ENGINEERING TRIPOS PART IIA

Friday 28th April 2006 9 to 12

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of the question is indicated in the right margin.*

There are no attachments

STATIONERY
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Compressible flow data book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain how the stagnation pressure measured by a pitot probe changes as the upstream flow varies from subsonic to supersonic. [20%]

(b) The inlet of a convergent divergent nozzle is connected to a large plenum of air at a stagnation pressure p_0 . The nozzle exhausts to a second large plenum of pressure p_e. The ratio of the throat area to exit area of the nozzle is 0.75. A pitot probe is mounted just downstream of the exit plane of the nozzle. The pressure measured by the pitot probe is p_{pitot}. Find the ratio p_e / p_0 and the ratio p_{pitot} / p_0 for the following two cases:

(i) The throat is sonic but the flow elsewhere is subsonic. [20%]

(ii) A normal shock wave is present at the exit plane of the nozzle. [20%]

(c) The ratio of p_e / p_0, of the nozzle in part (b) is set to 0.75. Calculate the ratio p_{pitot} / p_0. [20%]

(d) If the ratio of p_e / p_0 of the nozzle in part (b) is varied from 1 to 0.2 explain how the ratio p_{pitot} / p_0 will vary. [20%]

2 (a) A convergent nozzle is connected to a large reservoir filled with air at 1.25 times atmospheric pressure. A straight pipe is added to the end of the nozzle. The pipe exits to atmosphere. The nozzle and pipe flow are frictionless and adiabatic. Calculate the Mach number at the exit of the pipe. If the straight pipe is uniformly cooled while the nozzle remains adiabatic, explain what will happen to the Mach number variation through the straight pipe. [15%]

(b) Draw temperature-entropy diagrams (T-s) showing the variation of flow conditions through the convergent nozzle and the straight pipe with and without the pipe cooling described in part (a). Mark on the diagrams lines of constant pressure and the stagnation and the static conditions of the flow at the inlet to the nozzle and exit from the pipe. [30%]

(c) The straight pipe described in part (a) is cooled so that the ratio of the stagnation temperatures between its exit and inlet is 0.85. The stagnation pressure in the reservoir is altered so that the Mach number at the exit of the straight pipe remains the same as in the un-cooled case described in part (a). Calculate the Mach number at the inlet of the straight pipe and the stagnation pressure in the large reservoir. [30%]

(d) Calculate the percentage change in mass flow rate through the pipe when the cooling in part (c) is switched on. The large reservoir, convergent nozzle and straight pipe are restrained from moving by a single force. Discuss with explanation the effect of switching on the cooling in part (c) on this force. [25%]

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3 A cylinder of constant cross-sectional area is closed at one end and open at the other. The open end is sealed with a diaphragm. The cylinder initially contains air at a pressure p_0 and at atmospheric temperature. The surrounding atmospheric air is at pressure p_a . The initial pressure ratio p_a/ p_0 is 4.5. At time t=0 the diaphragm instantaneously bursts and a travelling shock wave forms.

(a) Calculate the strength of the initial shock wave. Draw a space-time (s-t) diagram of the shock wave as it travels through the cylinder and reflects off the closed end. Mark on the diagram a number of particle trajectories. [25%]

(b) Calculate the velocity of the air and the speed of sound immediately down stream of the initial shock wave in terms of the speed of sound in the air before the diaphragm bursts, a_0. [35%]

(c) Calculate the strength of the shock wave immediately after it reflects off the closed end of the cylinder. The velocity immediately down stream of the reflected shock is zero. Use may be made of the formula for the density change across a shock wave,

$$\frac{\rho}{\rho_s} = \frac{2}{(\gamma+1)M^2} + \frac{\gamma-1}{\gamma+1}$$

where ρ and M are the density and Mach number upstream of the shock, ρ_s is the density downstream of the shock and γ is the ratio of the specific heat capacities of the air. [40%]

4 Figure 1 shows a two-dimensional external compression jet engine intake that is designed for a supersonic aircraft. It employs three wedges of 8 degrees, to produce a focused system of four shocks. Conditions downstream of the final shock are subsonic.

(a) Draw a clearly labelled sketch of the intake and its shock system. [15%]

(b) Calculate the Mach Number downstream of the final shock. [25%]

(c) Calculate the total pressure recovery of the whole shock system. Express your answer as the ratio of the stagnation pressure downstream of the last shock P_{0f} to the stagnation pressure upstream of the intake $P_{0\infty}$. Assume that the shocks are the only non-isentropic flow features. [20%]

(d) It is proposed to reduce the cost and complexity of the design by removing the third wedge. The front two 8 degree wedges are retained (but appropriately re-scaled) and the downstream components are adequately redesigned so that the three remaining shocks form a new focused inlet. Calculate the stagnation pressure ratio $P_{0f}/P_{0\infty}$ for the new inlet. [10%]

(e) Calculate the stagnation pressure ratio $P_{0f}/P_{0\infty}$ for the new inlet, described in part (d), when the flight Mach number is reduced to 2.00. Comment on the relative magnitude of the stagnation pressure ratios calculated in parts (c), (d) and (e). [30%]

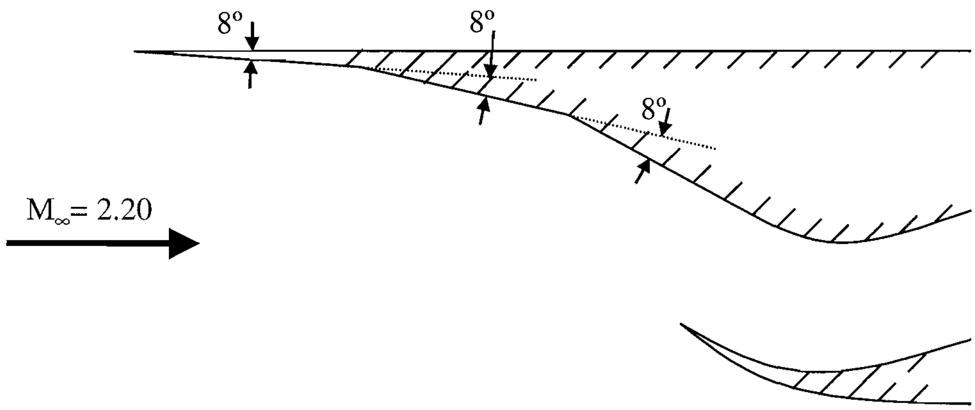

Fig. 1

5 Consider a steady, two-dimensional, isentropic flow with velocity components u and v described in a cartesian coordinate system (x,y).

(a) Write down differential equations for mass continuity and energy conservation. Why can the flow be assumed to be irrotational? [15%]

(b) By considering the momentum of the flow, derive the following expression:

$$\left(a^2 - u^2\right)\frac{\partial u}{\partial x} - 2uv\frac{\partial u}{\partial y} + \left(a^2 - v^2\right)\frac{\partial v}{\partial y} = 0$$

where a is the local speed of sound. [45%]

(c) By making the substitutions:

$$u = \left(U_\infty + \frac{\partial \varphi}{\partial x}\right) \text{ and } v = \left(\frac{\partial \varphi}{\partial y}\right) \text{ where } \frac{\partial \varphi}{\partial x} << U_\infty \text{ and } \frac{\partial \varphi}{\partial y} << U_\infty$$

show that, for cases where the flow is only *slightly* disturbed (as for thin airfoils at small incidence) the following expression can be derived:

$$\beta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$

Define β and *sketch* a plot of $(1/\beta)$ against Mach Number, M_∞, in the range $0<M_\infty<1$. [30%]

(d) An aircraft wing section is designed to operate at a lift coefficient, C_L = 0.50 at a flight Mach Number, $M_\infty = 0.60$. Estimate the correct lift coefficient for which this section should be designed in a low speed wind tunnel. [10%]

6 (a) Explain what is meant by *upwind differences* in the context of the numerical solution of hyperbolic partial differential equations. [15 %]

(b) The one-dimensional scalar convection equation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$$

where A is a positive constant, is to be modelled using the following difference equation

$$u_i^{n+1} = u_i^n - c\left(u_i^n - u_{i-1}^n\right)$$

where time $t = n\Delta t$, distance $x = i\Delta x$ and $c = A\Delta t/\Delta x$. Show that the equivalent differential equation of this scheme takes the form

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}$$

and find the value of the constant α. Hence determine the order of accuracy of this numerical scheme.

You may use the result

$$\frac{\partial u}{\partial t} = -A\frac{\partial u}{\partial x} \quad \Rightarrow \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(-A\frac{\partial u}{\partial x}\right) = -A\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) = A^2\frac{\partial^2 u}{\partial x^2}$$

[45 %]

(c) At time $t = 0$, u_i^0 is a square pulse as shown in Fig. 2. Describe, with the aid of sketches, the evolution of u_i^n for

(i) $c = 1$
(ii) $c = 0.5$
(iii) $c = 1.5$

[40 %]

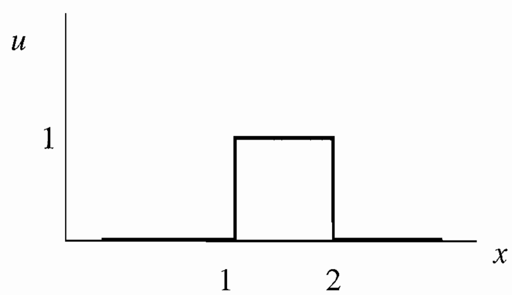

Fig. 2

7 (a) An axial flow turbine has an output power of 15 MW and a mass flow rate of 50 kgs^{-1}. The stagnation pressure and stagnation temperature at the inlet to the turbine are 8 bar and 900 K respectively. The turbine exhausts into the atmosphere (assumed to be at 1 bar) where the Mach number of the flow is 0.7. You may assume that the working fluid has the same properties as air.

(i) Calculate both the total-to-total and the total-to-static efficiencies of the turbine. [35%]

(ii) Explain the significance of the two different measures of efficiency and suggest how the turbine output power can be increased. [15%]

(b) (i) Explain what is meant by the *conservation form* of the unsteady Euler equations and why this form is the most common starting point for numerical methods to solve these equations. [20%]

(ii) The triangle shown in Fig. 3 is one cell of a finite-volume grid used to solve the two-dimensional, unsteady, Euler equations using conservative variables stored at the nodes. If the trapezium rule is used to evaluate integrals over the edges, show that the equation representing mass conservation reduces to

$$h^2 \frac{\partial \rho_{cell}}{\partial t} = -h\left\{\frac{(\rho V_x)_1 - (\rho V_x)_0}{2} + (\rho V_y)_2 - \frac{(\rho V_y)_1 + (\rho V_y)_0}{2}\right\}$$

[30%]

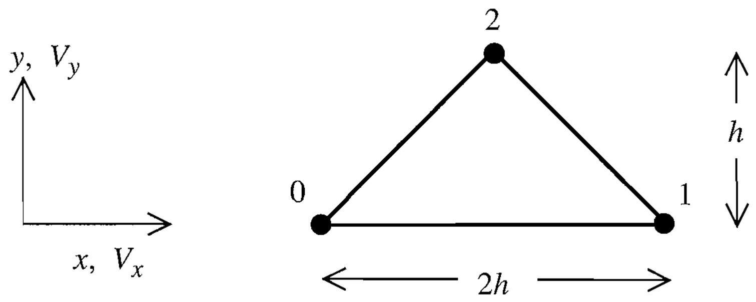

Fig. 3

8 A single-stage axial-flow compressor has a mean radius of 0.3 m and the flow has zero tangential velocity in the absolute frame ahead of the rotor. The (absolute) stagnation conditions at inlet to the compressor are 1 bar and 288 K. The compressor rotates at 6500 rpm and is designed to operate at a flow coefficient of 0.5 when the mass flow rate through the compressor is 25 kgs^{-1}. You may assume that the working fluid has the same properties as air.

(a) Calculate the necessary blade height at entry to the rotor blade row. [20%]

(b) Calculate the relative stagnation temperature, relative stagnation pressure and relative flow angle at inlet to the rotor blade. [20%]

At the design operating conditions, the stagnation pressure loss coefficient of the rotor blade is 0.05 and at rotor exit the relative flow angle is $-50°$.

(c) Calculate the relative stagnation pressure at rotor exit and, assuming that the blade height does not vary, the corresponding static pressure. [30%]

(d) By calculating the tangential velocity at the exit from the rotor, calculate the stage loading coefficient ($\Delta h_0/U^2$) and comment on its value. [20%]

(e) Comment on the design choice of not varying the blade height of the rotor blade. [10%]

END OF PAPER

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Answers to 3A3 2006

1. (b) (i) 0.84, 1, (ii) 0.65, 0.86.
   (c) 0.93.

2. (a) 0.572.
   (c) 0.74, 1.17.
   (d) 0.992.

3. (a) 2.
   (b) 1.25a_0, 1.30a_0.
   (c) 1.73.

4. (b) 0.77.
   (c) 0.95.
   (d) 0.87.
   (e) 0.93.

7. (a) 1.39 bar, 84.2%, 74.0%.

8. (a) 0.112m.
   (b) 308.7k, 1.275bar, -63.4°
   (c) 1.258 bar, 1,124bar.
   (d) 0.473.