ENGINEERING TRIPOS PART IIA

Tuesday 9 May 2006 2.30 to 4.00

Module 3D1

SOIL MECHANICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Soil Mechanics Data Book (19 pages)

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Figure 1 shows a layer of clay 5 m thick, submerged by 10 m of water, which is to receive a cylindrical caisson whose external dimensions are 15 m x 15 m. A thin layer of gravel is to be dumped over the clay prior to the sinking of the caisson. Once in place, the water levels inside and outside will be kept the same. It is required to evaluate the transient compression of the clay when rock-fill is placed inside the flooded caisson. Field trials have shown that the filling process achieves a dry unit weight $\gamma_d = 15.0$ kN/m^3. Both rock-fill and clay can be taken to have $G_s = 2.7$. In computing applied stresses, it will be sufficient to ignore the thickness and density of the caisson material itself relative to the rock-fill.

(a) Estimate the saturated unit weight of the rock-fill. Calculate the effective stress applied to the gravel layer by the caisson when it is filled with rock-fill

i) to a depth of 10 m, level with the water;

ii) to its full height of 15 m. [20%]

(b) Plot the profiles of pore water pressure through the clay immediately before and after the caisson is filled with rock-fill to a depth of 10 m, and in the long term, making carefully explained assumptions. Deduce the excess pore pressures that must dissipate by consolidation, and illustrate the process with a sketch of appropriate isochrones. [20%]

(c) Tube samples recovered from mid-depth in the clay showed a bulk unit weight of 20 kN/m^3. Oedometer tests indicated an effective preconsolidation pressure $\sigma'_c = 200$ kPa. Samples consolidated from 25 kPa to 125 kPa gave a confined modulus $E_o = 3300$ kPa and a coefficient of consolidation $C_v = 5$ m^2/year. Treating this data as representative of the clay layer as a whole, estimate the ultimate settlement of the caisson after filling to a depth of 10 m, and its value 6 months after filling. Explain whether your estimate is likely to be on the high or low side. [20%]

(d) It is suggested that the consolidation process might be speeded up if the caisson were initially filled to a depth of 15 m, with the extra surcharge being removed after a few months. The engineer realised that further oedometer test data would ideally be requested. Nevertheless, the engineer was prepared to extrapolate the existing data to derive new estimates of the required parameters. Do likewise, and advise on the required period of surcharging. [40%]

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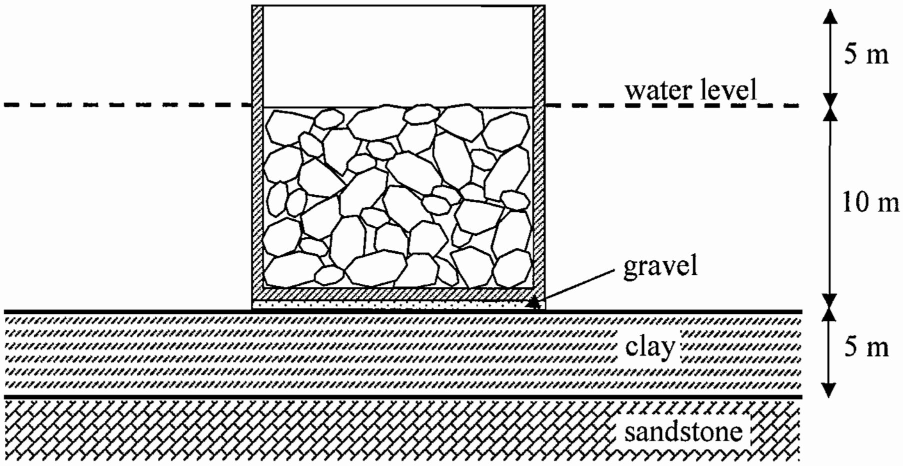

Fig. 1

2 (a) Use diagrams to show paths in (τ, σ', v) space for both "drained" and "undrained" tests on "loose" and "dense" soils. Thereby, define and explain the inter-relationship between "friction" and "cohesion" in soils. [50%]

(b) In the following cases, the elementary Cam Clay model is found to be inadequate. In each case explain the nature and practical impact of the macroscopic phenomenon referred to, suggest an appropriate micro-mechanism, and indicate whether the Cam Clay model might be extended to encompass it.

(i) anisotropy of soft clays,
(ii) sensitivity of "quick" clays,
(iii) progressive failure of overconsolidated clays,
(iv) seismic liquefaction of fine sands. [50%]

3 Figure 2 shows the outline of a proposed structure to buffer an existing mid-harbour bridge pier against potentially catastrophic collisions from ships. The structure will consist of a circular cofferdam formed of steel sheet piles driven concentrically around the pier, the annulus being filled with granular material. Design calculations will be based on sea level coinciding with the surface of the fill. The maximum resisting force of the buffer will chiefly depend on the resistance to deformation of the granular fill which will consist of uniform, medium grain size, sub-angular, quartz sand compacted to a relative density $I_D = 0.75$ corresponding to a saturated bulk density of 20 kN/m^3. Two different modes of deformation are to be considered, as shown: P – a passive mode analogous to a triaxial extension test failure, and S – a simple shear mode. The actual failure mode will depend on the height to diameter ratio (*H/D*) of the buffer, and also on the pattern of force applied by the colliding ship. Therefore, both modes P and S need to be investigated.

(a) Assuming that the granular fill remains fully drained, use appropriate values of peak friction angle to estimate the maximum passive resistance $\sigma'_{h,max}$ for mode P, and the maximum shear resistance τ_{max} for mode S, of soil elements at depths of 0, 5 m and 10 m inside the buffer. [60%]

(b) Estimate corresponding fully drained resistances based alternatively on the critical state angle of friction. [10%]

(c) Explaining your reasoning, select a design value of the fully drained resistance F in MN, for the particular case $H = D = 10$m. [10%]

(d) Discuss the proposition that the fill will not have time to drain during a ship collision, and that its design resistance F will be much greater than the estimate derived in (c). Make some appropriate calculations. [20%]

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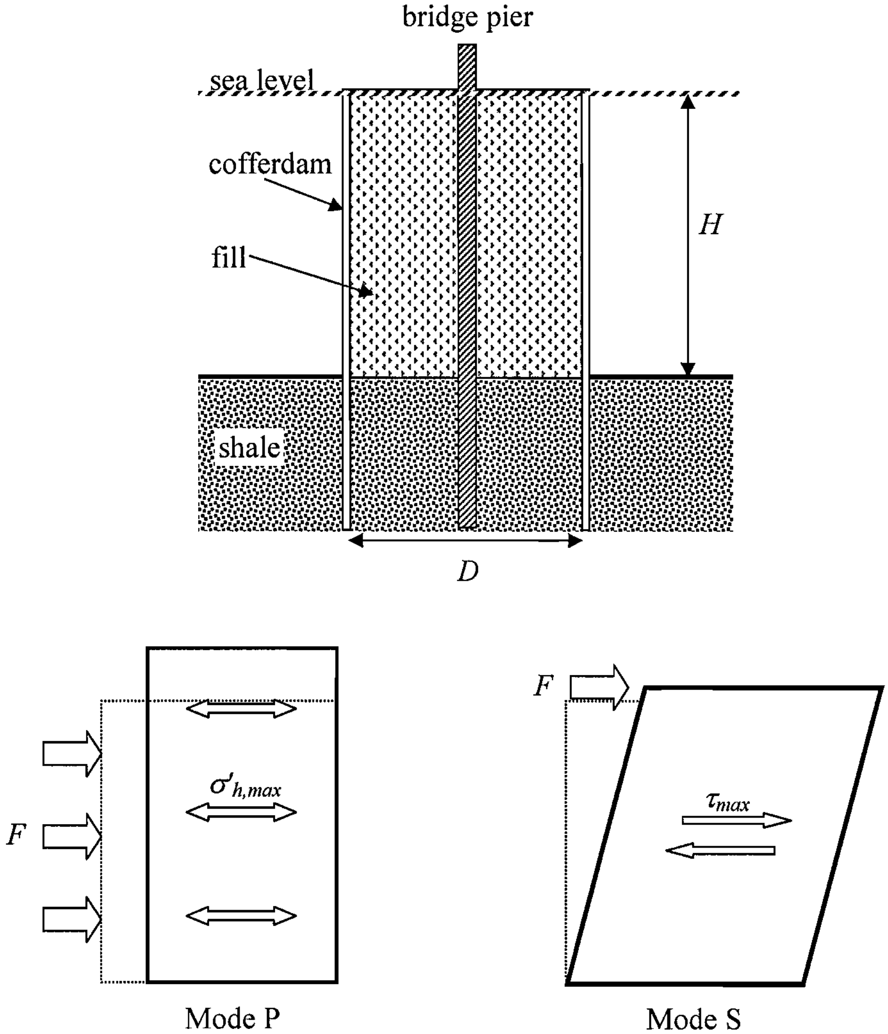

Fig. 2

(TURN OVER

4 The surface of a stratum of soft clay in the field is currently covered by shallow water, but it is thought that this current surface was produced by erosion many years ago. A saturated core of the clay was taken from a depth of 5 m and trimmed to suit a Simple Shear Apparatus capable of either compressing cylindrical soil samples along their axis or shearing them on planes perpendicular to their axis so as to maintain their cross-sectional shape. The initial sample height was 40 mm and its diameter was 100 mm. Trimmings gave a water content of 69% and a specific gravity for the grains of 2.70. The sample was then compressed one-dimensionally in several drainage stages to give the following data of height versus vertical effective stress.

σ'_v kPa	15	30	45	60	75	90
h mm	40.00	39.56	39.28	39.09	38.31	37.68

At the end of this process the sample was sheared very slowly at a constant vertical stress of 90 kPa while more pore water was allowed to drain out to atmospheric pressure. The shearing continued until the sample height had stopped changing; it was then 34.86 mm, and the ultimate shear stress was 44 kPa.

(a) Assuming that a critical state had finally been reached, find ϕ_{crit}. Plot compression data on axes of v versus $\ln\sigma'$. Find best-fit Cam Clay parameters Γ, κ and λ. [40%]

(b) Estimate the pre-consolidation pressure of the sample in the field, and deduce the magnitude of the reduction of effective overburden pressure which it must have experienced. You may neglect deviations of in situ density from that which can be deduced for the sample. Find an expression for the overconsolidation ratio n and the specific volume v of the clay in the field as a function of depth. [40%]

(c) Predict the undrained strength profile of the clay in situ from 0 to 10 m depth. Plot a few points, and comment on the shape of the curve. [20%]

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Soil Mechanics Data Book

Data Book 2005/2006

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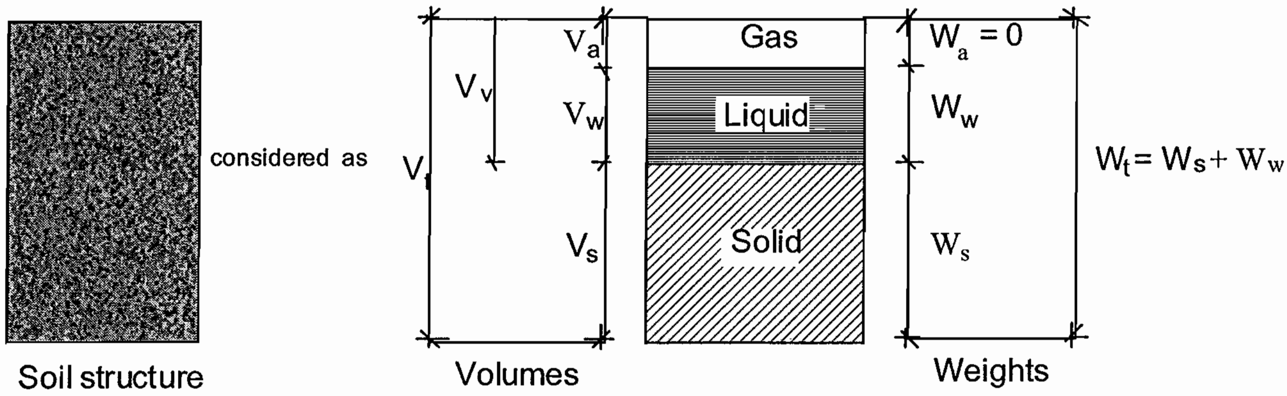
General definitions



Specific gravity of solid G_s

Voids ratio $e = V_v / V_s$

Specific volume $v = V_t / V_s = 1 + e$

Porosity $n = V_v / V_t = e/(1 + e)$

Water content $w = (W_w / W_s)$

Degree of saturation $S_r = V_w / V_v = (w\,G_s/e)$

Unit weight of water $\gamma_w = 9.81\ \text{kN/m}^3$

Unit weight of soil $\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right)\gamma_w$

Buoyant saturated unit weight $\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right)\gamma_w$

Unit weight of dry solids $\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right)\gamma_w$

Air volume ratio $A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$

(at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

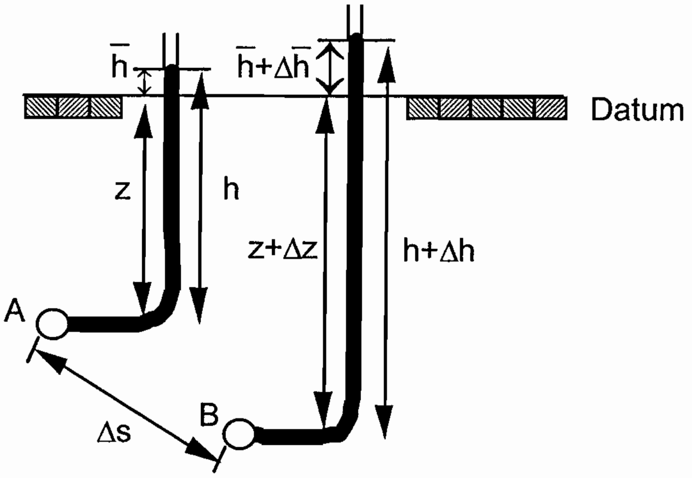
D — equivalent diameter of soil particle

D_{10}, D_{60} etc. — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U — uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta \bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla \bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} > 1\mu$m	:	$k \cong 0.01$ (D_{10} in mm)^2 m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

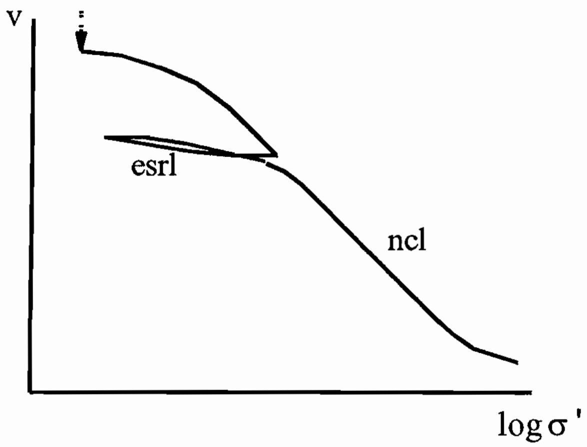
$$h_c = \frac{4T}{\gamma_w d}$$: capillary rise in tube diameter d, for surface tension T

$$h_c \approx \frac{3\times10^{-5}}{D_{10}} \text{ m}$$: for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

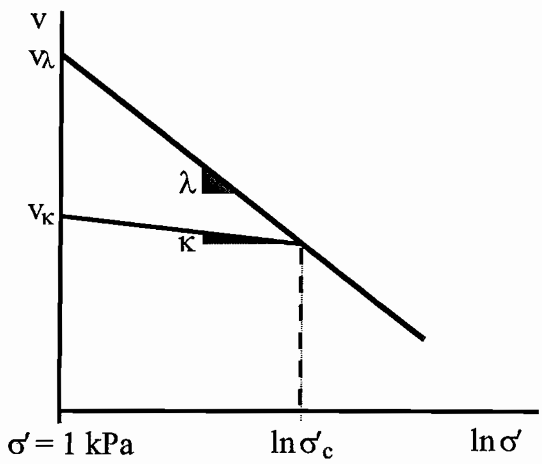
One-Dimensional Compression

- **Fitting data**

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

- **Deriving confined soil stiffnesses**

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

Tangent 1D plastic compression modulus $E_o = v\,\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\,\sigma' / \kappa$

One-Dimensional Consolidation

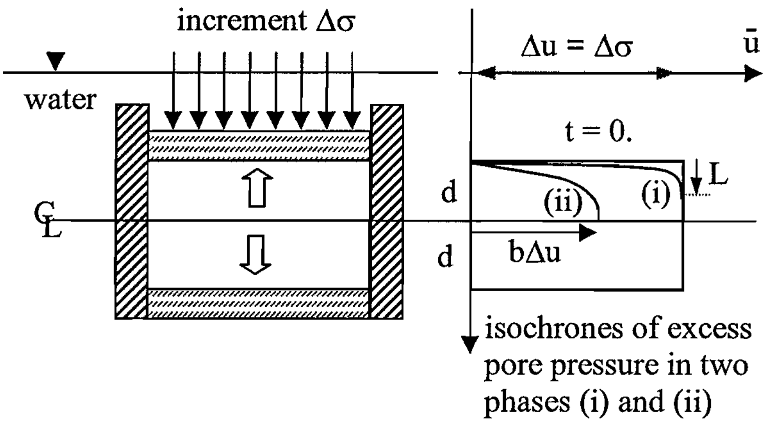
Settlement $\rho = \int m_v(\Delta u - \bar{u})\,dz = \int (\Delta u - \bar{u}) / E_o\,dz$

Coefficient of consolidation $c_v = \dfrac{k}{m_v \gamma_w} = \dfrac{kE_o}{\gamma_w}$

Dimensionless time factor $T_v = \dfrac{c_v t}{d^2}$

Relative settlement $R_v = \dfrac{\rho}{\rho_{ult}}$

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\,c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

- **Principal components of stress and strain**

sign convention	compression positive
total stress	σ_1, σ_2, σ_3
effective stress	σ_1', σ_2', σ_3'
strain	ε_1, ε_2, ε_3

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	s	$= (\sigma_1 + \sigma_3)/2$
mean effective stress	s'	$= (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	t	$= (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	ε_v	$= \varepsilon_1 + \varepsilon_3$
shear strain	ε_γ	$= \varepsilon_1 - \varepsilon_3$
work increment per unit volume	δW	$= \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	δW	$= s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

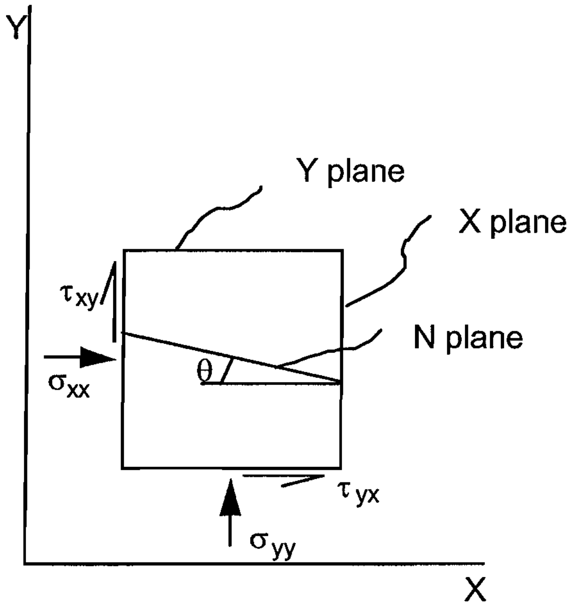
isotropic compression in which p' increases at zero q

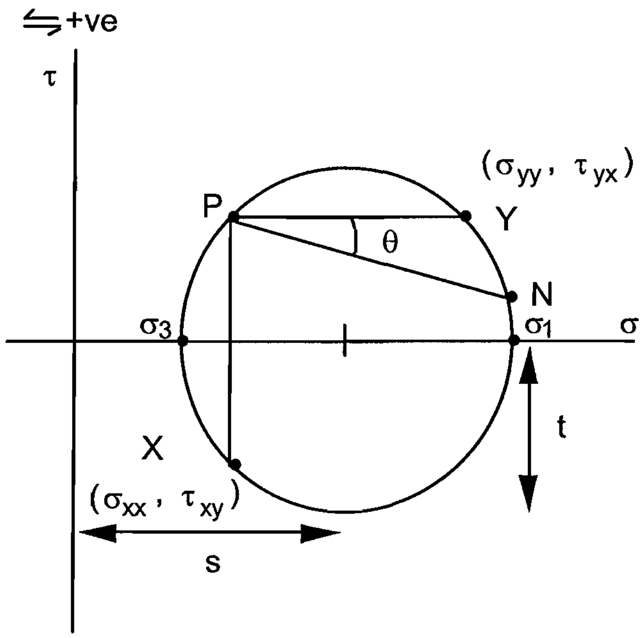
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

- **Interchangeable parameters for stress combinations at yield, and plastic strain increments**

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

- **General equations of plastic work**

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*} . \dfrac{d\gamma^*}{d\varepsilon^*} = -1$

- **General yield surface**

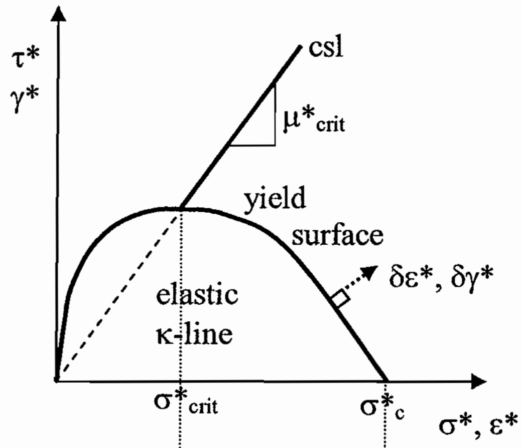
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit} . \ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

- **Parameter values which fit soil data**

	London Clay	**Weald Clay**	**Kaolin**	**Dog's Bay Sand**	**Ham River Sand**
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	-------------	-------------
w_P	0.26	0.18	0.42	-------------	-------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

- **The yield surface in (σ*, τ*, v) space**



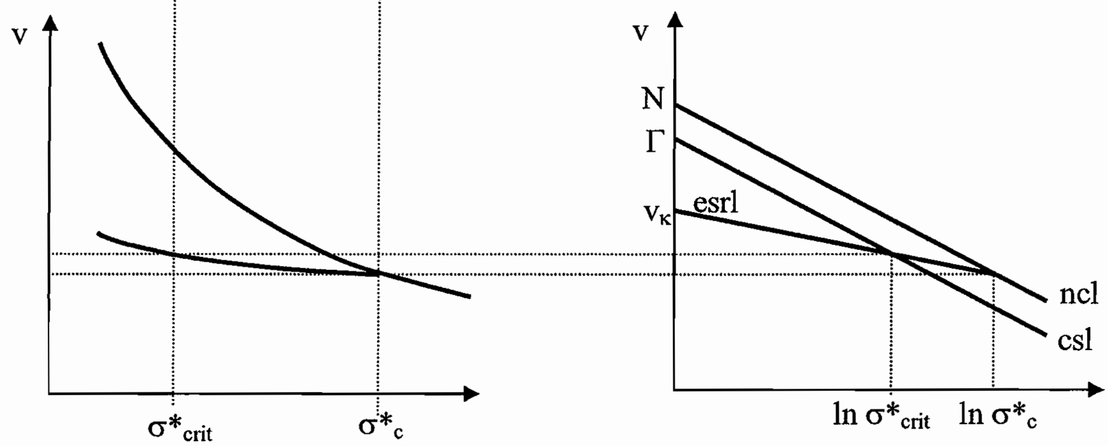
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

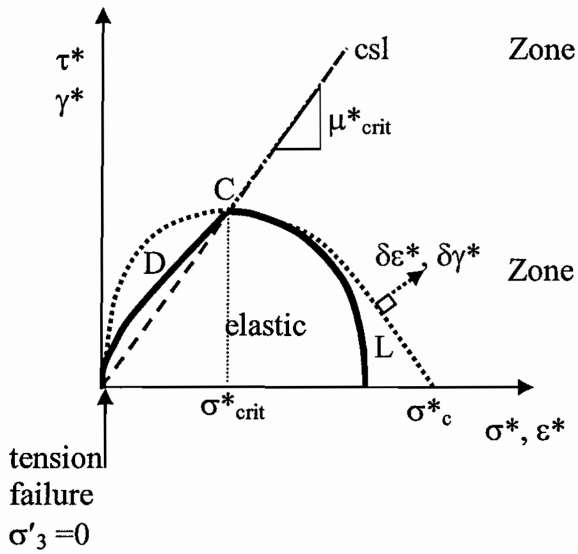
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



- **Regions of limiting soil behaviour**



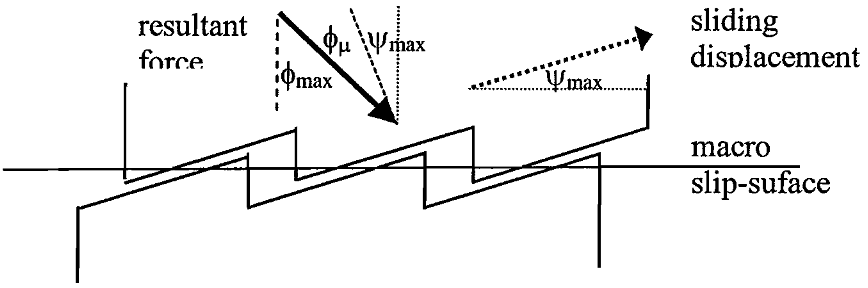
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

- **Friction and dilatancy: the saw-blade model of direct shear**

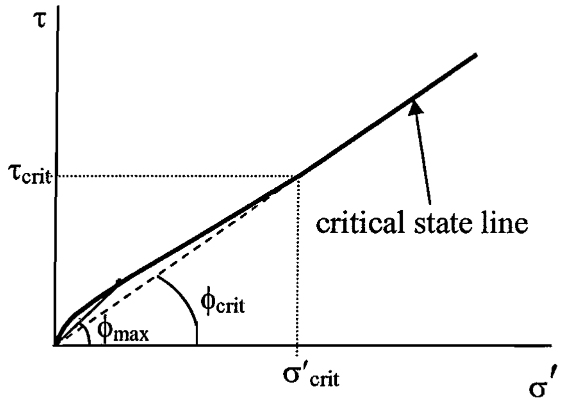


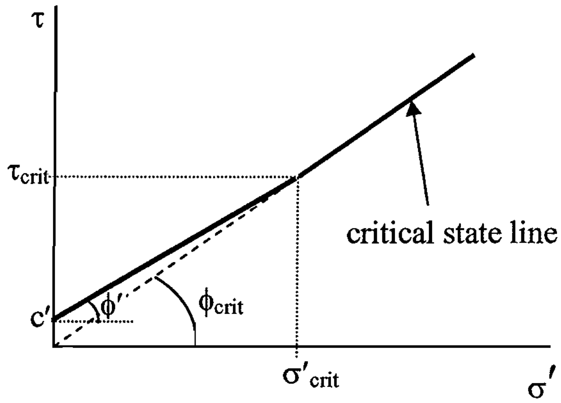
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

- **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f\,(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f\,(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

- **Friction and dilation: data of sands**

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $$I_D = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

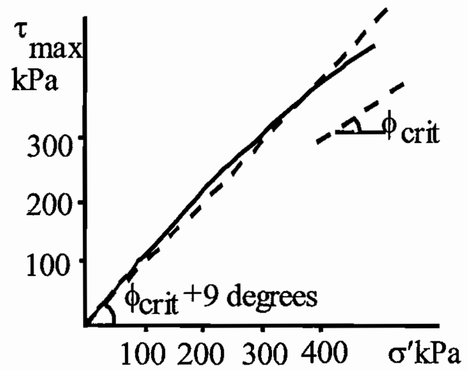
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

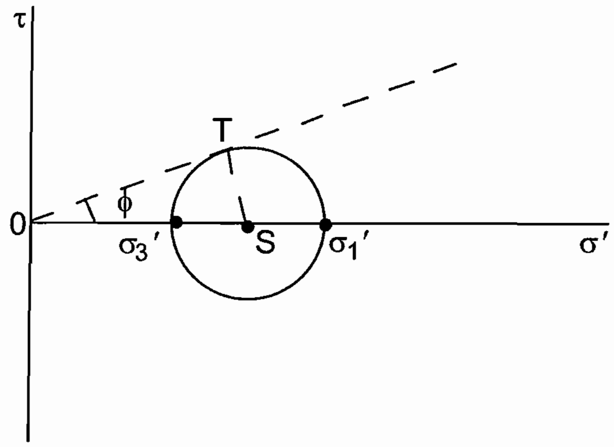
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1, \sigma' = 400$ kPa

• **Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane**



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

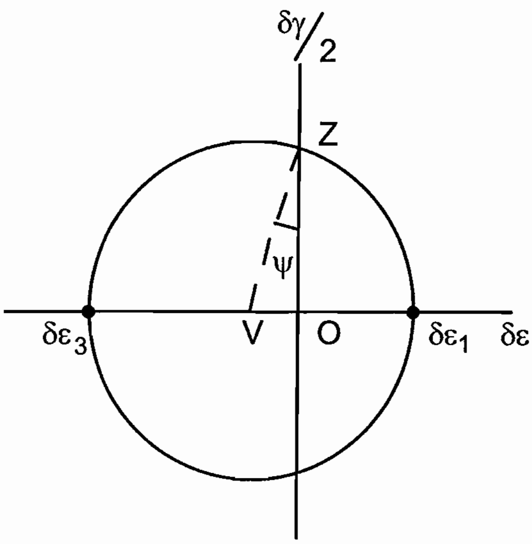
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• **Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane**



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3}\, q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

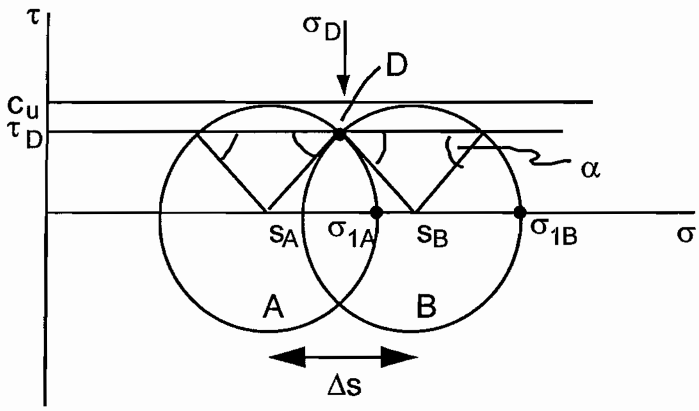
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u\, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

- **Stress conditions across a discontinuity**

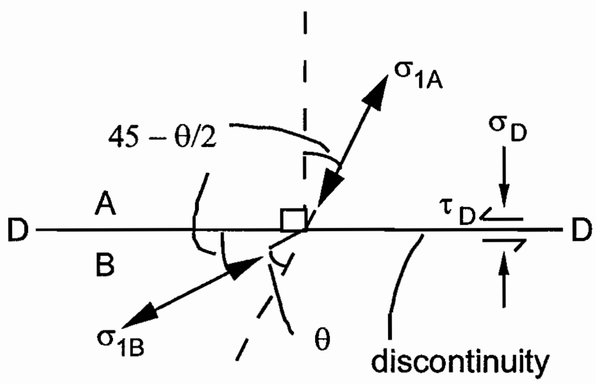


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u\, d\theta$$



Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = c_u$$

$$\tau_D / c_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi'$

• Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

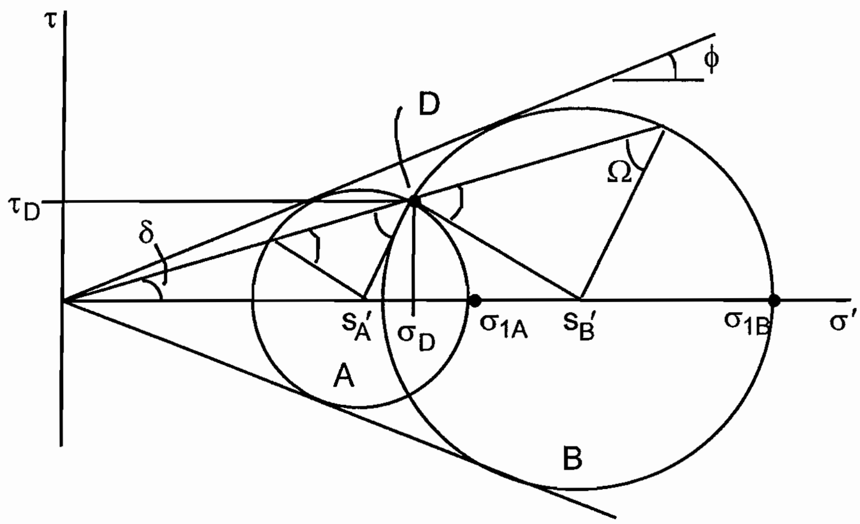
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

• Stress conditions across a discontinuity



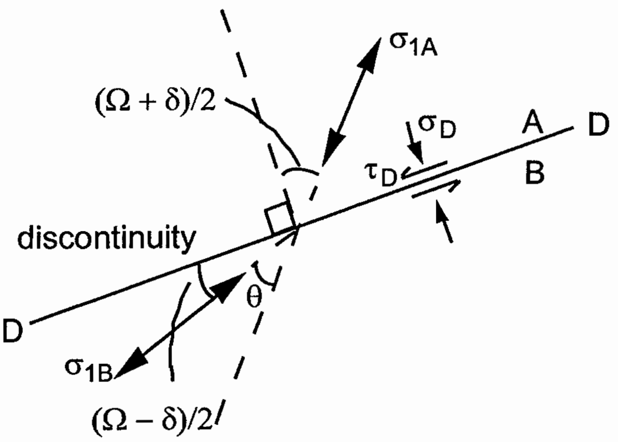
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha} - 1)}{(n_{max} - 1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \qquad \text{(Prandtl, 1921)}$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2\, B / L$$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$$d_c = 1 + 0.33 \tan^{-1}(h/B) \qquad \text{(or h/D for a circular foundation)}$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $H = H_{ult} = Bs_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0 \qquad \text{(Taiebet \& Carter 2000)}$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

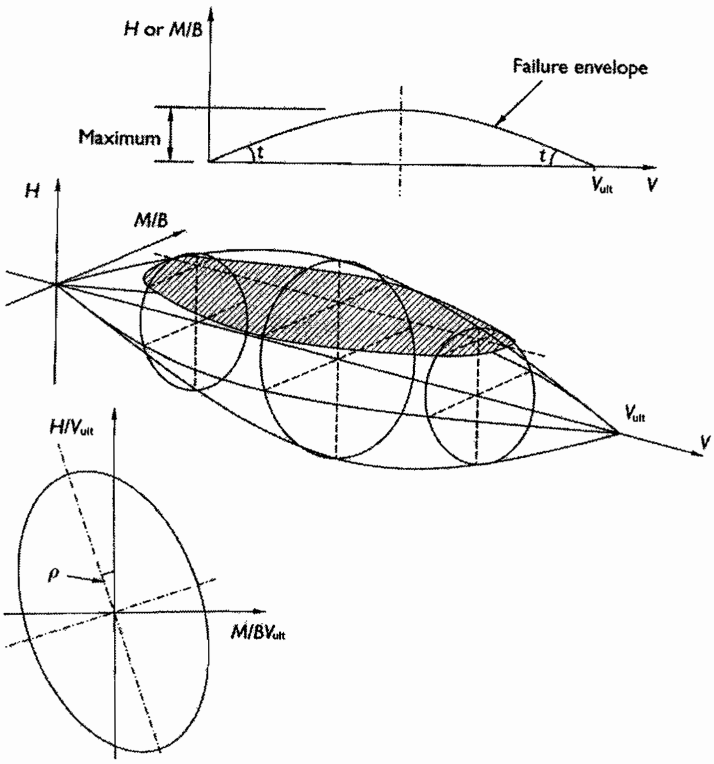
An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$



Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\dfrac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, $H/V = \tan\phi$, during sliding.

3D1 2006 Answers

1 (a) 19.2 kN/m3 (i) 94 kPa (ii) 169 kPa
(b) 94 kPa
(c) 0.142 m, 0.099 m
(d) 5 months

3 (a) $\sigma'_{h,max}$ = 0, 272 kPa, and 516 kPa approximately at 0, 5 and 10 m respectively
τ_{max} = 0, 68 kPa, and 123 kPa approximately at 0, 5 and 10 m respectively
(b) σ'_{hcrit} = 0, 193 kPa, and 385 kPa approximately at 0, 5 and 10 m respectively
τ_{crit} = 0, 36 kPa, and 73 kPa approximately at 0, 5 and 10 m respectively
(c) 5.7 MN in Mode S
(d) 17.1 MN in Mode S, based on cavitation

4 (a) 26°, 3.605, 0.047, 0.246
(b) 60 kPa, 31 kPa, $n = 1 + 5.3/z$, $v = \Gamma + \lambda - \kappa - \lambda \ln(5.8z + 31) + \kappa \ln(1 + 5.3/z)$
(c) $c_u \approx 4 + 1.4z$ kPa