ENGINEERING TRIPOS PART IIA

Tuesday 2 May 2006 2.30 to 4

Module 3D5

ENVIRONMENTAL ENGINEERING I

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Special datasheets (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper (4 sheets)

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Rain falls uniformly at 10 mm hr $^{-1}$ for one hour and then at 30 mm hr $^{-1}$ for the next hour over a rural catchment of 20 km^2 area. The soil has Horton f-capacity parameters f_o = 20 mm hr $^{-1}$, f_c = 5 mm hr $^{-1}$ and K_f = 0.7 hr $^{-1}$. The soil is initially dry. Determine the total volume of runoff. [40%]

(b) Rain falls uniformly for 4 hours over a small urban catchment of area 10 km^2. The distribution percentages of the outflow hydrograph over successive intervals of 4 hours are 4, 14, 30, 25, 16, 8, and 3. Assuming that the catchment is completely impervious, estimate the outflow hydrograph that would result from 2 hours of uniform rainfall at 10 mm hr $^{-1}$.

Sketch the hydrograph of this outflow and indicate your estimate of the peak instantaneous outflow in m^3 s^{-1}. [60%]

2 (a) An irrigation channel has a symmetric trapezoidal cross-section with a bed width of 3 m and side-slopes that make an angle of 30° with the horizontal. It is carrying a flow of 40 m^3 s^{-1} and at one section the flow depth is 2 m. Determine whether the flow at that section is supercritical or subcritical flow. [40%]

(b) Engineers have constructed a wide, flat, smooth, straight channel 120 km long from an inland city to a nearby ocean. The water depth in the channel is 3 m and is flowing at a steady average velocity of 0.3 m s^{-1} towards the ocean. The water surface everywhere across the channel and ocean may be assumed to be initially flat at time $t = 0$.

A storm surge arrives at the coast such that, over the next 8 hours, the ocean surface level rises above the original level by an extra amount Δh (in metres), where

$$\Delta h = 2\left(1 - \cos(\pi t/8)\right)$$

where t is in hours.

Determine the time t at which the water level in the channel at the city is a maximum. [60%]

3 A wide channel of uniform cross-section has a bed-slope of 0.0001 and a depth of 1.0 m. Suspended sediment can be considered to be the sediment carried above the bed layer, which is 0.01 m thick. The concentration of suspended sediment at the top of the bed layer is 13.6 kg m^{-3}.

Assume the Karman constant is 0.4, the bed roughness height k_s is 0.01 m, the specific gravity of the sediment is 2.65, all grain diameters are 0.1 mm, the kinematic viscosity of water ν is 10^{-6} m^2 s^{-1} and the water temperature is 20°C.

(a) Estimate the sediment transport rate in suspension per metre width of channel above the bed layer. [70%]

(b) What, according to various theories, would be the bed regime? [30%]

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4 Water flows through a 450 mm diameter cast iron gravity pipeline linking 2 reservoirs over a length of 1500 m, as shown in the simple pipe system in Fig. 1.

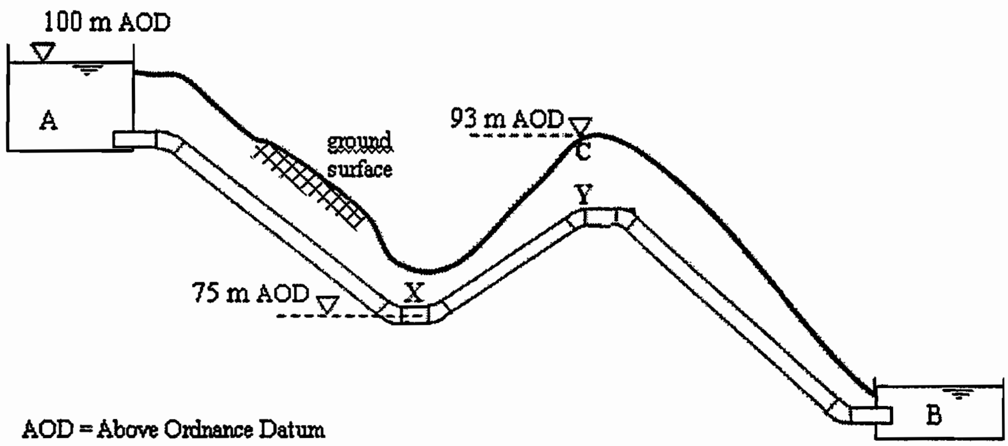

Fig. 1 (Not to scale)

Take: $k_{entry} = 0.5$, $k_{exit} = 1.0$, $k_{bend} = 0.25$
Roughness k for cast iron = 0.26 mm
Kinematic viscosity at 15°C = 1.14×10^{-6} m^2 s^{-1}
Vapour pressure head of water at 15°C = -10.1 m

Darcy-Weisbach equation:

$$h_f = \frac{\lambda L}{D} \cdot \frac{v^2}{2g}$$

The Moody diagram is in Fig. 2.

(a) On a sketch of the pipeline show the position of the total energy line and the hydraulic gradient. [10%]

(cont.

(b) If the pressure in the pipe at X (500 m along the pipeline from reservoir A) is 120 kN m^{-2} determine:

(i) The flow in the system from reservoir A to reservoir B. [20%]

(ii) The minimum acceptable depth of pipe at Y below C (800 m along the pipeline from reservoir A). [20%]

(iii) The water level in reservoir B. [20%]

(c) If a pump with the characteristics shown in the table below was installed on the pipeline at reservoir B, what flow rate could be delivered to reservoir A? (Assume the friction factor, λ, from part (b) is constant in the rough turbulent zone). [30%]

Discharge Q (m^3s^{-1})	0	0.2	0.4	0.6	0.8
Head H (m)	100	91.2	75.4	53.2	24.3

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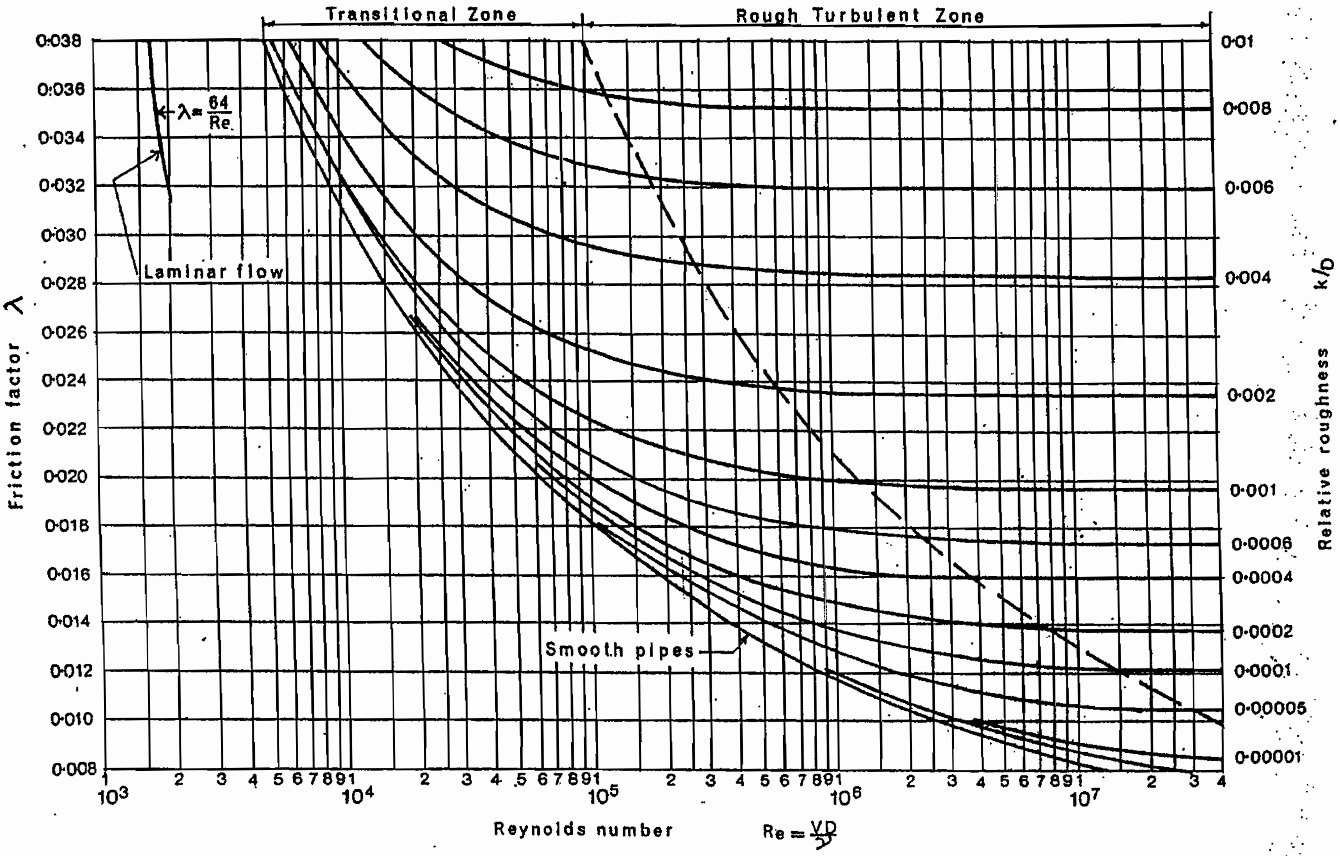

Moody diagram

Fig. 2

END OF PAPER

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Engineering Tripos Part IIA THIRD YEAR

Module 3D5: Environmental Engineering I

Data Sheet

(SI units throughout)

f-capacity equation

$$f = f_c + (f_0 - f_c)\, e^{-K_f t}$$

Spatially-varied flow (backwater curves)

$$\delta d = \ell \left(\frac{S - \overline{U}^2 C_f / 2gR}{1 - \overline{U}^2 / gd} \right)$$

Open-channel characteristics

$$\overline{U} + 2c - g(S - S_f)t = const \qquad \frac{dx}{dt} = \overline{U} + c$$

$$\overline{U} - 2c - g(S - S_f)t = const \qquad \frac{dx}{dt} = \overline{U} - c$$

Fall velocity of particles of sand in water (20°C)

For D < 0.0005 m $\quad W \doteq 56 \times 10^4 D^2 (\rho_s - \rho) / \rho$

For D > 0.002 m (and shape factor=0.7) $\quad W \doteq 3.3\, D^{1/2} ((\rho_s - \rho)/\rho)^{1/2}$

Initial motion of sediment on a flat bed

$$\frac{u_* k_s}{\nu} > 70 \qquad \frac{\tau_c}{(\rho_s - \rho) g D} = 0.05$$

Velocity in uniform flow in a channel

Chézy $\quad \overline{U} = C R^{\frac{1}{2}} S^{\frac{1}{2}}$

Manning $\quad \overline{U} = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$

Prandtl-Karman

Hydraulically smooth $\left(\frac{u_* k_s}{\nu} < 5\right)$

$$\frac{u}{u_*} = 2.5 \log_e\left(\frac{9.05 y u_*}{\nu}\right)$$

$$\frac{\overline{U}}{u_*} = 2.5 \log_e\left(\frac{3.66 R u_*}{\nu}\right)$$

Hydraulically rough $\left(\frac{u_* k_s}{\nu} > 70\right)$

$$\frac{u}{u_*} = 2.5 \log_e\left(\frac{30.2 y}{k_s}\right)$$

$$\frac{\overline{U}}{u_*} = 2.5 \log_e\left(\frac{12.1 R}{k_s}\right)$$

Variation of the concentration of sediment in suspension with distance from the bed

$$\frac{C}{C_a} = \left[\left(\frac{d-y}{y}\right)\left(\frac{a}{d-a}\right)\right]^{\frac{W}{Ku_*}}$$

Sediment load in suspension

$$\int_b^d Cu\,dy = 11.6 u_* C_b b\left[I_1 \log_e(Ad) + I_2\right]$$

where $A = \frac{9.05 u_*}{\upsilon}$ for a hydraulically smooth bed

$= \frac{30.2}{k_s}$ for a hydraulically rough bed

	$W/Ku_* = 0.2$		$W/Ku_* = 0.6$		$W/Ku_* = 1.0$		$W/Ku_* = 1.5$	
b/d	I_1	I_2	I_1	I_2	I_1	I_2	I_1	I_2
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113

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Limiting shear stress for a particle on a slope

$$\frac{(\tau_c)_\theta}{\tau_c} = \cos\theta\left(1 - \frac{Tan^2\theta}{Tan^2\phi}\right)^{\frac{1}{2}}$$

"Regime" formulae

$$\overline{U} = 0.635 \quad f^{\frac{1}{2}} R^{\frac{1}{2}}$$

$$P = 4.83 \quad Q^{\frac{1}{2}}$$

$$R = 0.4725 \quad Q^{\frac{1}{3}} f^{-\frac{1}{3}}$$

$$S = 0.000303 \quad f^{\frac{5}{3}} Q^{-\frac{1}{6}}$$

SYMBOLS

C Concentration of sediment. Chezy roughness coefficient $\left(= (2g / C_f)^{1/2}\right)$

C_f friction coefficient

D grain size or body diameter

D_{65} grain size for which 65% by weight of grains have a smaller diameter

f Lacey silt factor

f_0, f_c coefficients in f-capacity equation

h_r ripple height

K Karman constant

K_f coefficient in f-capacity equation

k_s roughness height

P wetted perimeter of a channel

Q total flow rate of water

R hydraulic radius $(= A/P)$

S channel slope

t time

u horizontal component of fluid velocity

$\overline{U}$ mean velocity

u_* $(\tau_0 / \rho)^{1/2}$

v vertical component of fluid velocity

W fall velocity

x,y co-ordinates

θ angle of a slope to the horizontal

φ angle of repose of sediment

ν kinematic viscosity

ρ density of fluid

ρ_s density of sediment

τ_o shear stress on the bed

τ_c critical value of τ_o for sand movement

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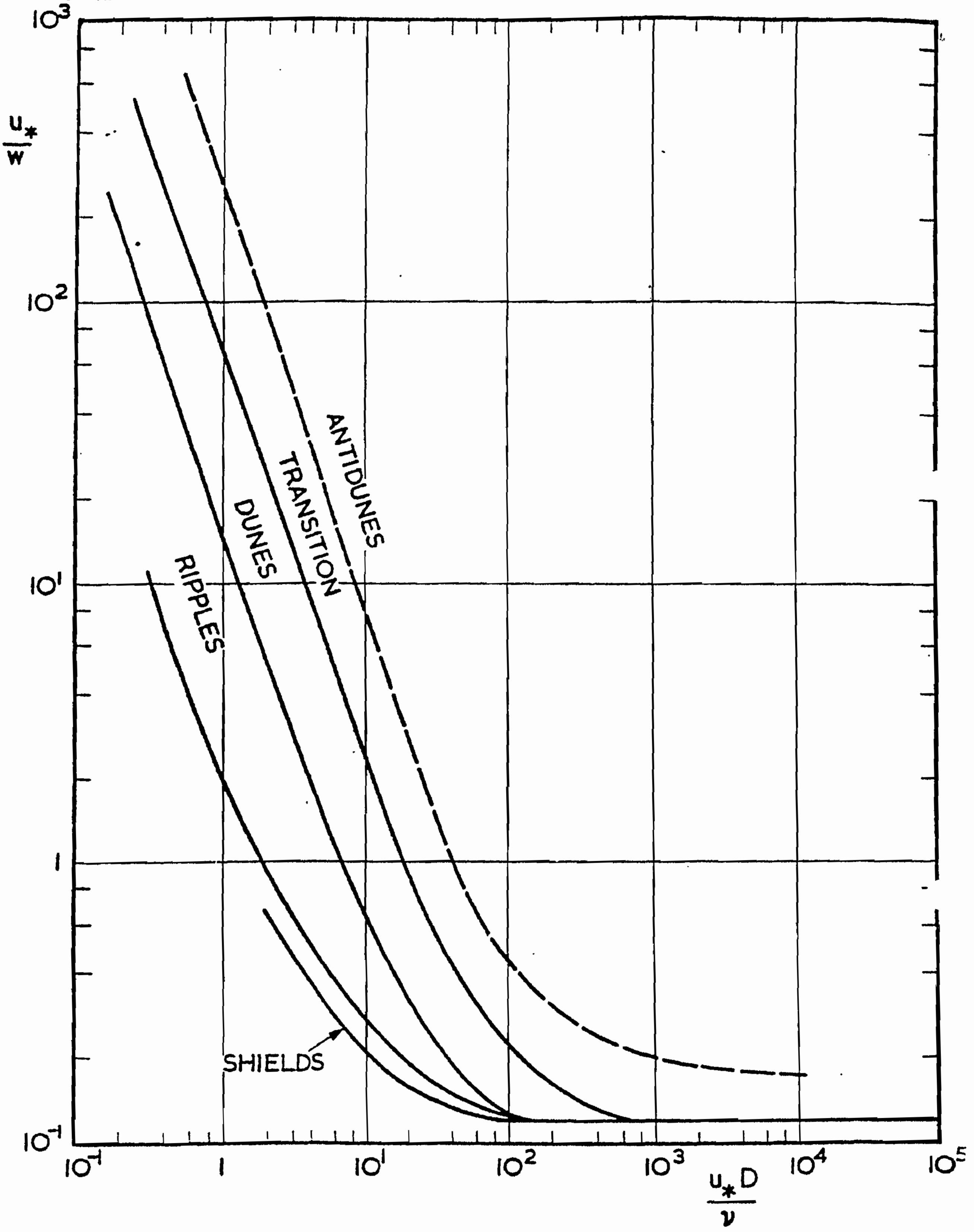

BED REGIMES IN OPEN CHANNEL FLOW (Albertson Simons & Richardson)

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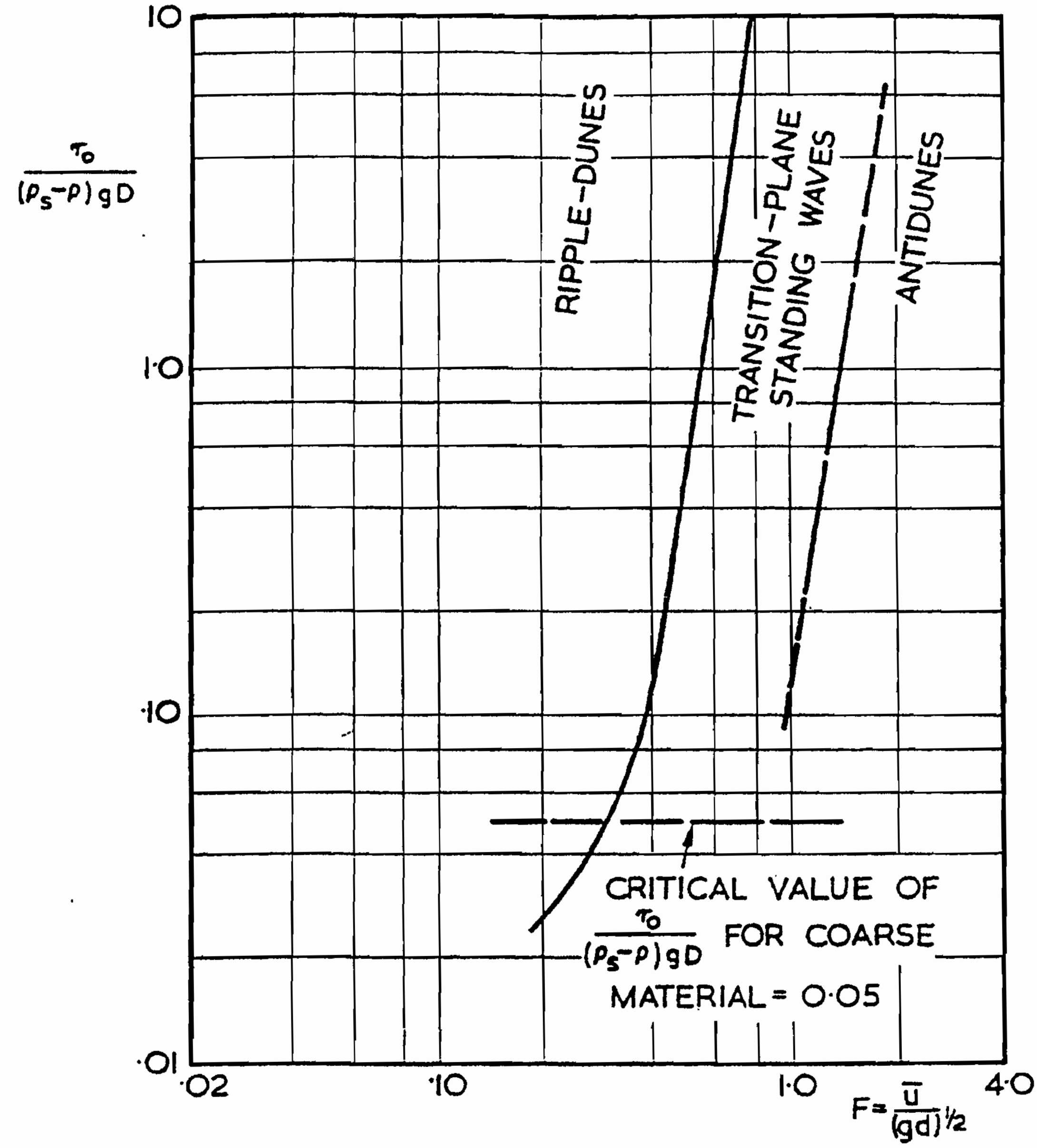

BED REGIMES IN OPEN-CHANNEL FLOW
(Garde & Albertson)

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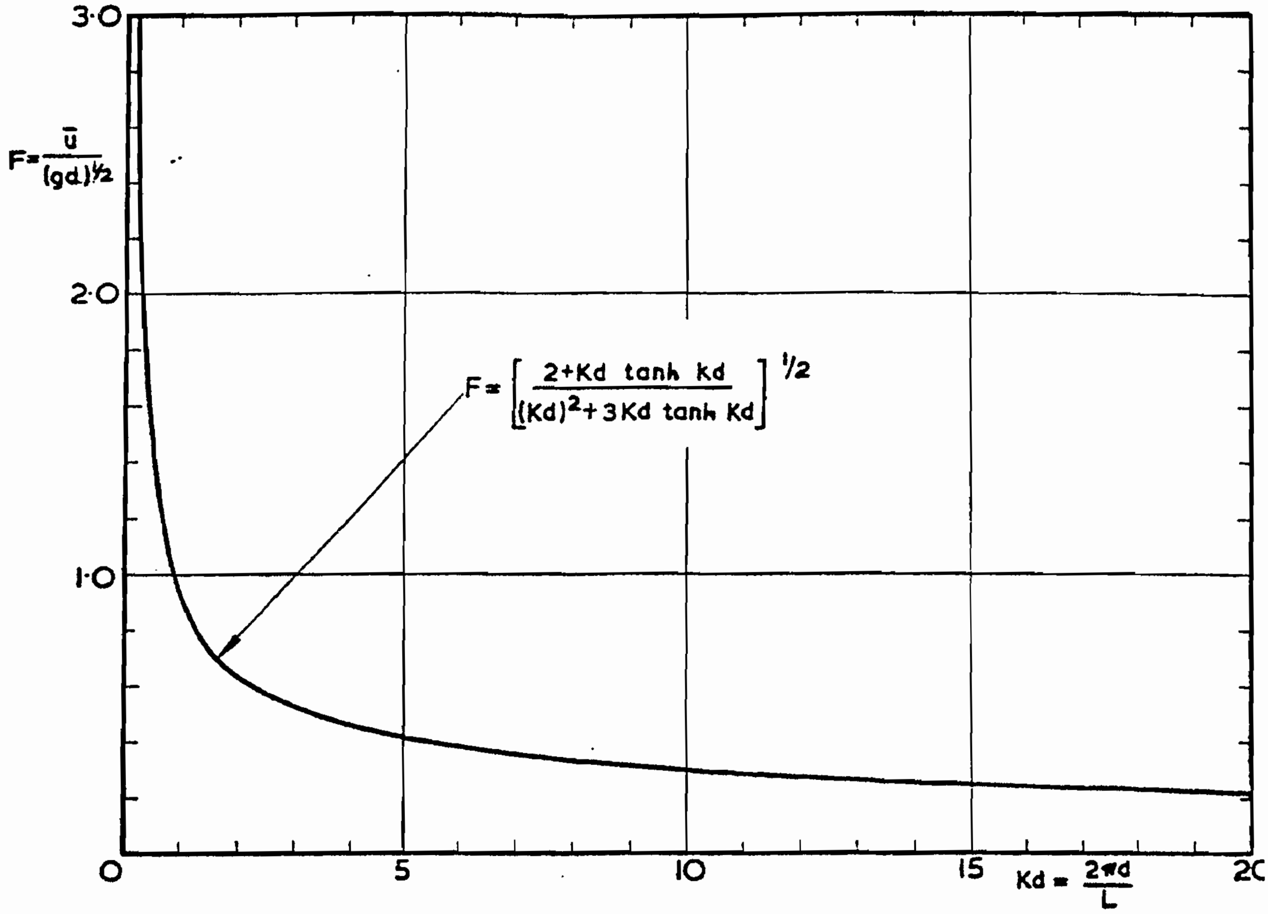
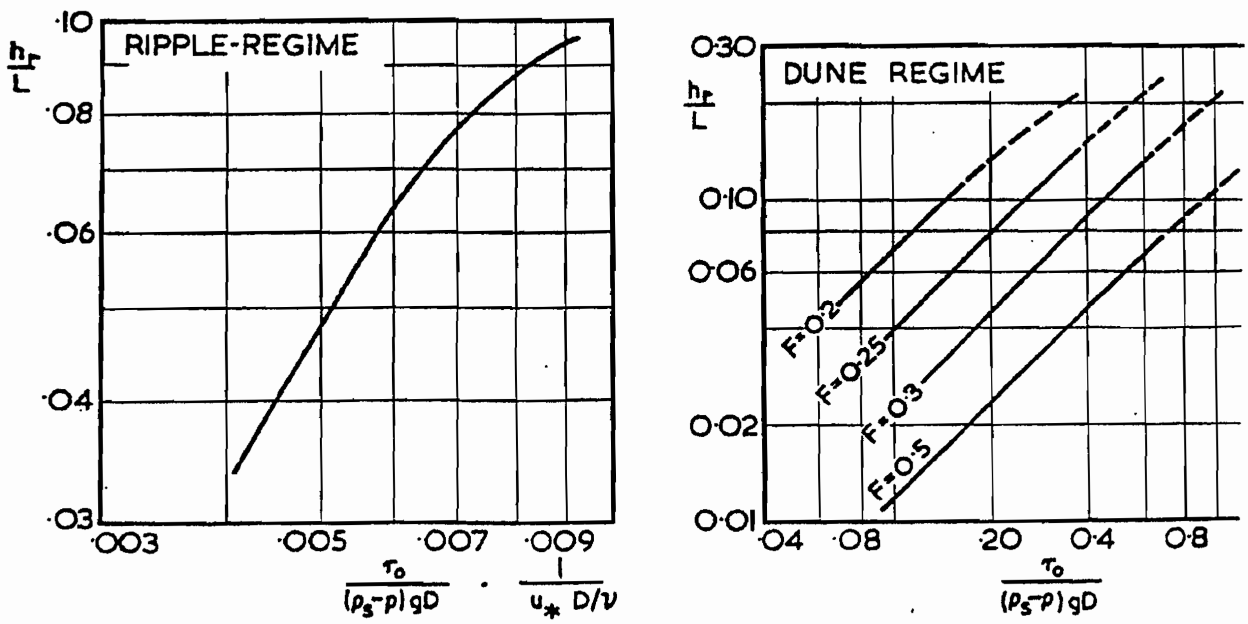


WAVELENGTH OF DUNES / ANTIDUNES (Kennedy)



RIPPLE / DUNE STEEPNESS (Garde & Albertson)

ENGINEERING TRIPOS PART IIA 2006

MODULE 3D5: ENVIRONMENTAL ENGINEERING I

NUMERICAL ANSWERS

1. (a) Volume of the runoff = 360 x 10^3 m^3.
   (b) Peak instantaneous outflow = 4.45 m^3/s

2. (a) Flow is super-critical.
   (b) Time = 10.43 hrs.

3. (a) Sediment transport rate = 0.84 kg/s per metre width of channel.
   (b) Albertson Simons and Richardson – Dunes
   Garde and Alberton – Ripple-Dunes.

4. (b) (i) Q = 0.543 m^3/s.
   (ii) Minimum acceptable depth at Y = 2.906m.
   (iii) Water level in reservoir B = 63.486m.

   (c) Flow rate that could be delivered to reservoir A = 0.483m^3/s.