ENGINEERING TRIPOS PART IIA

Wednesday 10 May 2006 2.30 to 4

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Data Sheet for Question 4

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The following section properties of a structural member (Fig.1) have been determined by means of a scanning program.

$\int dA = 7640\ \text{mm}^2$ $\int x\ dA = 3056 \times 10^3\ \text{mm}^3$ $\int y\ dA = 4584 \times 10^3\ \text{mm}^3$

$\int x^2\ dA = 1281 \times 10^6\ \text{mm}^4$ $\int xy\ dA = 1912 \times 10^6\ \text{mm}^4$ $\int y^2\ dA = 2899 \times 10^6\ \text{mm}^4$

In the following calculations you should work to at least 4 significant figures.

(a) Find the location of the centroid of the cross-section and calculate its section properties relative to the centroid. [40%]

(b) Plot, to scale and on graph paper, a Mohr's circle for the section and hence determine its major and minor principal second moments of area. [40%]

(c) The section is a slightly modified Universal Beam (Structures Data book pages 12/13). Suggest which section it might be and, without doing any detailed calculations, suggest what the modification might be. Draw a sketch to show clearly how the section is placed relative to the x- and y- axes. [20%]

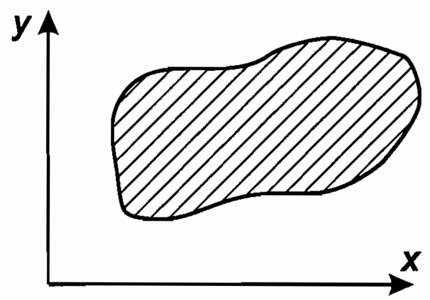


Fig 1

2 (a) A beam ABC, with flexural stiffness $EI = 10^7$ kNm^2, is continuous over two spans of 15 m and 20 m, as shown in Fig. 2. It must be designed to allow for differential settlement of the supports.

(i) Use Macaulay's method to analyse the beam if the internal support sinks by 10 mm relative to the other two supports. Find the moment induced in the beam at B and the deflection at the centre of the longer span. [40%]

(ii) Use the result of (i), in combination with a suitable rigid-body rotation, to determine the moment at B due to a settlement of support A by 10 mm, assuming that B and C do not move. [20%]

(iii) Sketch the range of bending moments for which the beam must be designed, in addition to those due to any applied loads, caused by any combination of support settlements up to 20 mm. [20%]

(b) A simply-supported beam 35 m long, with stiffness 2×10^7 kNm^2, has a point load of 10 kN applied 10 m from the left hand end. Use the reciprocal theorem, and the results of (a)(i) above, to determine the deflection at a point 15 m from the right hand end. [20%]

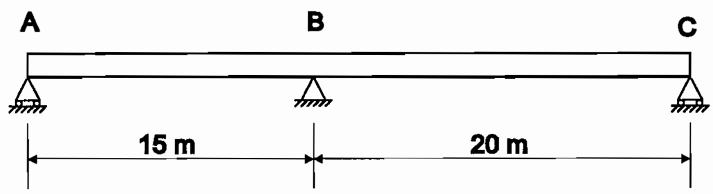


Fig. 2

(TURN OVER

3 (a) Briefly describe a situation where buckling of a structure occurs by bifurcation of the equilibrium path, and another where it occurs by reaching a limit point. [20%]

(b) Figure 3 shows a shallow "arch" structure consisting of two equal, initially straight pin-jointed bars with square cross-section of dimensions $b \times b$ and made of linear-elastic material with Young's modulus E. A vertical force V can be applied at the apex joint. In the unloaded configuration the apex joint is at $y = -H$, where $H << L$. The structure is constrained to lie in the plane of the diagram.

(i) Obtain expressions for the axial force, P, in the bars and for the load, V, in terms of the height of the apex, y. What are the maximum magnitudes of P and V? Sketch $P(y)$ and $V(y)$, and mark salient values on your sketch. [50%]

(ii) What condition has to be satisfied for Euler buckling of the individual bars to occur at the point where V_{max} is reached? Indicate on your sketches of $P(y)$ and $V(y)$ the effect of Euler buckling of the bars. [30%]

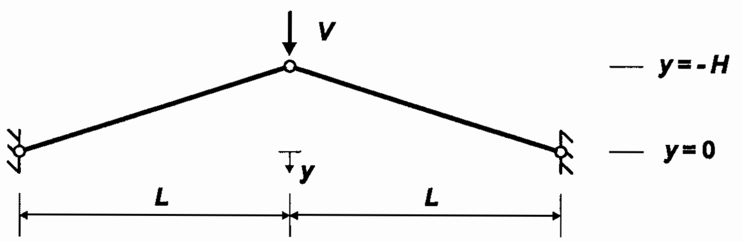


Fig. 3

4 (a) Briefly explain the procedure for determining the buckling load of a structure using the stability functions, stating any assumptions. How does the analysis of no-sway frames differ from that of frames allowed to sway? [30%]

(b) Figure 4 shows two structures consisting of linear-elastic members, all of length L, rigidly jointed together and to a rigid foundation. Out-of-plane displacements are not allowed. Each of the members, if loaded in compression through pinned ends, would buckle at a critical load P_E. Both structures are stress-free and perfectly aligned when they are unloaded.

Make use of the values of the stability functions provided on the attached data sheet to determine the critical buckling value of the load multiplier λ for the two cases below. An accuracy of one decimal place in P/P_E is sufficient; marks will be awarded for a clear explanation of the procedure.

(i) The two-member frame shown in Fig. 4(a); [30%]

(ii) the four-member frame shown in Fig. 4(b). [40%]

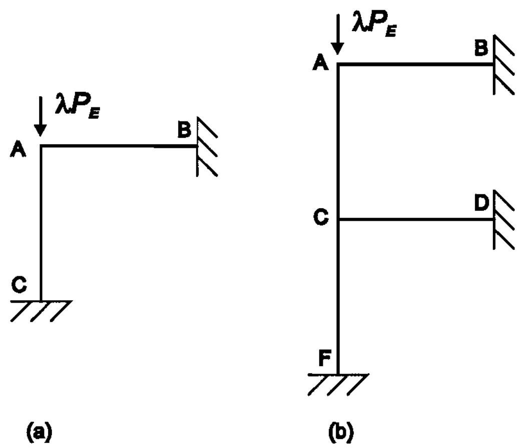
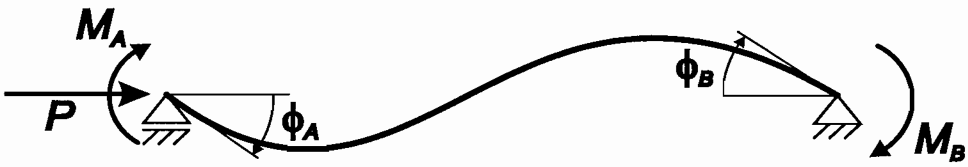


Fig. 4

END OF PAPER

Data Sheet for Question 4

Stability Functions



For a beam of length L , as shown in the figure, the following stiffness relationships apply.

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s & cs \\ cs & s \end{bmatrix} \begin{bmatrix} \phi_A \\ \phi_B \end{bmatrix}$$

where E is the Young's modulus and I the second moment of area of the member for bending within the plane of the structure. If the Euler buckling load of the member is P_E, the stability functions s and c are tabulated below.

P/P_E	s	c
1.0	2.47	1.00
1.1	2.28	1.11
1.2	2.09	1.25
1.3	1.89	1.42
1.4	1.68	1.66
1.5	1.46	1.97
1.6	1.22	2.43
1.7	0.98	3.17
1.8	0.72	4.50
1.9	0.44	7.66

P/P_E	s	c
2.0	0.14	24.68
2.1	-0.18	-21.07
2.2	-0.52	-7.51
2.3	-0.89	-4.62
2.4	-1.30	-3.37
2.5	-1.75	-2.67
2.6	-2.25	-2.23
2.7	-2.81	-1.93
2.8	-3.44	-1.71
2.9	-4.18	-1.54

3D4 Structural Analysis and Stability – Examination 2006

Numerical answers

1. (a) $\bar{x} = 400$ mm, $\bar{y} = 600\,\text{mm}$ $I_{yy} = 58.60.10^6\,\text{mm}^4$, $I_{xx} = 148.6.10^6\,\text{mm}^4$, $I_{xy} = 78.40.10^6\,\text{mm}^4$ (b) $194.0.10^6$ mm^4, $13.21.10^6$ mm^4 (c) 356×171×67 UB

2. (a)(i) 1000 kNm (sagging); 7.50 mm (ii) 571 kNm (hogging)
(iii) ±2000 kNm at B (b) 0.321 mm

3. (b)(i) $P(y) = \dfrac{Eb^2}{2L^2}\left(y^2 - H^2\right)$; $V(y) = \dfrac{Eb^2}{L^3}\left(H^2 y - y^3\right)$; $\dfrac{Eb^2 H^2}{2L^2}$; $\dfrac{2Eb^2 H^3}{3\sqrt{3}L^3}$
(ii) $\dfrac{H}{b} = \dfrac{\pi}{2}$

4. (b)(i) 2.86; 2.1

C J Burgoyne
1st June 2006