ENGINEERING TRIPOS PART IIA

Tuesday 9 May 2006 9 to 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: 8

1 (a) The *Einstein relation* states that

$$D = \left(\frac{kT}{e}\right)\mu .$$

(i) Identify the terms in this equation and explain its physical significance. [20%]

(ii) Derive the *Einstein relation* by using the example of the graded pn junction shown in Fig. 1. [30%]

(b) The graded junction shown in Fig. 1 is 1 μm thick and fabricated from silicon which has a band gap of 1.12 eV, an effective density of states in the conduction band $N_C = 2.8\times10^{25}\ \text{m}^{-3}$ and an effective density of states in the valence band $N_V = 1.04\times10^{25}\ \text{m}^{-3}$. The acceptor and donor doping densities, N_A and N_D respectively, at either end of the junction are given in Table 1.

(i) Calculate the electric field inside the graded junction. State any assumptions made. [40%]

(ii) Sketch how the acceptor and donor doping densities vary as a function of position x across the junction. [10%]

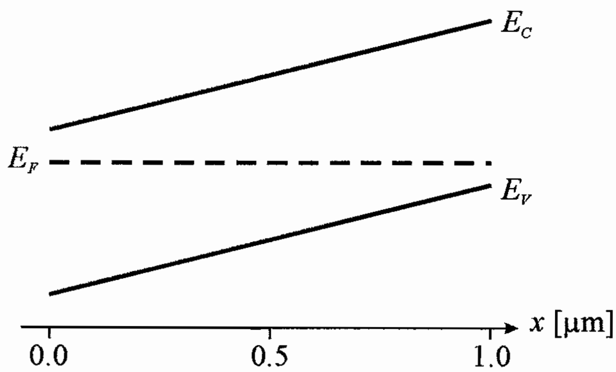

Fig. 1

Position x [μm]	N_D [m^{-3}]	N_A [m^{-3}]
0	10^{23}	10^{22}
1	10^{22}	10^{23}

Table 1

2 (a) Sketch the band diagram for a p^+n junction diode under the condition of no applied bias and with a forward bias voltage applied. [10%]

(b) Show that the excess minority carrier (hole) concentration $p - p_{n0}$ varies with distance x away from the edge of the depletion region on the n-type side of the p^+n junction as

$$p - p_{n0} = p_{n0}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_h}\right),$$

stating any assumptions made. p_{n0} is the equilibrium concentration of holes in the bulk of the n-type semiconductor, V is the applied bias, $L_h = \sqrt{(D_h \cdot \tau_h)}$ is the diffusion length of holes and the injected minority carrier concentration at $x = 0$ is given by

$$p(x = 0) = p_{n0}\exp\left(\frac{eV}{kT}\right).$$

[35%]

(c) Sketch the variation in the excess minority carrier concentration as a function of distance x away from the edge of the depletion region on the n-type side of the p^+n junction. What physical process causes this variation? [10%]

(d) Show that the current density across the p^+n junction is given by

$$J = \frac{eD_h n_i^2}{L_h N_D}\left[\exp\left(\frac{eV}{kT}\right) - 1\right],$$

where N_D is the donor doping density in the n-type semiconductor. What will cause the current density in a real p^+n junction diode to deviate from this ideal expression? [45%]

Note that the Continuity equation for holes is

$$\frac{\partial(\Delta p)}{\partial t} = \frac{-\Delta p}{\tau_h} - \mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}.$$

3 (a) Explain the meaning of the *Fermi energy* in a metal and how it is related to the Pauli exclusion principle. Explain the meaning of the Fermi energy in a semiconductor. [15%]

(b) The density of states in the conduction band of a metal (that is the band with the highest energy that is occupied by electrons) is given by

$$g(E) = \frac{4\pi}{h^3}\left(2m^*\right)^{3/2}\left(E - E_C\right)^{1/2},$$

where E_C is the energy of the bottom of the conduction band of the metal. Derive the relationship between $E_F - E_C$ and the total number of electrons N in the band. [15%]

(c) The effective density of states in the conduction band of a semiconductor is given by

$$N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}.$$

Assuming that the equation for the density of states in part (b) is also valid for a semiconductor, show that the following relationship holds for any semiconductor when $E_C - E_F >> kT$:

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right).$$ [40%]

(d) The resistivity values for an intrinsic semiconductor are found to be 4.16×10^7 Ω cm at $T = 250$ K and 4.59×10^5 Ω cm at $T = 300$ K. Estimate the energy gap of the semiconductor. State all assumptions made. [30%]

Note: $$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \;;\; \int_0^\infty x^{1/2}\exp(-x) = \frac{\sqrt{\pi}}{2}$$

4 An electron beam, accelerated through a certain electric potential, is directed onto a crystal. The angle between the beam direction and the atomic planes of the crystal is α. The separation beween the crystal planes is d.

(a) By considering the phase difference between beam 1 and beam 2 of Fig. 2, deduce the Bragg diffraction law. [20%]

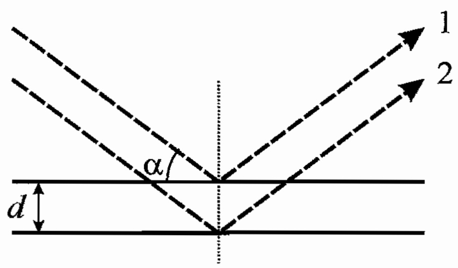

Fig. 2

(b) The angle α is fixed at 50°, $d = 2\times10^{-10}$ m and the accelerating potential is increased while the electrical current associated with the diffracted beam is measured. What is the minimum voltage for which the detected current shows a maximum? [30%]

(c) Discuss the *quasi-free electron approximation* for a one-dimensional solid with lattice parameter d. In particular,

(i) Identify the First and Second Brillouin Zones;

(ii) Show by symmetry argument or otherwise that the wavefunctions at the zone boundaries are standing waves;

(iii) Explain the formation of energy gaps. [50%]

END OF PAPER

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Answers to numerical questions

Q1

b (i) ε=851kVm^{-1}

Q4

(d) E_g= 1.1eV