ENGINEERING TRIPOS PART IIA

Wednesday 10 May 2006 9 to 10.30

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Special datasheets (10 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Give three examples of geometrical ratios that are of importance in the initial design of structures, explaining briefly in each case why they are significant. For one of the ratios, give a more detailed derivation of its limiting value. [25%]

(b) Figure 1 shows a plan view of a balcony at the corner of a large building supported on columns at B, C, D etc. There is no column at A. The floor consists of a uniform concrete slab 200 mm thick, continuous over beams AC, CD, BD etc. whose centrelines coincide with the gridlines – notice that there is no beam AB. The concrete weighs 25 kNm^{-3}, and the required load factors at the ultimate limit state are 1.4 on dead load and 1.6 on live load. In the worst case there is a working live load of 4 kNm^{-2} on the entire balcony plus a downward point live load of 20 kN at A. The self-weight of the beams can be ignored.

(i) For what ultimate hogging bending moment must the edge beam AC be designed, if all the uniform loads on the slab are taken by cantilever action rooted along BD, and the slab provides no assistance in carrying the point load at A? [10%]

(ii) Assume now that top steel is provided all along BD, in direction AB, such that the ultimate moment of resistance per m width of the slab along BD is 70 kNm. If the slab is considered as a series of identical cantilever strips parallel to AB, each providing maximum support to the beam AC, for what hogging moment must AC be designed? Sketch the bending moment diagrams for beam AC and a typical cantilever strip. [35%]

(iii) What other equilibrium checks would you carry out, to arrive at a satisfactory design against the loading on the balcony? Does any member have to be designed to carry torque? What assumptions underlie the use of simplified and apparently arbitrary equilibrium systems of this kind for design purposes, and in what circumstances are these assumptions valid? [30%]

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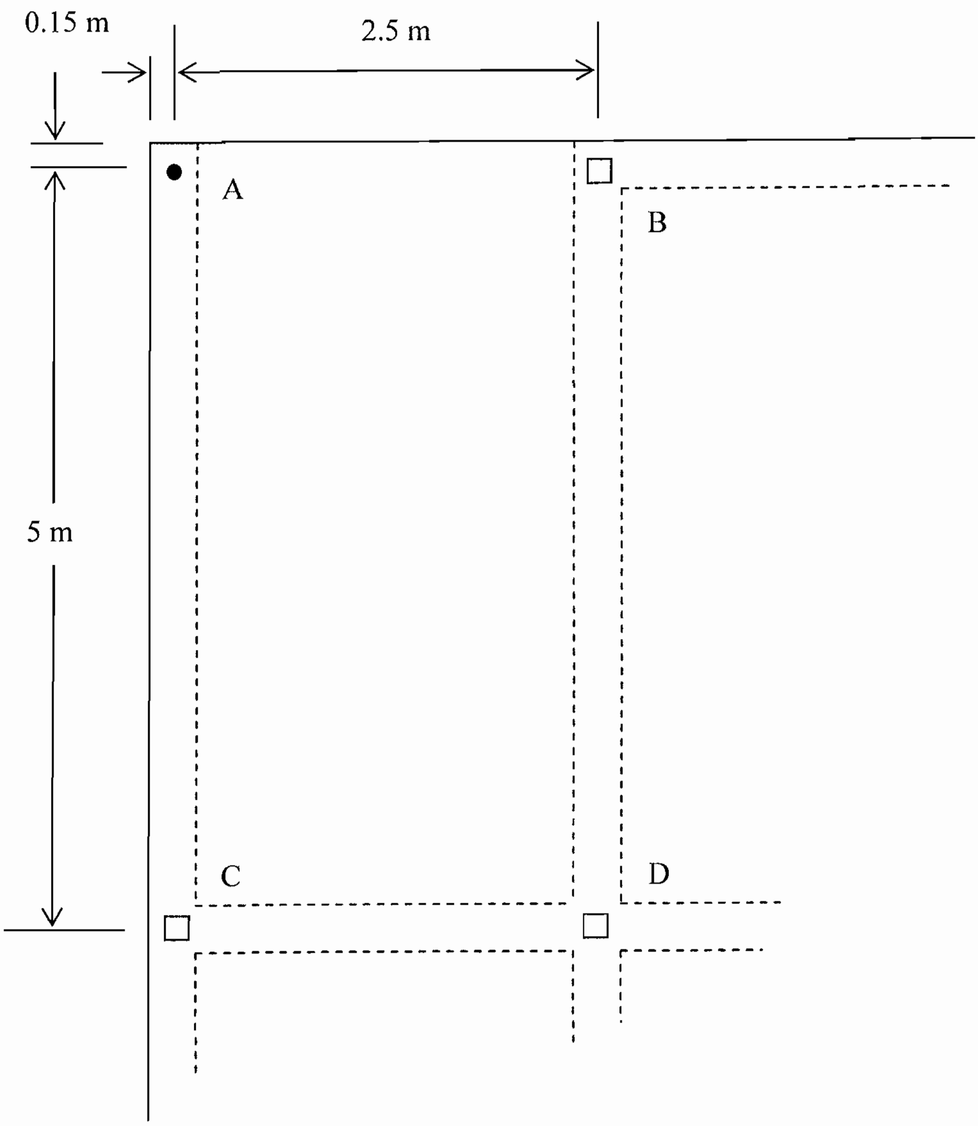


Fig. 1

(TURN OVER

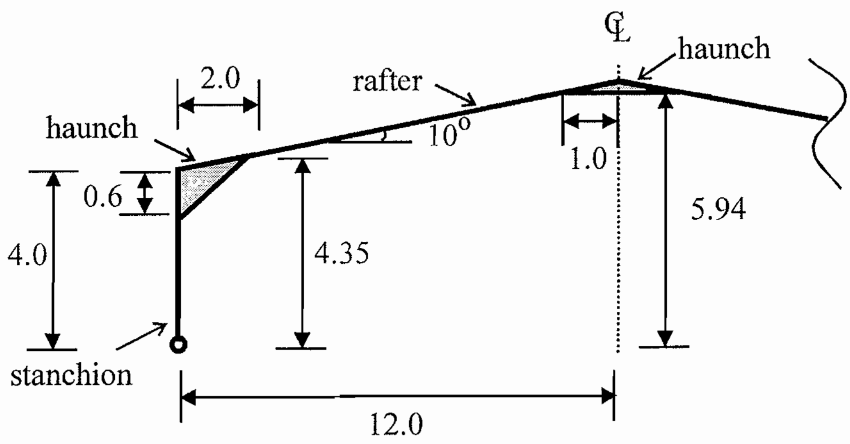
2 (a) A column may be regarded as a vertical cantilever of height 5 m, fixed at its base but otherwise unrestrained. At the ultimate limit state it is to carry an axial compressive force of 1300 kN, applied at the top of the column. Select an efficient Universal Column section in the 305 by 305 group in S355 steel with material safety factor $\gamma_m = 1.05$ (using curve b on the relevant chart). [20%]

(b) Figure 2(a) shows schematically in elevation the main features of a symmetrical single-bay pinned-foot pitched-roof portal frame in S275 steel, with strengthening haunches at the corners. ULS design is governed by vertical loading – a total (including an allowance for self-weight and appropriate load factors on live and dead load) of 10 kN per horizontal metre, uniformly applied to the rafters across the 24 m span.

(i) By considering an appropriate mechanism of deformation, outline briefly, without doing any calculations, how you would find an equilibrium system with equal magnitudes of bending moment in the rafter at the four ends of the haunches. [10%]

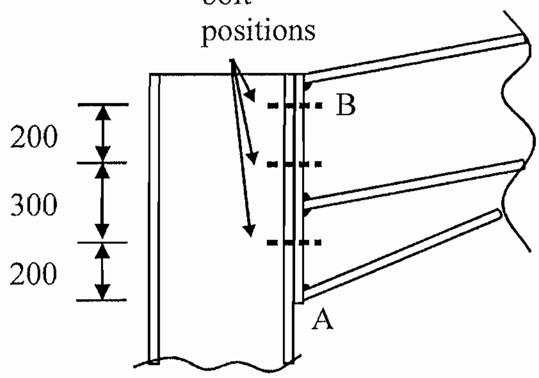
(ii) Assuming that the horizontal reaction at the frame feet is an inward force of 90 kN, sketch the bending-moment diagram for the frame, giving values at salient points. Suggest an initial choice of a section for the stanchion. What further considerations would be required to refine the design? [40%]

(iii) Figure 2(b) shows some features of the bolted joint between the stanchion and a haunched end of the rafter, with its welded-on end plate. Without carrying out any further calculations, briefly discuss what stiffeners and lateral restraints would be needed in this zone, and in the rest of the frame? Make a rough estimate of the force(s) to be transmitted by the bolts at position B. [30%]



all dimensions in m

(a)



all dimensions in mm

(b)

Fig. 2

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3 (a) Briefly discuss the basis for the design plots shown on the 3D3 Advanced Composites Datasheet and explain how the design plots can be used in the initial design of composite structures. [25%]

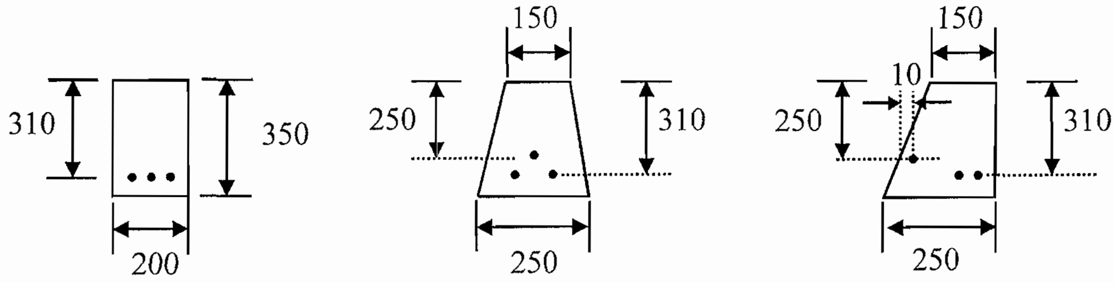
(b) A one-way spanning simply-supported reinforced concrete beam has a span of 9 m. The reinforcing steel consists of three 20 mm diameter bars. The characteristic yield stress of the steel is 460 MPa and the concrete has a characteristic compressive cube strength of 45 MPa. The partial material safety factors for concrete and steel are 1.5 and 1.15 respectively and $\varepsilon_{cu} = 0.0035$ and $\varepsilon_y = 0.002$.

(i) Find the ultimate design moment capacity of a rectangular beam with cross-section shown in Fig. 3(a). The effective depth to the reinforcing steel is 310 mm and the section can be assumed to be under-reinforced. For the given section, would you expect the serviceability limit state to be satisfied? Explain your reasoning but do not carry out any detailed calculations. [20%]

(ii) During casting, the formwork is inadvertently displaced resulting in a slope on the side faces of the beam as shown in Fig. 3(b). The height of the beam is unchanged but the width is now tapered symmetrically about a vertical centreline. In addition, one of the reinforcing bars also moved during casting resulting in a new effective depth for the central bar of 250 mm. Calculate the ultimate moment capacity of the tapered beam. [35%]

(iii) There is a further problem with another beam where only one side of the formwork is displaced and one of the side bars has moved resulting in the section shown in Fig. 3(c). Do not carry out any further calculations but briefly discuss factors that may concern you relating to the structural performance. How will the performance differ from that of the beam in Fig. 3(b)? [20%]

(cont.



all dimensions in mm

(a) (b) (c)

Fig. 3

(TURN OVER

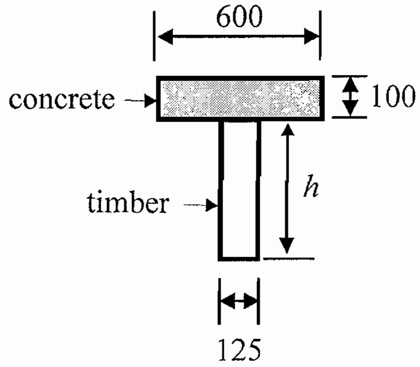
4 (a) Calculate the design bending strength of a C18 rectangular timber beam of height 220 mm and width 75 mm subjected to long-term loading. The beam is outside and fully exposed to the elements (moisture content > 20%). The elastic critical buckling moment can be approximated as $M_{crit} = \dfrac{\pi E_{0.05} b^3 h}{24 l_{ef}}$ and the effective span over which the beam is subjected to a uniform moment can be taken to be 5 m. Assume γ_m = 1.3, k_{ls}=1 and k_h=1. Relevant timber properties can be found on the 3D3 Timber Datasheet. [30%]

(b) A new system that combines unreinforced concrete and timber is proposed as shown in Fig. 4. All partial material safety factors and timber modification factors can be taken to be 1. The concrete has a Young's modulus of 20 GPa and the timber has a Young's modulus of 9 GPa. The concrete can be assumed to behave compositely with the timber. Shear deflections can be ignored.

(i) A designer wishes to ensure that the neutral axis depth is at 100 mm from the top of the concrete (at the interface between the concrete and timber). If both the concrete and the timber behave linear-elastically, find the height h of the timber beam to meet this requirement. Why might the designer wish to set the neutral axis depth at the interface level? [15%]

(ii) Using the height h found in part (i), and assuming the materials behave linear-elastically, compare the curvatures of the timber/concrete beam with a timber beam of height h when subjected to a total SLS bending moment of 20 kNm . Comment on the relative values. Find the maximum compressive stress for each case. [30%]

(iii) Briefly discuss possible advantages and disadvantages of the timber/concrete section as a structural system. Discuss whether it might be more efficient to use a high strength concrete or a low strength concrete in combination with a softwood timber beam. [25%]



all dimensions in mm

Fig. 4

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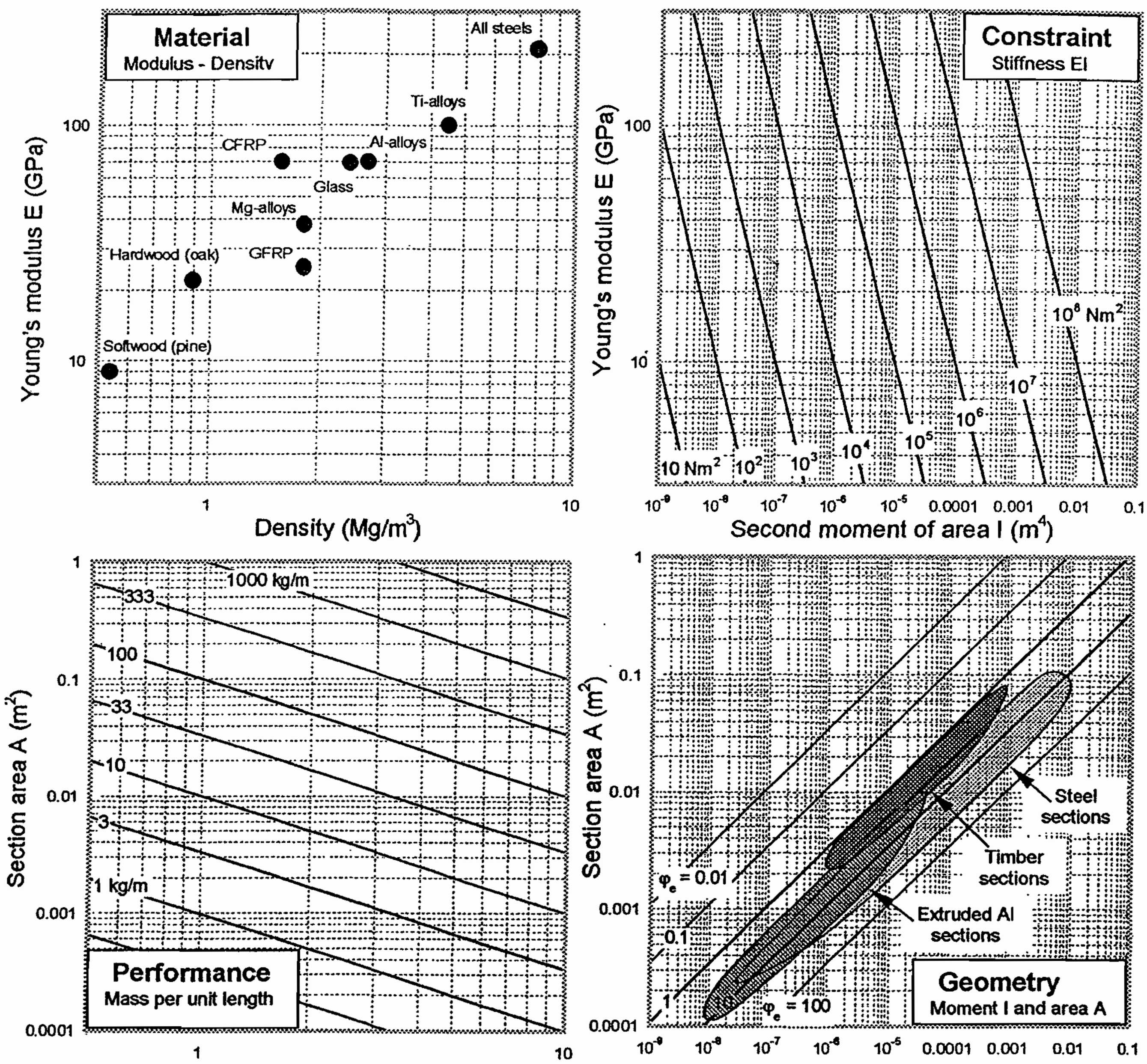

Figure 1. The chart-assembly for exploring structural sections for stiffness limited design. Each chart shares its axes with its neighbours.

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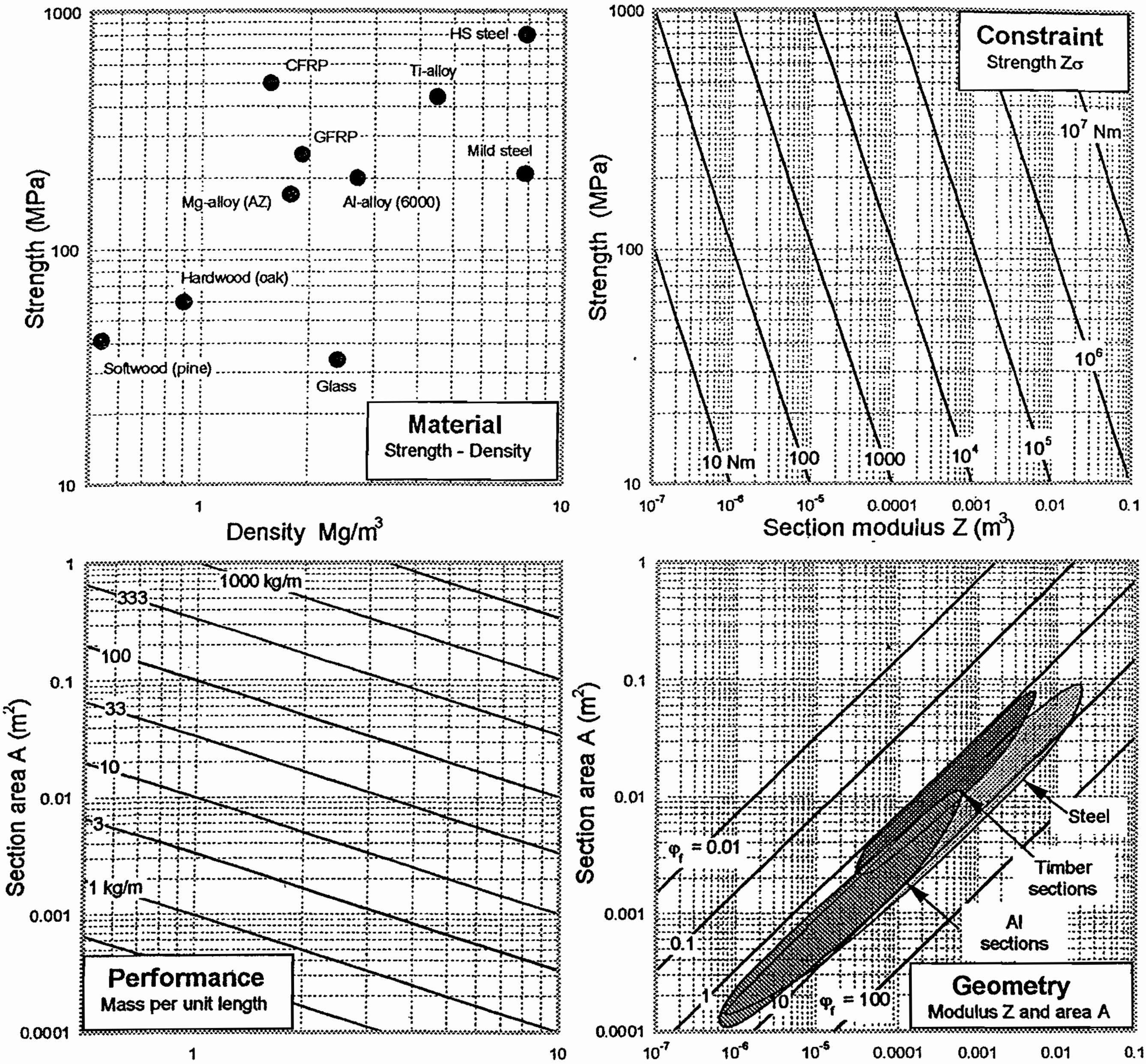


Figure 2 The chart-assembly for exploring structural sections for strength limited design. Like that for stiffness, each chart shares its axes with its neighbours.

Note: concept and charts are due to Prof. Michael Ashby, 2002
(see M.F. Ashby 'Materials selection in mechanical design' 2nd Edition, Butterworth 1999, JA163).

Module 3D3 – Structural Materials and Design **Steel Datasheet**

Materials available (see Structures Data Book 1999 pp. 1 and 11)

The two common structural steels BS EN – S275 and S355 have characteristic yield strengths σ_y of 275 and 355 MPa respectively. Both satisfy the usual criteria for plastic design (adequate ratio of UTS to yield strength; adequate elongation to fracture). In design, the calculated strength (e.g. buckling resistance) for either material would be divided by a specified partial safety factor γ_m, often 1.1. Strength design is at ULS, with specified partial safety factors γ_f on loads, often 1.4 on dead load, 1.6 on live.

Tension members (axial force only)

Gross area A; net area A_n is A minus hole(s). Effective section A_e is KA_n but not greater than A, where factor K is 1.2 for S275, 1.1 for S355.

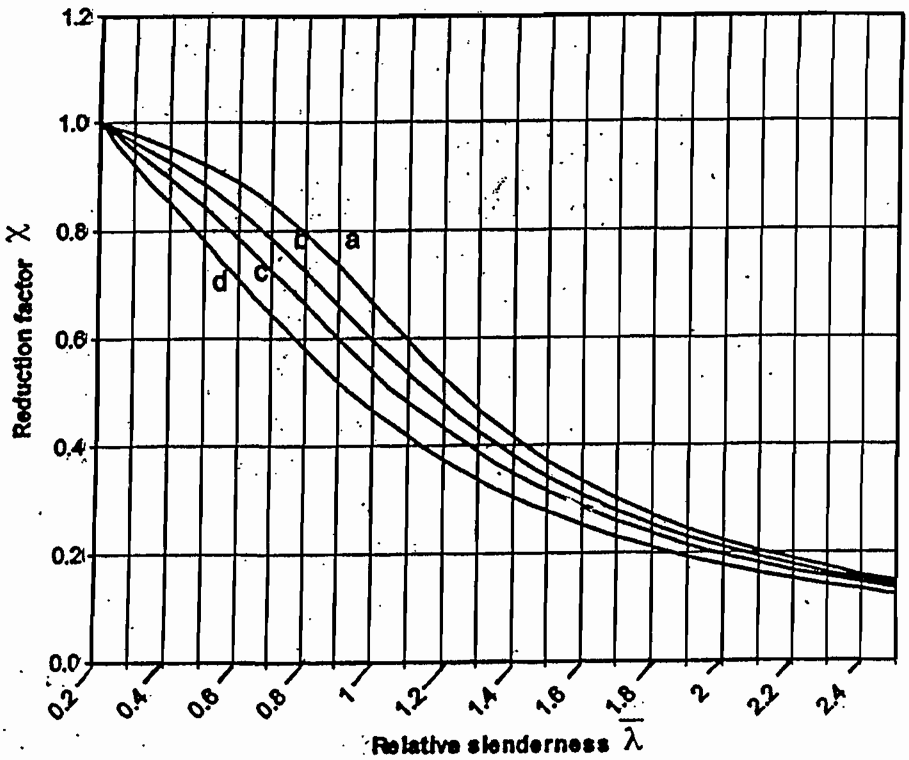
For eccentric connection, with area a_2 not connected at joint, effective area is often taken as $A_e - ca_2$, where factor c is 0.5 for bolted connections, 0.3 for welds.

Compression members (axial force only)

Joints do not normally control design, though criteria above should be checked. Base design against buckling on gross area A, yield strength σ_y, radius of gyration r, and column effective length L between points of contraflexure. Slenderness $\lambda = L/r$.

Define λ_0 as the slenderness at which the elastic critical stress for a perfect column equals the yield strength, so $\lambda_0 = \pi \{ E/\sigma_y \}^{1/2}$. Relative slenderness $\bar{\lambda} = \lambda/\lambda_0$

To allow for interaction between yield and buckling, use curves of reduction factor χ (on the full yield axial strength) plotted against the relative slenderness $\bar{\lambda}$. Typical curves are as shown below (these from the IStructE EC3 (Steel) Design Manual). Choice of empirical curve a to d depends on section type (extreme fibre distance y/r) and typical imperfection and residual stress magnitudes.



Beams (without axial force)

Moment - check maximum moment less than $\sigma_y Z_p$. Sections are Class 1 to 4, for ability to provide enough ductility for full plastic behaviour without local buckling.

Shear – yield strength q_w in shear often taken as $0.6\sigma_y$ for simplicity. But check for buckling in thin webs, for depth/thickness d/t and aspect ratio a/d, e.g. for a/d > 1, elastic critical average stress $q_{cr} = (0.75 + \{d/a\}^2)\,\{1000t/d\}^2$ in MPa.

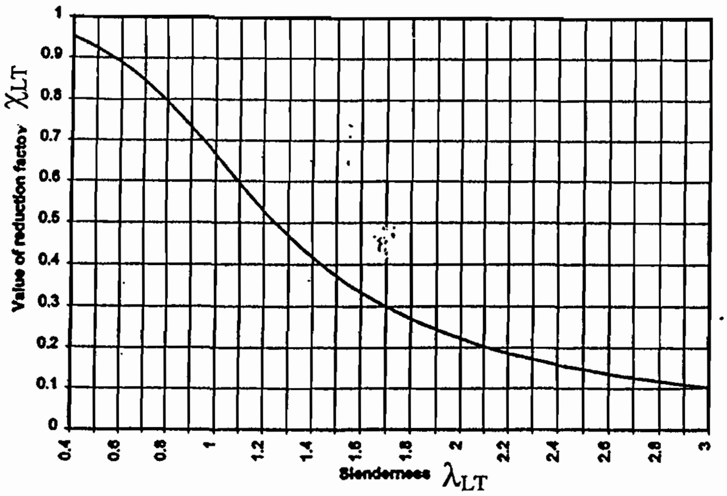
Bearing – check, using special formulae for the various failure modes, at supports and under high local loads, in case stiffener(s) are required.

Lateral-torsional buckling - for uniform bending moment M over a distance L between points where lateral displacement and rotation about beam axis are prevented, elastic critical value, with significant torsional stiffness due to restraint of warping,

$$M_c = \frac{\pi}{L}\sqrt{EI_{yy}\left\{GJ + \frac{\pi^2}{L^2}EC_w\right\}}.$$

For typical I-beam, basic torsion constant $J = \Sigma\, bt^3/3$ where b is the width of component plates of thickness t; and warping-restraint factor $C_w = D^2 I_{yy}/4$ where D is the distance between centres of flanges.

To allow for interaction between buckling and yield, again use empirical curves of the capacity reduction factor χ_{LT} against relative slenderness $\lambda_{LT} = \{M_p/M_{cr}\}^{1/2}$ (here M_{cr} is the hinge-position moment for elastic buckling).



Joints

Ductility of steel allows a reasonably simple equilibrium system to be envisaged for initial design – often with a transmitted force uniformly distributed across the various fasteners involved (particularly if they have similar properties). For a bolted joint in shear, a couple C about its centre can be taken simply by forces F_i on each bolt, perpendicular to the line to the centre and proportional to the distance d_i from the centre, so that $F_i = Cd_i/\Sigma d_i^2$, but other equilibrium systems can be envisaged.

Applied shear forces F on bolts are commonly checked against the shear strength (say $0.6\sigma_y$) of the bolt, depending on the number of shear planes activated; and against the bearing strength $\sigma_b dt$ in each plate (d is bolt diameter, t is plate thickness), despite the fact that bolts may not actually transmit force in this way. For M20 bolts, σ_y is typically 600 MPa, and in Codes nominal σ_b is typically of order 400 MPa in S275 steel.

(information mainly from Eurocode 3) CTM Jan. 2003

3D3 – Structural Materials and Design – Concrete Datasheet

Structural system	Span/effective depth ratio: IStructE* beam	IStructE* slab	EC2** high	EC2** light
1. Simply supported beam	15		18	
One-way or two-way spanning simply supported slab		21		25
2. End span of continuous beam:	20		23	
one-way continuous slab; or two-way spanning slab continuous over one long side		27		32
3. Interior span of: beam;	21		25	
one-way or two way spanning slab		30		35
4. Slab supported on columns without beams (flat slab), based on longer span	n/a	26	21	30
5. Cantilever	6	8	7	10

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 4.2 [10.1], ** Table 4.14 [10.2]

Table 10.1 Span versus depth ratio

Member	Fire resistance	Minimum dimension, mm: 4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width	240	200	200
	cover	70	50	45
Slabs with plain soffit	thickness	170	125	100
	cover	45	35	35

Extracts from Table 4.1 [10.1]

Table 10.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

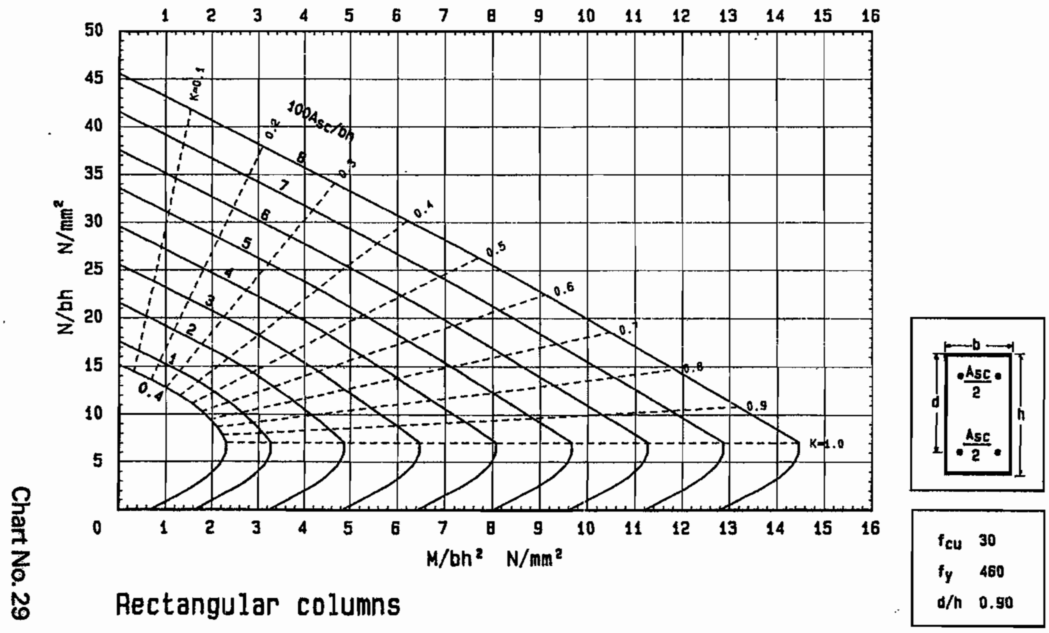


Fig 10.14 Interaction diagram from [10.3]

[10.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[10.2] Eurocode 2: Design of concrete structures DD ENV 1992-1-1:1992 – note** draft document
[10.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure

Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1 - 0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if $x/d = 0.5$

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Bond anchorage

$$l_b = \frac{f_y r}{2\beta_a \sqrt{f_{cu}}}$$

Cracking

$$w_k = \beta s_{rm} \varepsilon_{sm}$$

$$s_{rm} = 50\text{ mm} + 0.25 k_1 k_2 \phi / \rho_r$$

where: w_k is the design crack width

β is a statistical variation coefficient, usually 1.7

s_{rm} is the average final spacing

ε_{sm} is the mean steel strain (cracked-elastic theory)

ϕ is the average bar size

ρ_r is the effective steel ratio A_s/A_{ceff}

Shear

Concrete contribution

$V_{Rd1} = \{\tau_{Rd} k (1.2 + 40\rho_l)\} b_w d$

where: τ_{Rd} is the basic shear strength of the concrete.

k is a factor that varies with effective depth, d ($k = 1.6 - d$ but $k \geq 1$ and d in metres),

b_w is the minimum width

ρ_l is the reinforcement ratio $= A_s/b_w d$ but $\rho_l \leq 0.02$.

Shear stirrup contribution

$v_s = f_y A_v / \gamma_s b_w s$

Columns

Axial loading only

$$\sigma_u = 0.6 \frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm)

6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 – Structural Materials and Design – Timber Datasheet

			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens $\|\|$	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens $\perp$	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp $\|\|$	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp $\perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod $\|\|$	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod $\perp$	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	Service class 2	Service class 3
Solid and glued laminated timber and plywood			
Permanent	0.60	0.60	0.50
Long-term	0.70	0.70	0.55
Medium-term	0.80	0.80	0.65
Short-term	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – Design shear stress

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – Design bearing stress

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$\lambda_{rel,m} = \sqrt{f_{m,k} / \sigma_{m,crit}}$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e)"

$$k_{crit} = \begin{cases} 1 & \text{for} \quad \lambda_{rel,m} \le 0.75 & (5.2.2c) \\ 1.56 - 0.75\lambda_{rel,m} & \text{for} \quad 0.75 < \lambda_{rel,m} \le 1.4 & (5.2.2d) \\ 1/\lambda_{rel,m}^2 & \text{for} \quad 1.4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm^2

For bolts up to 30 mm diameter at an angle α to the grain:

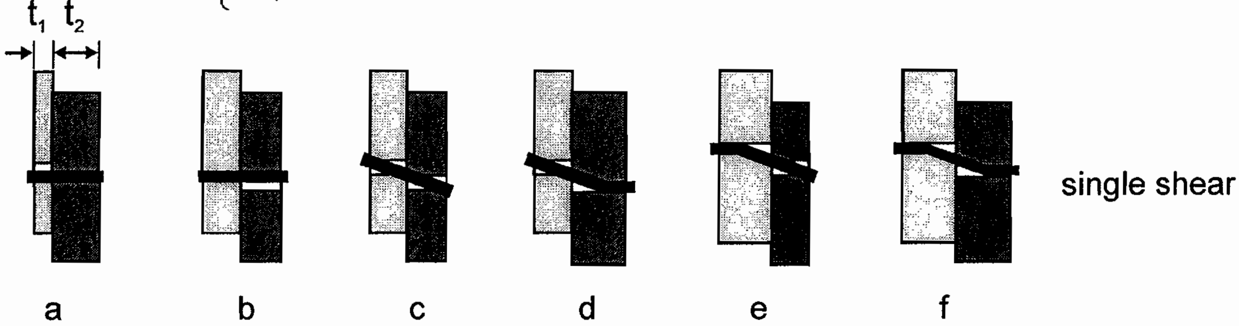
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear

$$R_d = \min.\begin{cases} f_{h,1,d}\, t_1 d & (6.2.1a) \\ f_{h,1,d}\, t_2 d\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}\, t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \dfrac{t_2}{t_1} + \left(\dfrac{t_2}{t_1}\right)^2\right] + \beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1 + \dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1.1\dfrac{f_{h,1,d}\, t_1 d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}\, d t_1^2}} - \beta\right] & (6.2.1d) \\ 1.1\dfrac{f_{h,1,d}\, t_2 d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}\, d t_2^2}} - \beta\right] & (6.2.1e) \\ 1.1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d} f_{h,1,d}\, d} & (6.2.1f) \end{cases}$$



Extract from [11.2] – Timber-to-timber and panel-to-timber joints

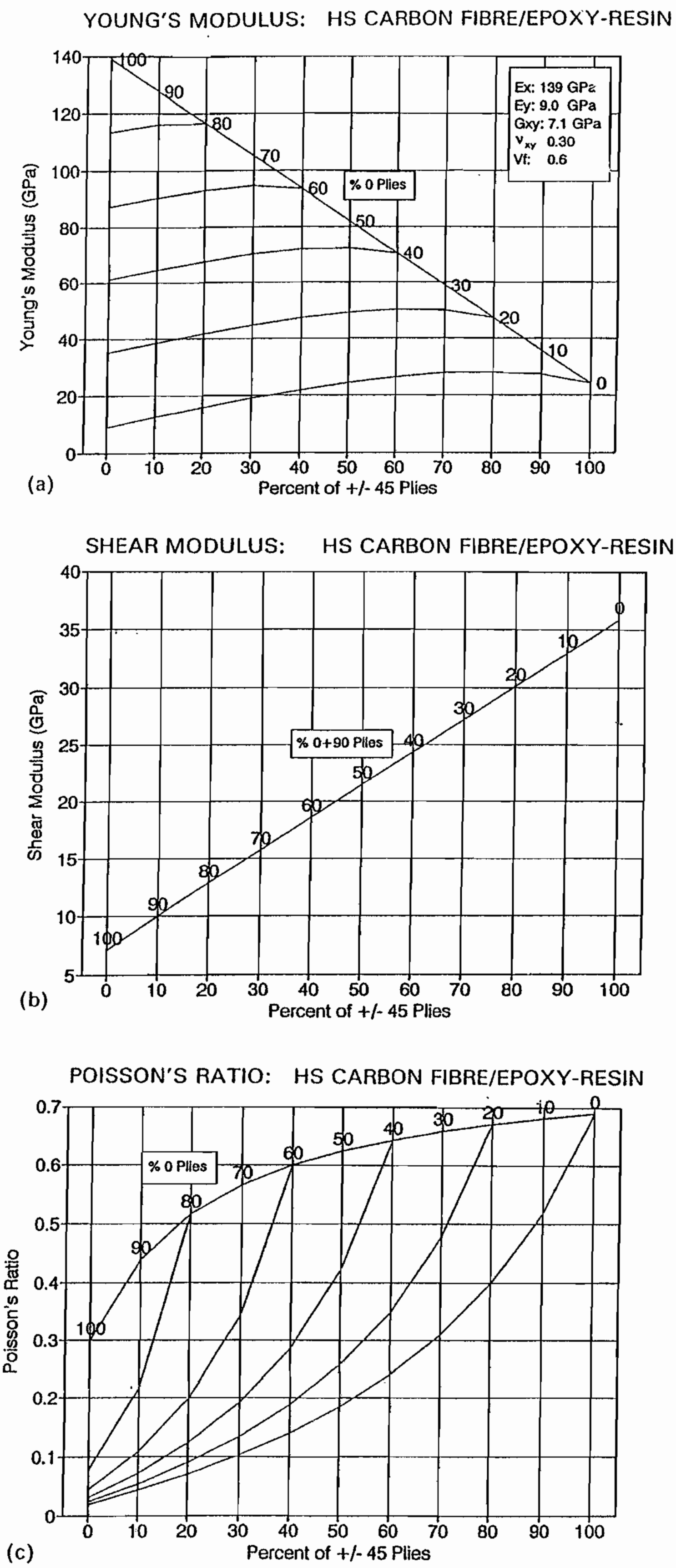


Figure 1. HS carbon/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

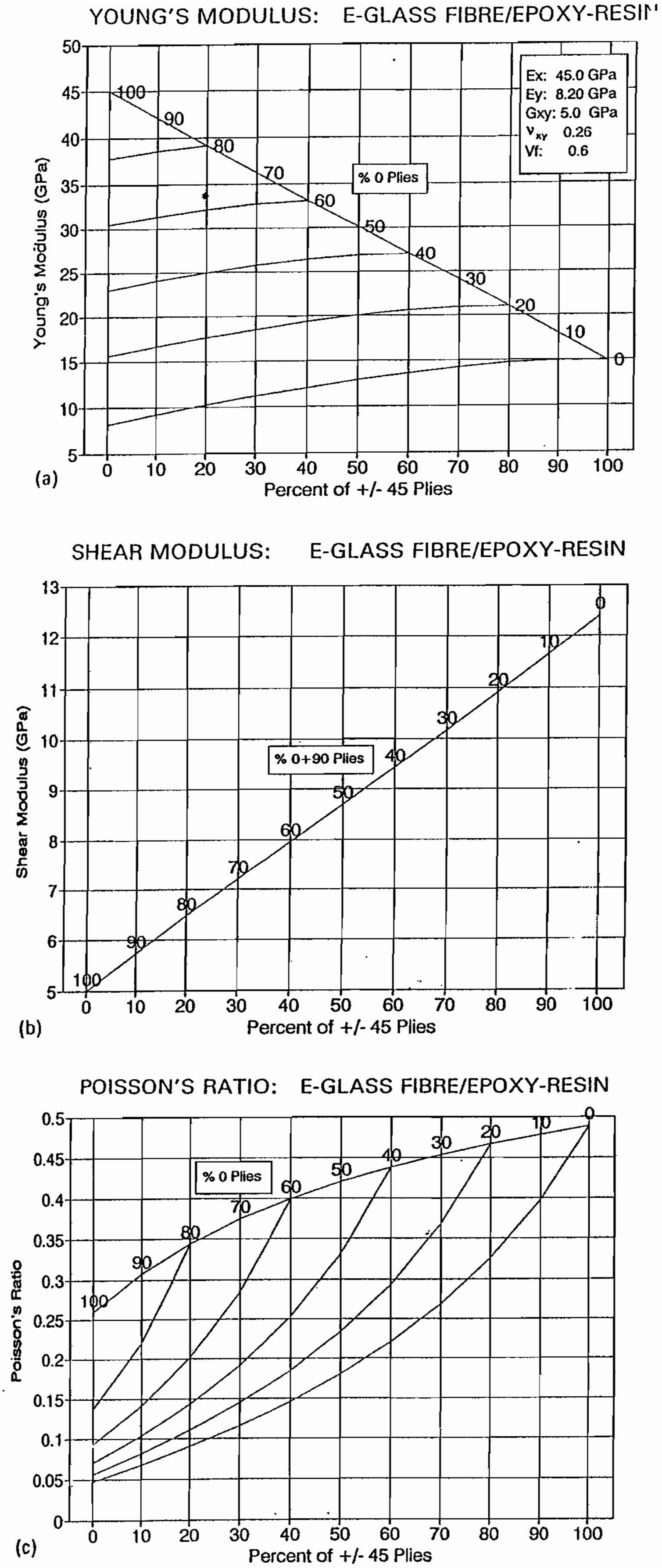


Figure 2. E-glass/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

[12.3] M.G. Bader, Materials selection, preliminary design and sizing for composite laminates. Journal Paper: Composites Part A, 27A, (1996), pp 65-70.

Engineering Tripos Part IIA, 2006

Paper 3D3 Structural Materials and Design

Answers

1. (b) (i) $M_c = 160$ kNm
   (ii) $M_{max} = 51.1$ kNm

2. (a) 305 × 305 × 118 UC ($\chi \sim 0.29$, $P_{provided} = 1471$ kN)
   (b) (ii) Z_p (required) = 1168 cm^3, possible section 356 × 171 × 67 UB

3. (b) (i) $M_U = 97.1$ kNm , L/d ~ 29 ∴ SLS unlikely to be satisfied
   (ii) $M_U = 86.6$ kNm

4. (a) (i) $f_{m,d} = 6.94$ MPa
   (b) (i) h = 327 mm, if neutral axis is at the interface, the concrete remains in compression
   (ii) $\kappa = 1.17 \times 10^{-6}$ mm^{-1} (timber/concrete) < $\kappa = 6.11 \times 10^{-6}$ mm^{-1} (timber)
   $\sigma_{comp} = 2.33$ MPa (timber/concrete), $\sigma_{comp} = 8.98$ MPa (timber)

J.L. June, 2006