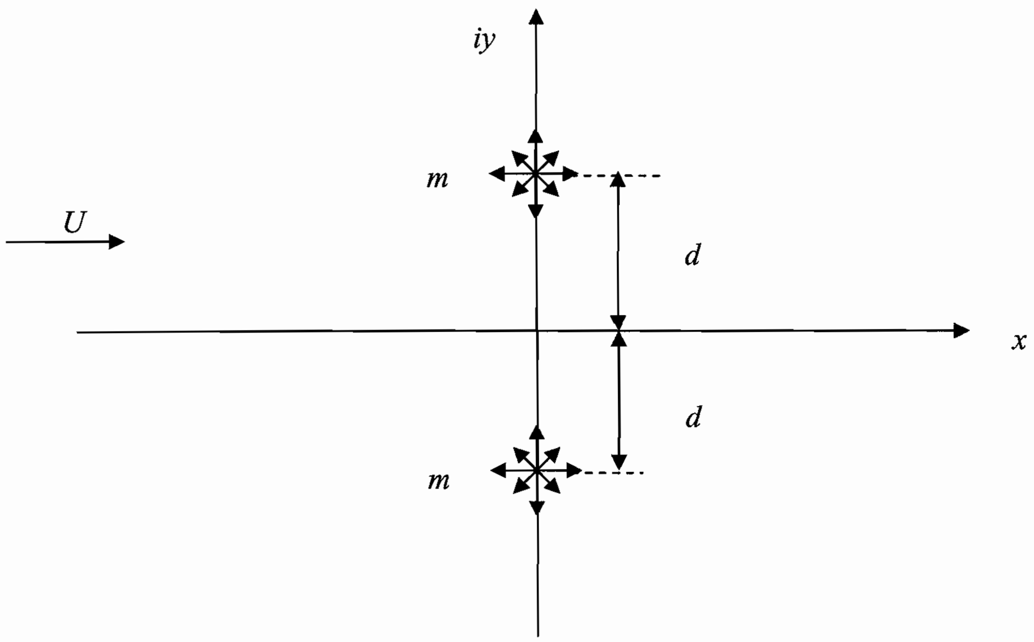

Fig. 3

4 A good approximation for the velocity variation, u, across an incompressible turbulent boundary layer profile on a flat plate (at zero angle of attack) was given by Prandtl as:

$$\frac{u}{U_\infty} = \left(\frac{Y}{\delta}\right)^{\frac{1}{7}}$$

where Y is the wall normal distance, δ is the boundary layer thickness and U_∞ the free stream velocity.

(a) Although a good representation of the actual velocity profile of a turbulent boundary layer, the approximate equation given above is unphysical in at least one aspect. Suggest what this might be. [20%]

(b) For a flat plate of length L and width W, calculate the friction drag incurred on one side using a suitable control volume and the steady flow momentum equation. Hence, show that this can be expressed as

$$C_F = \frac{7}{36}\frac{\delta}{L}.$$

[60%]

(c) Prandtl also proposed the following relationship for the friction drag coefficient:

$$C_F \approx 0.05 \quad Re_\delta^{-\frac{1}{4}}$$

Using this and the result of (b) determine the boundary layer thickness as a function of L and Re_L. [20%]

5 (a) What is a boundary layer and why is it a useful concept in fluid mechanics and heat transfer? [30%]

(b) Compare and contrast the following flows, each with the same free-stream velocity:

(i) a laminar boundary layer on a smooth flat plate;
(ii) a turbulent boundary layer on a smooth flat plate;
(iii) a turbulent boundary layer on a flat plate with a rough surface.

Your answers should include comments on:

(i) the mechanisms of momentum transfer, [10%]

(ii) the mechanisms of heat transfer (if the plates were at a constant temperature above ambient), [10%]

(iii) the relative growth rates of the boundary layer, [10%]

(iv) the relative surface shear stresses, [10%]

(v) the relative heat fluxes from the surfaces (if the plates were at a constant temperature above ambient), [10%]

(c) What criterion is used to decide if a surface is “rough” or “smooth” with respect to the flow in the boundary layer over it? [20%]

6 An architect plans to install a large 3m wide by 2m high window in the wall of a house that has electric heating and needs to estimate the cost of the heat loss (due to convection effects alone) through the window for the 3 months of winter. It has been determined that the average temperature of the outside of the glass is 5° C over the three months and the room temperature is 20° C at all times. The glass is thin and hence the temperature difference across the glass may be neglected. To simplify the calculations it is assumed that the flow velocity over the window inside the room is uniform normal to the window surface and has a magnitude of 1 m/s in the vertical direction. That is, assume there is no momentum boundary layer and neglect buoyancy.

(a) Sketch the development of the thermal boundary layer and the temperature profiles over the 2m height of the window. [20%]

(b) Assume that the temperature profile can be approximated by a parabolic form. By considering the appropriate boundary conditions determine the unknown constants in the parabolic form. [20%]

(c) Calculate the surface heat flux density as a function of distance down the window (in the direction of the flow). [40%]

(d) Estimate the surface heat flux for the window in kilowatts and determine the cost of the heat loss over the 3 months of Dec, Jan and Feb (28 days) if the cost of electricity is 12p per kilowatt-hour. [20%]

Use the relevant data books if fluid properties are required.

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7 (a) Describe the physics associated with boundary layer separation. [20%]

(b) For a two-dimensional airfoil, describe the main types of boundary layer separation observed in practice. Illustrate your answer with sketches of airfoil pressure distributions at various angles of attack. How does this influence the stalling behaviour of the airfoil? [60%]

(c) How is the behaviour of the boundary layers altered on swept wings? [20%]

8 Consider the front wing of an open wheel racing car operating in ground effect. Using simple approximations, show that the sensitivity of the downforce, $\Delta C_L / C_L$, where C_L is the lift coefficient, is related to the wing – ground distance, h, by the following:

$$\frac{\Delta C_L}{C_L} \sim \frac{1}{\mathit{AR}\left(1 + 4(h/b)^2\right)^{1/2}}$$

where AR is the wing aspect ratio and b the wing semi-span. [80%]

Estimate this sensitivity for typical values of the parameters and comment on the result. [20%]

END OF PAPER

Module 3A1 – Fluid Mechanics I
Incompressible Flow Data Card

Continuity equation $\nabla \cdot \boldsymbol{u} = 0$

Momentum equation (inviscid) $\rho \dfrac{D\boldsymbol{u}}{Dt} = -\nabla p + \rho \boldsymbol{g}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

Vorticity $\boldsymbol{\omega} = \text{curl}\,\boldsymbol{u}$

Vorticity equation (inviscid) $\dfrac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \boldsymbol{u}$

Kelvin's circulation theorem (inviscid) $\dfrac{D\Gamma}{Dt} = 0,\quad \Gamma = \oint \boldsymbol{u} \cdot d\boldsymbol{l} = \int \boldsymbol{\omega} \cdot d\boldsymbol{S}$

For an irrotational flow

velocity potential (ϕ) $\boldsymbol{u} = \nabla \phi$ and $\nabla^2 \phi = 0$

Bernoulli's equation for inviscid flow,

$$\frac{p}{\rho} + \tfrac{1}{2}V^2 + gz + \frac{\partial \phi}{\partial t} = \text{constant throughout flow field}, \quad V = |\boldsymbol{u}|.$$

TWO-DIMENSIONAL FLOW

Streamfunction (ψ)

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad u_\theta = -\frac{\partial \psi}{\partial r}$$

Lift force Lift / unit length $= \rho U(-\Gamma)$

Complex potential $F(z)$ for irrotational flows, with $z = x + iy$, $F(z) = \phi + i\psi$ and $\dfrac{dF}{dz} = u - iv$

Examples of complex potentials

(i) uniform flow in x direction, $F(z) = Uz$

(ii) source at z_0, $F(z) = \dfrac{m}{2\pi}\ln(z - z_0)$

(iii) doublet at z_0, with axis in x direction, $F(z) = \dfrac{\mu}{2\pi(z - z_0)}$

(iv) anticlockwise vortex at z_0, $F(z) = -\dfrac{i\Gamma}{2\pi}\ln(z - z_0)$

TWO-DIMENSIONAL FLOW

Summary of simple 2 - D flow fields

	ϕ	ψ	circulation	u
Uniform flow (towards $+x$)	Ux	Uy	0	$u = U,\ v = 0$
Source at origin	$\frac{m}{2\pi}\ln r$	$\frac{m}{2\pi}\theta$	0	$u_r = \frac{m}{2\pi r},\ u_\theta = 0$
Doublet at origin θ is angle from doublet axis	$\frac{\mu\cos\theta}{2\pi r}$	$-\frac{\mu\sin\theta}{2\pi r}$	0	$u_r = -\frac{\mu\cos\theta}{2\pi r^2},\ u_\theta = -\frac{\mu\sin\theta}{2\pi r^2}$
Anticlockwise vortex at origin	$\frac{\Gamma}{2\pi}\theta$	$-\frac{\Gamma}{2\pi}\ln r$	Γ around origin	$u_r = 0,\ u_\theta = \frac{\Gamma}{2\pi r}$

THREE-DIMENSIONAL FLOW

Summary of simple 3 - D flow fields

	ϕ	u
Source at origin	$-\frac{m}{4\pi r}$	$u_r = \frac{m}{4\pi r^2},\quad u_\theta = 0,\quad u_\phi = 0$
Doublet at origin θ is angle from doublet axis	$\frac{\mu\cos\theta}{4\pi r^2}$	$u_r = -\frac{\mu\cos\theta}{2\pi r^3},\quad u_\theta = -\frac{\mu\sin\theta}{4\pi r^3},\quad u_\phi = 0$

Module 3A1 – Fluid Mechanics I

VISCOUS FLOW AND BOUNDARY LAYERS DATA CARD

Navier-Stokes equation:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mathbf{f} + \mu \nabla^2 \mathbf{u}$$

where D/Dt denotes the material derivative, $\partial/\partial t + \mathbf{u}.\nabla$ and where $\mathbf{f}$ is a volume force density (gravity, electromagnetic, Coriolis, ...)

Convection-diffusion of heat:

$$\frac{DT}{Dt} = \alpha \nabla^2 T$$

Boussinesq approximation:

$$\rho \mathbf{g} = \rho_0 [1 - \beta (T - T_0)] \mathbf{g}$$

Prandtl' equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\rho^{-1} \frac{dP_\infty}{dx} + f_x + \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Displacement thickness:

$$\delta^* = \frac{\int_0^\infty (U_\infty - u) dy}{U_\infty}$$

Momentum thickness:

$$\theta = \frac{\int_0^\infty u (U_\infty - u) dy}{U_\infty^2}$$

Shape factor:

$$H = \frac{\delta^*}{\theta}$$

Integral momentum equation:

$$U_\infty^2 \frac{d\theta}{dx} + U_\infty \frac{dU_\infty}{dx} \theta (H + 2) = \nu \left(\frac{\partial u}{\partial y} \right)_0$$

Paper 3A1 - Applications to external flows

Data Sheet

Coefficients:

Pressure coefficient $C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$

Section (local) lift coefficient $c_l = \dfrac{l}{c \times \frac{1}{2}\rho_\infty V_\infty^2}$ (Section chord **c**)

Wing lift coefficient $C_L = \dfrac{L}{A \times \frac{1}{2}\rho_\infty V_\infty^2}$ (Wing area **A**)

Section and wing drag coefficients $c_d = \dfrac{d}{c \times \frac{1}{2}\rho_\infty V_\infty^2}$ $\quad c_D = \dfrac{D}{A \times \frac{1}{2}\rho_\infty V_\infty^2}$

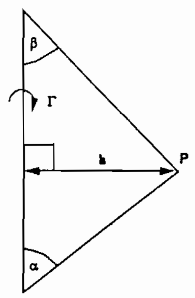
Vortices:

Biot-Savart:

$$\mathbf{dv} = \frac{\Gamma}{4\pi}\frac{\mathbf{dl} \times \mathbf{r}}{|\mathbf{r}|^3}$$

Line vortex:

$$u = \frac{\Gamma}{4\pi h}(\cos\alpha + \cos\beta)$$



Lifting Line Theory:

Lift / unit length and wing lift (span b) $l = \rho\, V\, \Gamma(z) \quad L = \rho V \int_{-b/2}^{b/2} \Gamma(z)\, dz$

Induced drag $D = \rho \int_{-b/2}^{b/2} \Gamma(z)\, w(z)\, dz$

Downwash $w(z_0) = \dfrac{1}{4\pi}\displaystyle\int_{-b/2}^{b/2} \dfrac{d\Gamma(z)}{dz}\dfrac{dz}{z_0 - z}$

Lift curve slope of aerofoil section $a_0 = \dfrac{dc_L}{d\alpha}$

Local circulation $\Gamma(z) = \frac{1}{2} V\, c\, a_0 \left(\alpha - \dfrac{w(z)}{V}\right)$

Elliptic Lift Distribution:

Circulation $\Gamma(z) = \Gamma_0 \sqrt{1 - (2z/b)^2}$

Lift $L = \dfrac{\pi}{4}\rho\, V\, b\, \Gamma_0$

Downwash $w(z) = \dfrac{\Gamma_0}{2\,b}$

Induced drag coefficient $C_D = \dfrac{C_L^2}{\pi\, A_R}$ (Aspect ratio $A_R = b^2/A$)

March 1, 2004