ENGINEERING TRIPOS PART IIA

Saturday 13 May 2006 2.30 to 4

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Thermal barrier coatings for improving resistance to high temperatures are created by impinging molten ceramic particles onto the treated part. A plasma jet generates a very high temperature gas, which heats up the small alumina particles entrained in the jet stream during their flight towards the part to be treated. The time of flight must be sufficient to heat the particles from ambient temperature to their melting point, and on to complete fusion. The plasma jet stream can be considered to remain at a constant temperature of T_∞ = 10,000 K during the particle flight. The particles and the ambient are initially at T_o = 300 K. The convection coefficient between particle and hot gas is h = 30,000 W/m^2 K. The particles can be considered spherical throughout the process, with a diameter of 50 microns. The relevant properties of alumina are shown below.

density	ρ	3970 kg/m^3
thermal conductivity	k	10.5 W/m K
specific heat	c	1560 J/kg K
melting point	T_{mp}	2318 K
heat of fusion	h_{sl}	3.577 x 10^6 J/kg

(a) Assuming that the particles are at uniform temperature T, and that particle radiation is negligible:

(i) Obtain an expression for the time t_1 required to heat the particle to its melting point as a function of the parameters given above. [20%]

(ii) Obtain an expression for the time interval t_2 starting from the end of interval t_1 to completely melt the particle. [20%]

(iii) Sketch the temperature history of the particle and calculate numerical values for t_1 and t_2. [10%]

(b) Determine and justify a criterion for the validity of the assumption of uniform temperature using the variables and values given. [25%]

(c) Determine the validity of neglecting radiation to the environment, which is assumed to be a large enclosure at 300 K. [25%]

2 Fluid at 500 K flows through a long cylindrical pipe. The pipe outer surface is at the same temperature as the fluid. The pipe surface has an emissivity of 0.10, and is surrounded by an outer shield 50 mm in diameter, with an emissivity of 0.02. The outer shield is kept at a temperature of 300 K. The space between the inner tube and outer shield is evacuated. The heat loss in the current configuration is considered to be excessive. Two methods are proposed to reduce the heat loss: (A) use a concentric, thin inner shield with emissivity 0.03 at the midpoint between the inner and outer pipe, or (B) fill the annular gap with high grade insulation of thermal conductivity 0.03 W/m K.

The pipe diameter is D=20 mm.

(a) Calculate the present heat loss rate per unit length q without modifications. [30%]

(b) Calculate the heat loss rate per unit length q for method (A). [30%]

(c) Calculate the heat loss rate per unit length q for method (B). [20%]

(d) Compare the numerical answers for the two proposed alternatives, and discuss the best course of action. [10%]

(e) Discuss whether the shield should be placed closer to or further from the midpoint of the annulus to reduce the heat loss. [10%]

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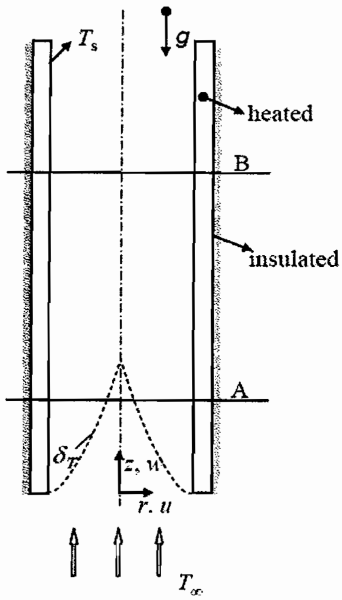
3 Consider the natural convection inside a vertical cylinder of length L and of inner radius R. The cylinder wall is heated so that the inner surface temperature is maintained at T_S while the outer surface is insulated as shown in the figure below. This cylinder is placed in ambient air at temperature $T_\infty < T_S$, and pressure P_∞. The exit temperature of the air is uniform and approximately equal to T_S. The thermal boundary layer thickness is δ_t. The acceleration due to gravity is g. Take the properties of air to be constant.



(a) Carefully sketch the radial variation of the velocity, w, and the temperature, T, at sections A and B shown in the figure above. [5%]

(b) For the fully developed region, the momentum equation in the z direction is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = \frac{-g\beta}{\nu}(T - T_\infty),$$

after the Boussinesq approximations are used. The volumetric expansion coefficient of air is β and its kinematic viscosity is ν. What is the physical meaning conveyed by this equation? What are the boundary conditions to this equation? [10%]

(c) Obtain the mass flow rate through the tube in terms of T_S, T_∞, R, ν, β and the density, ρ_∞, of air by integrating the governing equation in (b). You may like to use $(T - T_\infty) = (T_S - T_\infty) - (T_S - T)$. [40%]

(cont.

(d) Show that the Nusselt number based on L, in the fully developed region is

$$Nu_L = \frac{Ra_R}{16},$$

where $Ra_R = g\beta(T_s - T_\infty)R^3/(\alpha\nu)$, is the Rayleigh number and α is the thermal diffusivity of air. [45%]

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4 A carbon (C) particle, which can be approximated as a sphere of radius R_o, is placed in pure oxygen (O_2) environment at temperature of 1500 K and pressure of 1 bar. The environment is stagnant and the oxygen present near the particle surface is consumed via the following reaction

$$C + O_2 \rightarrow CO_2$$

forming carbon dioxide (CO_2). The carbon dioxide diffuses out from the surface. The binary diffusion coefficient for oxygen and carbon dioxide is $D = 1.71 \times 10^{-4}\mathrm{m}^2/\mathrm{s}$.

(a) Neglecting the change in the particle radius because of chemical reaction, determine the steady state radial distribution of O_2 and CO_2 when

(i) the surface reaction is infinitely fast,

(ii) the reaction is finite and first order with a rate of $\dot{\Omega}_{O_2} = -k_1 [O_2]_s$, where k_1 is the rate constant in units of m/s and $[O_2]_s$ is oxygen concentration at the surface.

Express your answers in terms of the particle radius, R_o, the radial distance from the centre of the particle, r, and the oxygen concentration, $[O_2]_\infty$, at $r = \infty$. [50%]

(b) Considering the conservation of particle mass, show that the time, t, required for the particle diameter to change from its initial value of R_o to R is

$$t = \frac{(R_o - R)\rho_c}{W_c k_1 [O_2]_\infty},$$

when the condition $(D/k_1 R) \gg 1$ is satisfied. Take the reaction to be finite and first order with a rate of $\dot{\Omega}_{O_2} = -k_1 [O_2]_s$. The symbols ρ_c and W_c are density and molecular weight of carbon respectively. If you make any assumptions in your analysis, state them clearly. [45%]

(c) Calculate the duration for the particle diameter to change from 0.1 mm to 0.05 mm when placed in the above oxygen environment. Take $k_1 = 0.1$ m/s, and $\rho_c = 1950$ kg/m^3. [5%]

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