ENGINEERING TRIPOS PART IIA

Tuesday 2 May 2006 2.30 to 4

Module 3C3

MACHINE DESIGN – TRIBOLOGY

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachment: Data sheet (10 pages).*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| Single-sided script paper | Engineering Data Book |
| | CUED approved calculator allowed |

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 Figure 1 shows a simple seal formed where a circular rod of diameter $D$ moves linearly through the wall of a pressure vessel. The contained pressure is of magnitude $p_1$ and the speed of the rod is $U$. The seal consists of two parallel sections each of length $B$ as shown. On the pressurised side, the gap between the rod and the housing is $h_1$ and on the atmospheric side it is of dimension $h_0$. The diameter of the rod is large compared to the film thicknesses. The pressure on the atmospheric side of the vessel can be taken as zero.

(a) Sketch the expected form of the pressure distribution within the fluid as it varies from $p_1$ to 0 if it can be assumed to be incompressible and to exhibit Newtonian viscosity. [20%]

(b) If $h_1 = 5h_0$, find an expression for the pressure at the step in the wall profile and hence for the leakage rate across the seal gap in terms of $U$, $B$, $h_0$, $p_1$, $D$ and $\eta$ the viscosity of the fluid. [30%]

(c) If the contribution to the leakage due to the pressure $p_1$ is half that due to the motion of the rod find an expression for $h_0$ in terms of $U$, $B$, $p_1$, and $\eta$. [15%]

(d) In a particular case, $B = 10$ mm, $U = 1\ \mathrm{m\,s^{-1}}$, $\eta = 0.01\ \mathrm{Pa\,s}$, $p_1 = 6$ MPa and $D = 50$ mm. What is the required value of $h_0$? Do you think this a satisfactory design for sealing the rod? [20%]

(e) What happens when the direction of $U$ reverses? [15%]

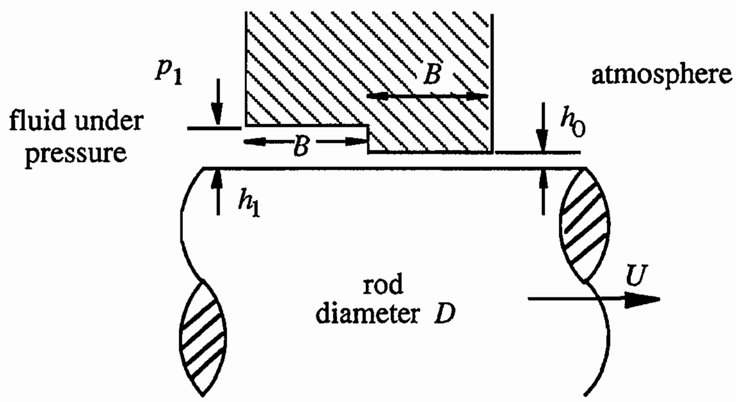


Fig. 1

2 A lubricated journal bearing of radius $R$, length $L$ and radial clearance $c$ has a length to diameter ratio of 2. The journal rotates at a steady speed $\omega$ and carries a steady radial load $W$. The lubricant has viscosity $\eta$, density $\rho$ and specific heat $\kappa$.

The normalised quantities $W^*$, $Q^*$ and $M^*$ are defined as

$$W^* = \frac{W}{2RL\eta\omega}\left\{\frac{c}{R}\right\}^2, \quad Q^* = \frac{Q}{LR\omega c} \quad \text{and} \quad M^* = \frac{Mc}{2\eta\omega LR^3}$$

where $Q$ is the volumetric flow rate of the lubricant and $M$ is the torque required to maintain rotation. The table shows how these vary with the eccentricity ratio $\varepsilon$.

| $\varepsilon$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|
| $W^*$ | 0.285 | 0.587 | 0.920 | 1.305 | 1.782 | 2.433 | 3.438 | 5.359 | 11.13 | 22.51 |
| $Q^*$ | 0.0537 | 0.104 | 0.153 | 0.199 | 0.243 | 0.285 | 0.329 | 0.369 | 0.406 | 0.422 |
| $M^*$ | 3.000 | 3.06 | 3.128 | 3.261 | 3.493 | 3.894 | 4.503 | 5.572 | 8.681 | 13.51 |

(a) Three important operational factors for the bearing are (i) the minimum film thickness $h_{\min}$, (ii) the effective coefficient of friction $\mu$ defined as $M/RW$, and (iii) the increase in temperature $\Delta T$ of the fluid which occurs as a result of the shear within the film. For any value of $\varepsilon$ show how $h_{\min}$, $\mu$ and $\Delta T\rho\kappa$ can all be related to the tabulated non-dimensional groups and the particular values of $R, L, \omega, \eta$ and $W$. [45%]

(b) By sketching a curve of $\mu \times \sqrt{W/2L\eta\omega R}$ versus $h_{\min} \times \sqrt{W/2L\eta\omega R^3}$ suggest a design value of $\varepsilon$ which would be a satisfactory compromise between the requirements to minimise friction but maximise the minimum film thickness. [25%]

(c) In a particular case, the fluid within the bearing has density $1.57 \times 10^3$ kg m$^{-3}$ and a specific heat 0.96 kJ kg$^{-1}$ K$^{-1}$. The bearing carries a radial load of 300 N and has an axial length of 20 mm. Estimate the value of $\Delta T$ on the basis of the choice of conditions made in part (b). [30%]

3 (a) A ball of radius $R$ is spinning in contact with a plane surface under a contact force $P$. Show that the motion is resisted by a spin torque of magnitude

$$\frac{3\pi\mu P}{16}\left(\frac{3PR}{4E^*}\right)^{1/3}$$

where $\mu$ is the coefficient of sliding friction between the ball and the plane and $E^*$ is the elastic contact modulus. Hertzian conditions may be assumed.

Note that $$\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta = \frac{\pi}{16}$$ [30%]

(b) Figure 2 shows a linear bearing which consists of two rails each with a 90° groove symmetrically ground in their surface. The upper rail is supported by 10 balls each of radius 5 mm which roll in the track formed by the opposing grooves. The supported normal load, which can be assumed to be equally shared between the balls, is 50 N. The lower rail is stationary and the upper rail is moving with a speed of $5\,\mathrm{m\,s^{-1}}$.

(i) What is the value of the maximum Hertz pressure developed at each contact? The contact modulus can be taken as 115 GPa. [15%]

(ii) What is the magnitude of the spin velocity at each of the points of contact between ball and rail? [15%]

(iii) Estimate the rate of energy consumption in the bearing if the coefficient of sliding friction between steel surfaces is 0.2. [30%]

(iv) What is the effective value of the coefficient of friction for the bearing as a whole? [10%]

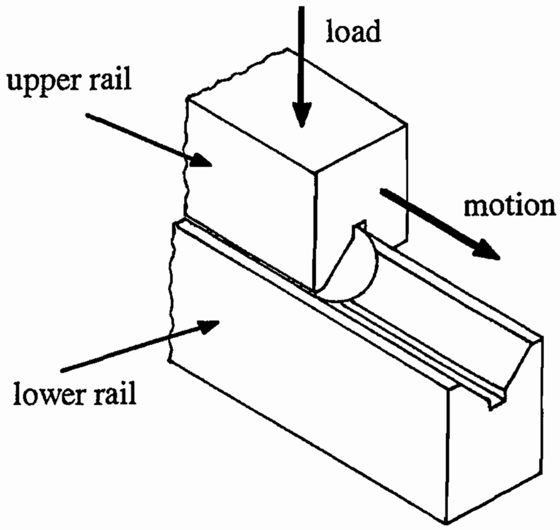


Fig. 2



(TURN OVER

4 (a) Explain what is meant by 'balanced design' of spur gears. [10%]

(b) A pinion with $N$ teeth of module $m$ engages with a rack. The pinion has standard teeth with the addendum equal to the module. The pressure angle is $20°$ and the tooth width is equal to $6m$.

(i) Show that if contact ratio is to be unity for $N = 16$ then the addendum on the rack must be of magnitude $0.253m$. [30%]

(ii) Assuming this to be the case, find an expression in terms of $m$ for the maximum allowable force in the direction of rack motion that can be safely transmitted by the rack. Take the properties of the tooth material to be equal to the maximum values given on the data sheet for through hardened and tempered steel with an effective contact modulus $E^* = 115$ GPa. What is the critical failure mode? [35%]

(iii) By what factor will the allowable load change if the number of teeth on the pinion is doubled, all other factors remaining unchanged? [25%]

END OF PAPER