ENGINEERING TRIPOS PART IIA

Thursday 11 May 2006 2.30 to 4.00

Module 3D2

GEOTECHNICAL ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Special datasheet (19 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 5

1 Figure 1 outlines a mechanism that can be used to predict the short-term stresses and strains in the ground due to the excavation of an unlined cylindrical tunnel of radius a, with its axis at depth b within uniform clay of undrained shear strength c_u and unit weight γ. A circular segment around the vertical plane of symmetry can be considered to be in an approximate state of cylindrical cavity reduction.

(a) Using a free body diagram, show that the differential equation for equilibrium inside the segment is:

$$\frac{d\sigma_r}{dr} + \frac{(\sigma_r - \sigma_\theta)}{r} = -\gamma$$

where σ_r is the radial stress, σ_θ is the circumferential stress and r is the radial distance from the centre of the cavity. [20%]

(b) Develop expressions for the shear strain $\varepsilon_{\gamma,a}$ of the clay at the cavity boundary in terms of the depression ρ_a at the cavity crown, and for shear strain ε_γ at any radius as a function of $\varepsilon_{\gamma,a}$. [20%]

(c) The shear stress-strain relation for the clay prior to ultimate shearing at $\tau = c_u$ for $\varepsilon_\gamma < \varepsilon_{\gamma,f}$ can be taken to be:

$$\frac{\tau}{c_u} = \left[\frac{\varepsilon_\gamma}{\varepsilon_{\gamma,f}}\right]^\beta$$

Derive an expression linking maximum ground settlement ρ_b to parameters a, b, γ, c_u, β and $\varepsilon_{\gamma,f}$. [50%]

(d) Simplify this for the particular case $\beta = 0.5$, and use this to estimate the maximum settlement caused by boring an unlined 5 m diameter tunnel with its axis at 10 m depth in clay with a shear strength of 100 kPa mobilised at a shear strain of 2%.

[10%]

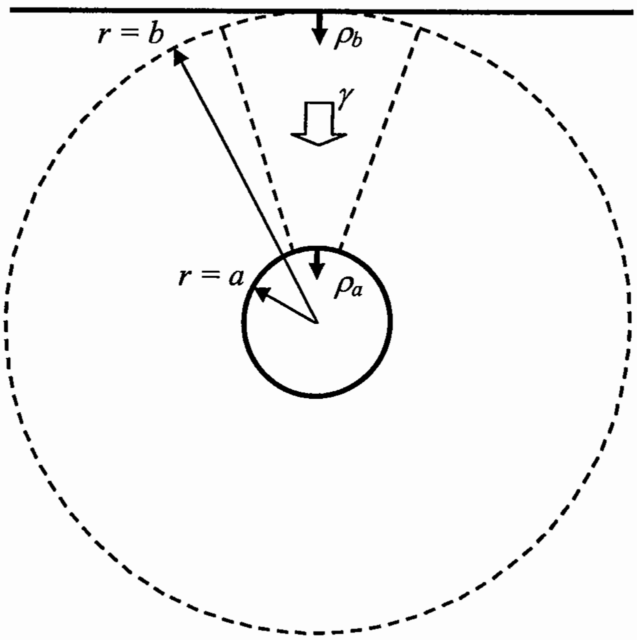


Fig. 1

(TURN OVER

2 (a) A clay has properties closely resembling those of London Clay given in the Data Book. A triaxial element of the clay is one-dimensionally normally consolidated from a slurry state O′ to state A′ (σ_v' = 200 kPa) and is then permitted to swell one-dimensionally to state B′ (σ_v' = 50 kPa). Calculate the corresponding horizontal effective stresses at A′ and B′ and sketch the state path O′→A′ →B′ on both (σ_v', σ_h') and (q, p') diagrams. Mark critical state stress ratios on both diagrams, distinguishing in each case between compression and extension. [30%]

(b) When the soil at B′ is subjected to an undrained compression test it remains quasi-elastic up to a deviatoric stress q = 65 kPa at state C′ before yielding, and ultimately shears in state D′ at constant q_u = 70 kPa. Mark these state points on both (σ_v', σ_h') and (q, p') diagrams. [30%]

(c) The same soil is at effective stress state B′ in the field at a depth of 2 m, with a water table at a depth of 3 m below the horizontal surface of the clay. A 3 m deep vertical cut is excavated. The clay adjacent to the cut at a depth of 2 m comes to a new effective stress state E′. Show total stress states B and E, and effective stress state E′, on each of the (σ_v', σ_h') and (q, p') diagrams. [25%]

(d) Discuss the stability of the face in the long term, extending your stress paths as appropriate. [15%]

3 The offshore structure shown in Fig. 2(a) is supported on 2 strip foundations, of width, b, and length, l, which can be idealised as imposing plane strain ($l >> b$). The seabed is uniform clay, with undrained strength c_u. The vertical load due to the weight of the structure, $V = 3blc_u$.

(a) If the foundation-leg connections are idealised as pin-joints, and the foundations cannot sustain tension, calculate the horizontal load, H, applied at a distance, a, above the seabed, which will cause undrained failure when the vertical load, V, acts centrally. You may ignore interaction between the foundations, and assume that the foundations mobilise equal horizontal reactions. State the mode of failure, and draw the load paths on a V-H interaction diagram. [35%]

(b) To increase the maximum horizontal load that can be resisted, the weight of the structure can be increased and redistributed by pumping water into ballast tanks. Calculate the value of H that will cause failure, if the weight is increased to $V = 4blc_u$, and the line of action of V is moved by a distance c, (see Fig. 2(a)), where $c = 3a/8$. State the mode of failure, and draw the load paths on a V-H interaction diagram. [35%]

(c) To upgrade the foundation capacity, each strip foundation is equipped with an impermeable skirt, penetrating a distance $b/2$ into the seabed as shown in Fig. 2(b). Explain two mechanisms by which this modification will alter the foundation capacity. [15%]

(d) Write an expression for the pure vertical load that will cause failure of the modified structure. [15%]

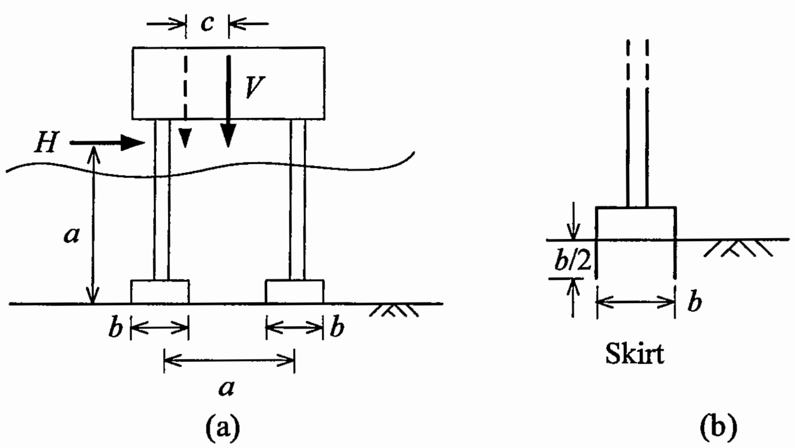


Fig. 2

4 The rigid cantilever gravity wall shown in Fig. 3 is made from reinforced concrete and is embedded in dry sand. The sand has a friction angle, $\phi = 35°$, and a unit weight of $\gamma = 20$ kN m^{-3}. The wall has a unit weight of $\gamma_{wall} = 25$ kN m^{-3}. The base of the wall has a concrete-soil friction angle of $\delta_{conc} = 20°$.

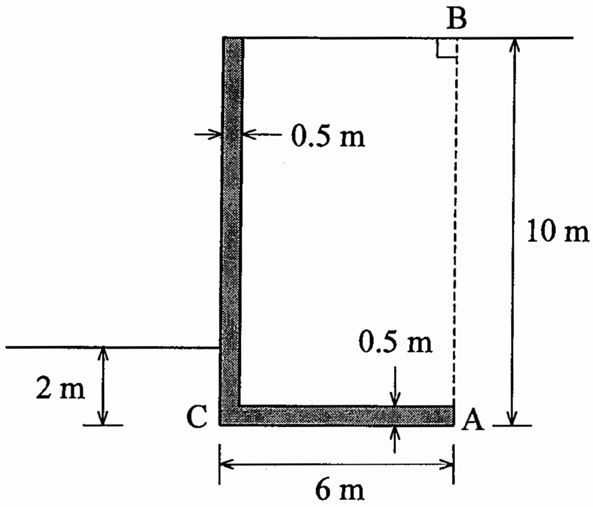


Fig. 3

(a) Using Rankine's lower bound method, and assuming that the wall has a 'virtual back' AB, calculate the net overturning moment acting about C due to the lateral earth pressures. [20%]

(b) By considering the restoring moments due to the weight of the wall and the enclosed soil, calculate the factor of safety against overturning failure about C. [25%]

(c) Calculate the effective contact width, using Meyerhof's method. Discuss and illustrate, using a failure envelope, how the foundation AC of the wall should be checked for stability under this combination of loads. [30%]

(d) Explain why the active pressure at failure could be less than the value calculated in part (a). Sketch and describe an alternative stress field behind the 'virtual back' of the wall, but do not perform any calculations. [25%]

END OF PAPER

Engineering Tripos Part IIA

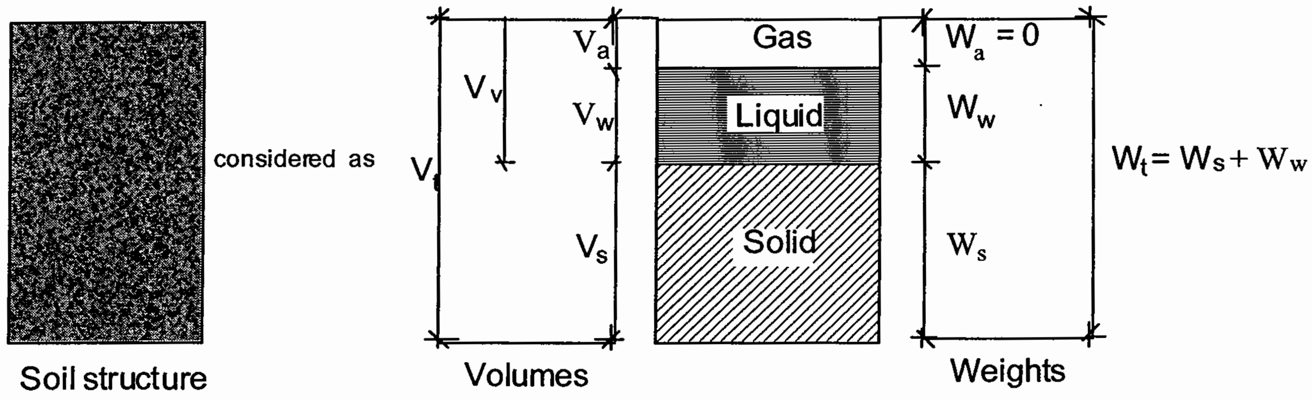
3D1 & 3D2
Soil Mechanics Data Book

Data Book 2005/2006

Contents **Page**

Contents	Page
General definitions	2
Soil classification	3
Seepage	4
One-dimensional compression	5
One-dimensional consolidation	6
Stress and strain components	7, 8
Elastic stiffness relations	9
Cam Clay	10, 11
Friction and dilation	12, 13, 14
Plasticity; cohesive material	15
Plasticity; frictional material	16
Empirical earth pressure coefficients	17
Cylindrical cavity expansion	17
Shallow foundation capacity	18, 19

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\frac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit	w_L
Plastic Limit	w_P
Plasticity Index	$I_P = w_L - w_P$
Liquidity Index	$I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than 2 }\mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

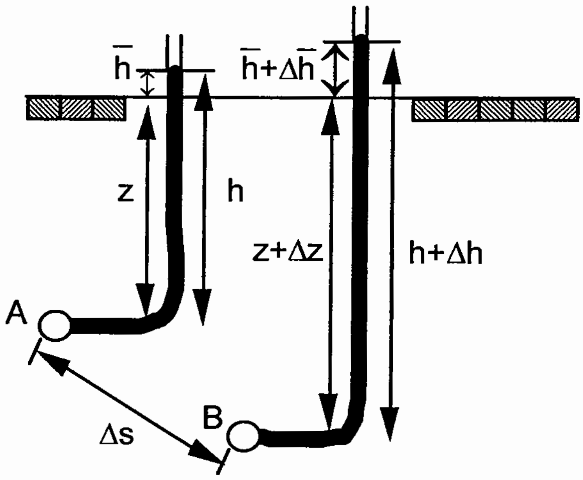
Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc.	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.
C_U	uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} > 1\mu$m	:	$k \cong 0.01\,(D_{10}$ in mm$)^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

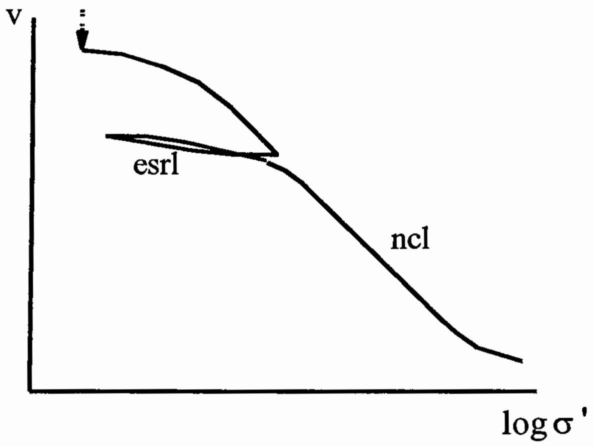
$$h_c = \frac{4T}{\gamma_w d}$$: capillary rise in tube diameter d, for surface tension T

$$h_c \approx \frac{3\times10^{-5}}{D_{10}}\ \text{m}$$: for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

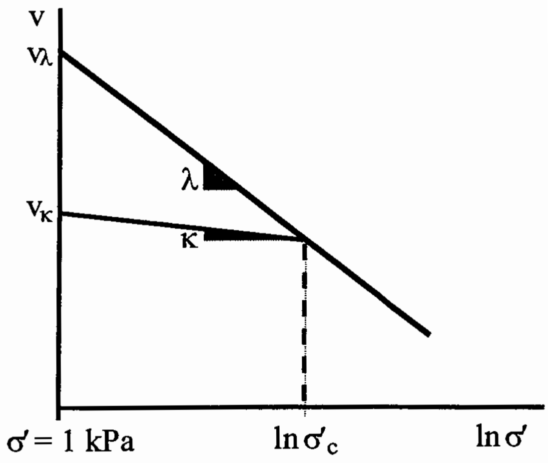
One-Dimensional Compression

- **Fitting data**

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

- **Deriving confined soil stiffnesses**

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

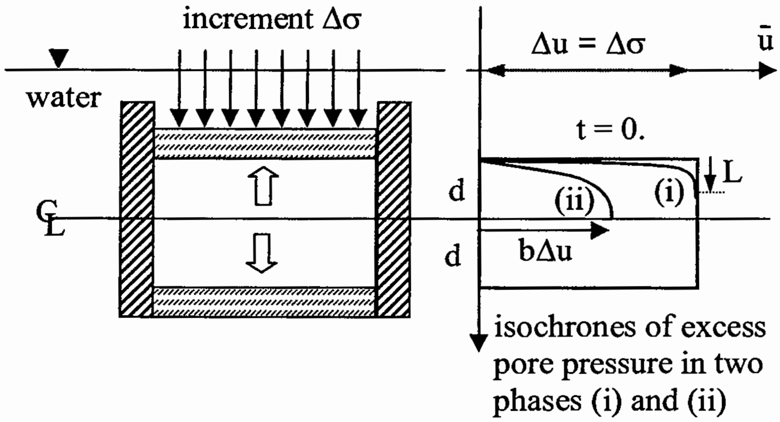
Tangent 1D plastic compression modulus $E_o = v\,\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\,\sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v(\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

- **Principal components of stress and strain**

sign convention	compression positive
total stress	σ_1, σ_2, σ_3
effective stress	σ_1', σ_2', σ_3'
strain	ε_1, ε_2, ε_3

- **Simple Shear Apparatus (SSA)** (ε_2 = 0; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

- **Biaxial Apparatus - Plane Strain (BA-PS)** (ε_2 = 0; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	$= \sigma'_a + u$
total radial stress	σ_r	$= \sigma'_r + u$
total mean normal stress	p	$= (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	$= (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	$= \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	$= q/p'$
axial strain	ε_a	
radial strain	ε_r	
volumetric strain	ε_v	$= \varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	$= \frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	$= \sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	$= p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

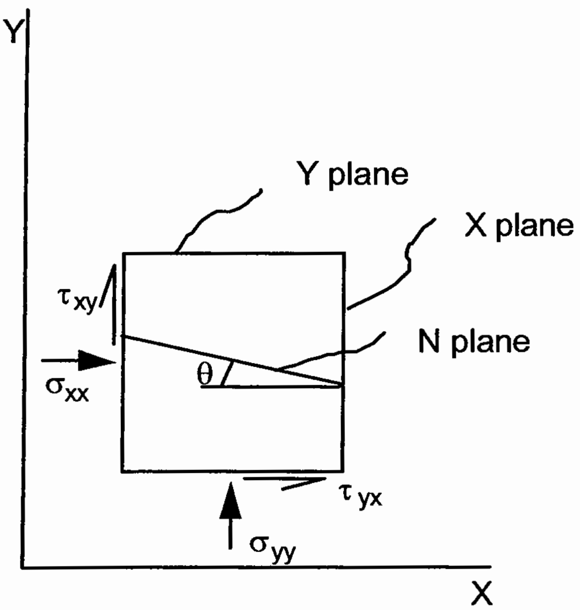
isotropic compression in which p' increases at zero q

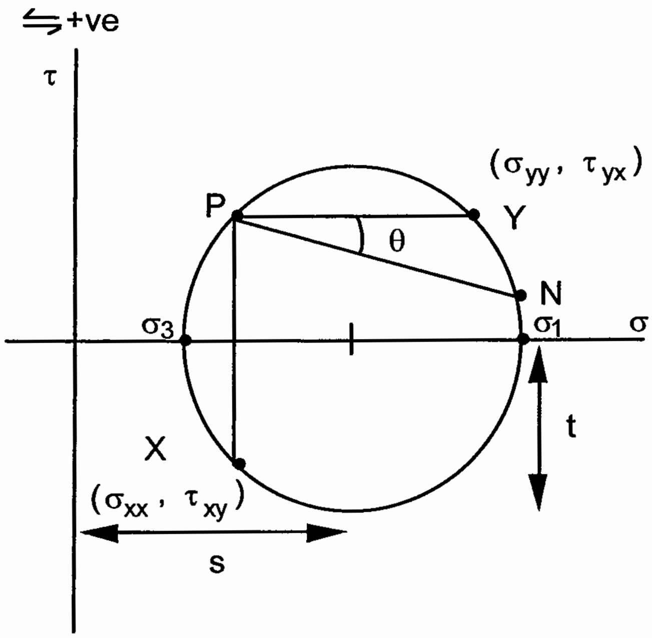
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P: the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

- **Interchangeable parameters for stress combinations at yield, and plastic strain increments**

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

- **General equations of plastic work**

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

- **General yield surface**

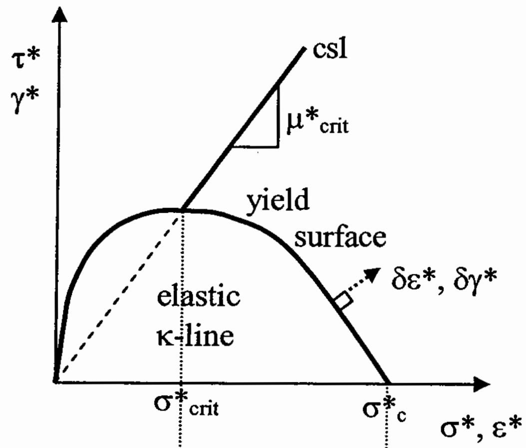
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

- **Parameter values which fit soil data**

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

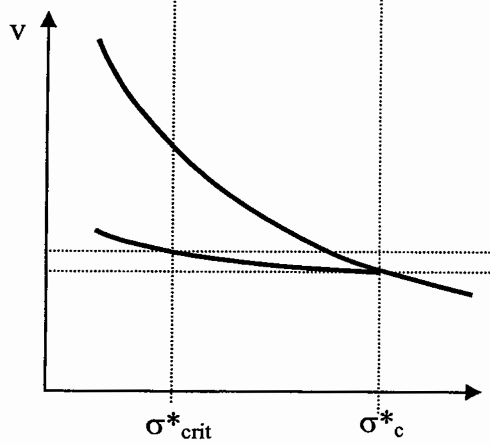
- **The yield surface in (σ*, τ*, v) space**

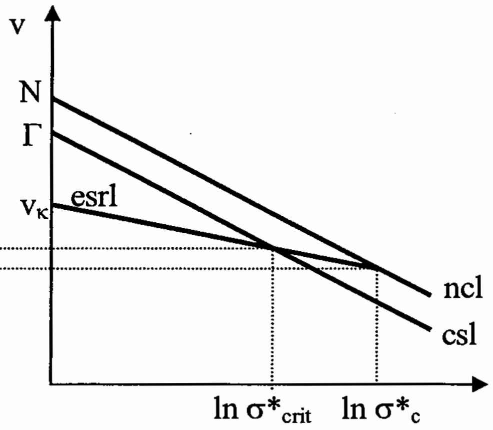


ncl: normal compression line
$$v = N - \lambda \ln \sigma^*$$

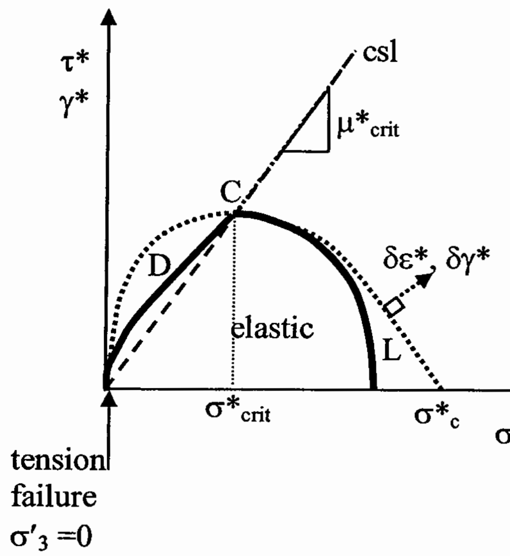
csl: critical state line
$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$





- **Regions of limiting soil behaviour**



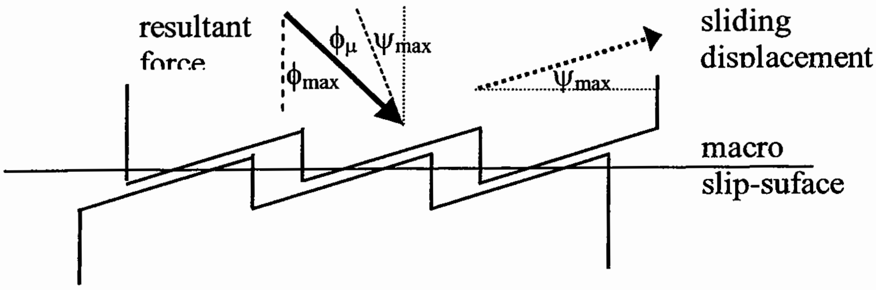
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

- **Friction and dilatancy: the saw-blade model of direct shear**

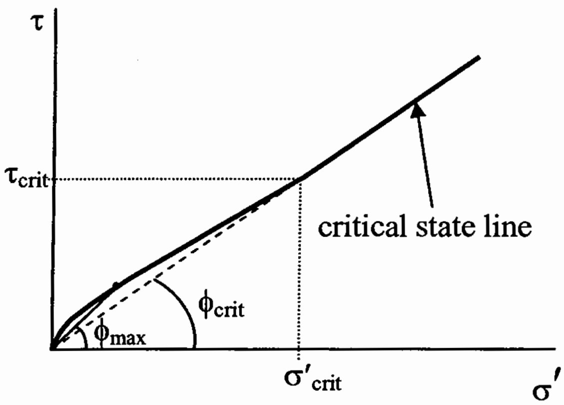


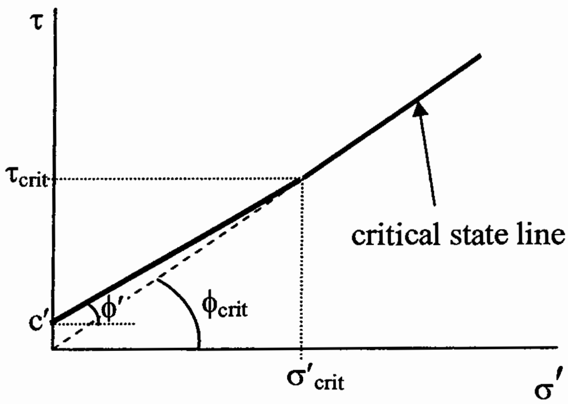
Intergranular angle of friction at sliding contacts ϕ_{μ}

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_{\mu} + \psi_{max}$

- **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^{\alpha}$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\ \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} (± 2°) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

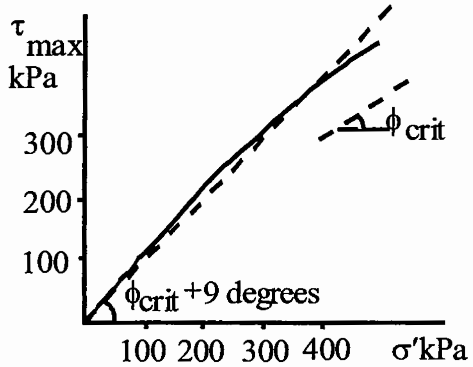
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

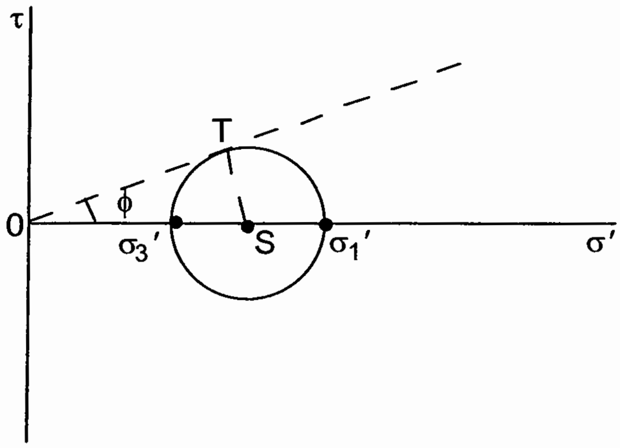
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1,\ \sigma' = 400$ kPa

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

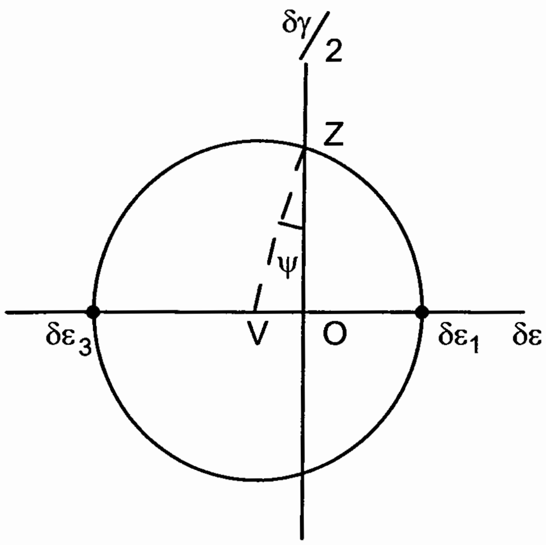
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ = -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2} = -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

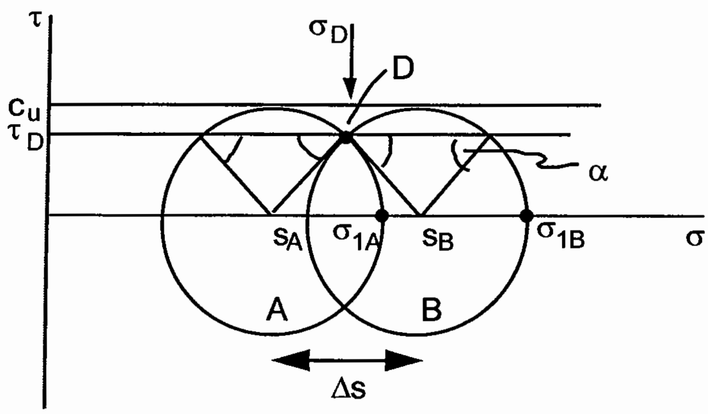
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

- **Stress conditions across a discontinuity**

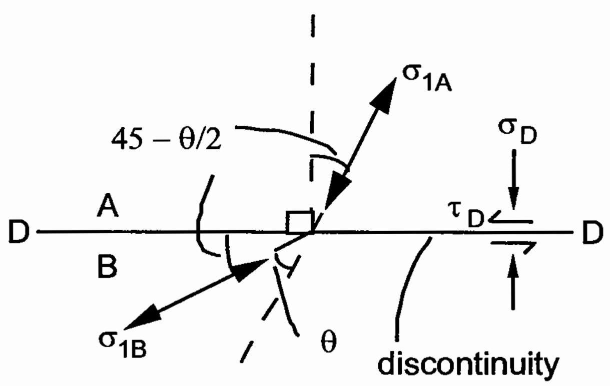


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$$\theta = 30°$$

$$\sigma_{1B} - \sigma_{1A} = c_u$$

$$\tau_D / c_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi'$

- **Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

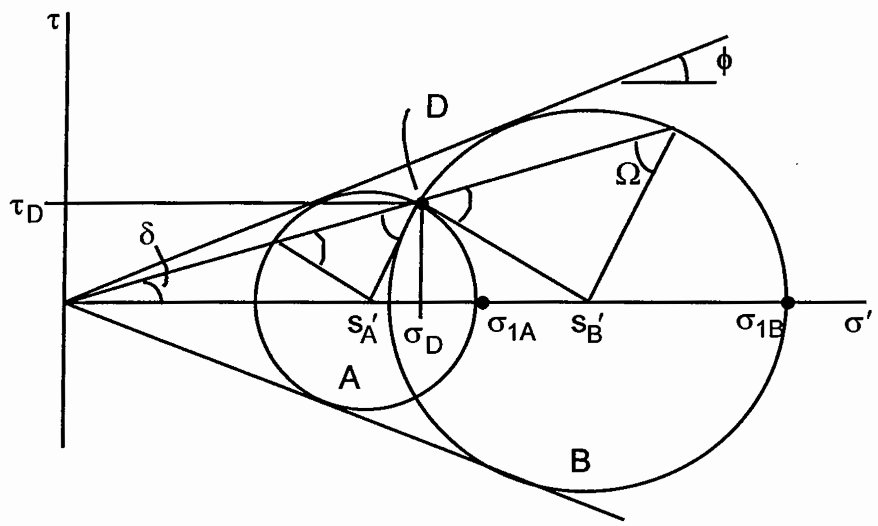
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

- **Stress conditions across a discontinuity**



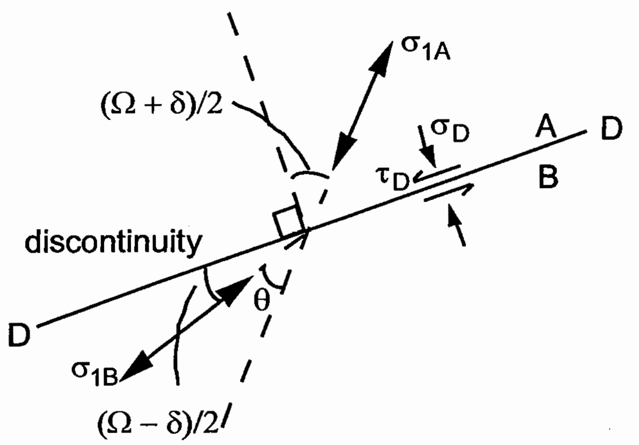
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin\phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha} - 1)}{(n_{max} - 1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin\phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \qquad \text{(Prandtl, 1921)}$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2\, B / L$$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$$d_c = 1 + 0.33 \tan^{-1}(h/B) \qquad \text{(or h/D for a circular foundation)}$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $$H = H_{ult} = Bs_u$$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0 \qquad \text{(Taiebet \& Carter 2000)}$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan\phi)} \qquad \text{(Prandtl 1921)}$$

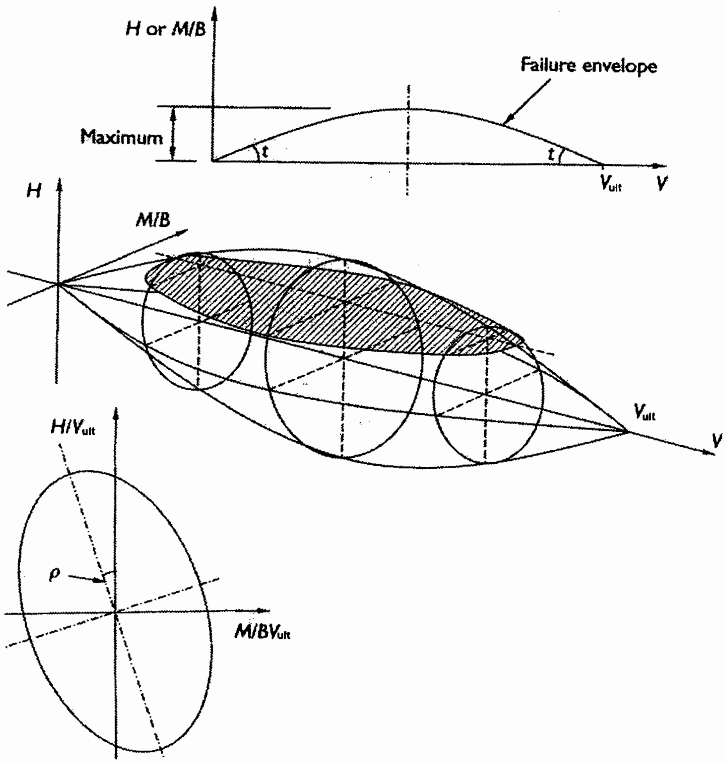
An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1)\tan\phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$



Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B\sin\phi)/L$$
$$s_\gamma = 1 - 0.3\,B/L$$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, $H/V = \tan\phi$, during sliding.

3D2 Geotechnical Engineering
Exam answers 2006

Question 1: Cylindrical cavity collapse analysis of tunneling-induced settlement

(a) Proof

(b) $\varepsilon_{\gamma,a} = -\frac{2\rho_a}{a}$, $\varepsilon_\gamma = -\frac{2\rho_a a}{r^2} = -2\varepsilon_{\gamma,a}\frac{a^2}{r^2}$

(c)
$$\frac{2\rho_b}{b\varepsilon_{\gamma,f}} = \left[\frac{\gamma}{c_u}\beta\frac{b-a}{\left\{\left(\frac{b}{a}\right)^{2\beta}-1\right\}}\right]^{1/\beta}$$

(d) $\rho_b = 6.25$ mm

Question 2: Stress paths in clay, collapse of a vertical cut

(a) A′: $\sigma'_h = 121.8$ kPa, $p' = 147.9$ kPa, $q = 78.2$ kPa
B′: $\sigma'_h = 58.5$ kPa, $p' = 55.6$ kPa, $q = -8.5$ kPa

$K_a = 0.44$, $K_p = 2.28$

(b) C′: $q = 65$ kPa, $p' = 55.6$ kPa, $\sigma'_h = 33.9$ kPa, $\sigma'_v = 98.9$ kPa
D′: $q = 70$ kPa, $p' = 78.9$ kPa, $\sigma'_h = 55.3$ kPa, $\sigma'_v = 125.3$ kPa

(c) B: $q = -8.5$ kPa, $p = 45.6$ kPa, $\sigma_h = 48.5$ kPa, $\sigma'_v = 40$ kPa
E: $q = 40$ kPa, $p' = 20$ kPa, $\sigma_h = 0$ kPa, $\sigma_v = 40$ kPa

Question 3: Combined V-H loading of a two-footing structure

(a) $H = 3blc_u/2$

(b) $H = 2blc_u$

(c) $V = 2bl\,(2+\pi)\,d_c c_u + \gamma' b/2$

Question 4: Stability of a cantilever gravity wall in dry sand

(a) $M_{overturning} = 801$ kNm/m

(b) $M_{restoring} = 3651$ kNm/m , FoS = 4.56

(c) $B' = B - 2e = B - 2(M/V) = 4.6$ m

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May 2006