ENGINEERING TRIPOS PART IIA

Wednesday 3 May 2006 2:30 to 4

Module 3E8

MODELLING DATA AND DYNAMICS IN MANAGEMENT

*Answer one of the first **two** questions and one of the last **two** questions*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Statistical tables (3 pages).

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Imagine a village in rural Norfolk with 3000 inhabitants. The population is initially evenly distributed between children aged 0 to 20 years, childbearing adults aged 20 to 45 years, and seniors aged 45 to 75 years (assume that no one lives past the age of 75). For every marriage, a couple will have, on average, 2 children together. You are interested in studying how the demographics of the small town will change over time. You can assume for the purposes of this model that all deaths are related to old age. The model will contain the elements listed below.

- maturing
- years as a child-bearing adult
- children
- number of child-bearing couples
- deaths
- years as a child
- aging
- births
- seniors
- babies per couple
- child-bearing adults
- adults per couple
- years as a senior
- total population (sum of the populations of all age groups)

(i) Identify each element as a stock, flow, or constant, and label its units. For each stock, determine its inflows and outflows. [20%]

(ii) Combine the elements to represent the structure of the system. Please include the model diagram. [20%]

(iii) Using the description of the system provided above, define the equations for all model elements. [10%]

(iv) Draw reference modes for all stocks in the model. [10%]

(b) Imagine a small agricultural village in central China. Again, a population of 3000 people is initially evenly distributed between children aged 0 to 20 years, childbearing adults aged 20 to 45 years, and seniors aged 45 to 75 years. In this village, however, each couple only has one child.

(cont.

Change the original population model to represent the new scenario.

(i) What do you expect will happen to the population distribution? [10%]

(ii) Draw reference modes for each stock. Include graphs of the behaviour of all stocks and of total population over a 100-year time horizon. [10%]

(c) Imagine a small village in Bangladesh. A population of 3000 people is also initially evenly distributed between children aged 0 to 20 years, childbearing adults aged 20 to 45 years, and seniors aged 45 to 75 years. In this village, however, each couple has four children. Change the original population model to represent the new scenario.

(i) What happens to the population distribution now? Draw reference modes for each stock. Then simulate the model over 100 years. Include graphs of the behaviour of all stocks and of total population. [10%]

(ii) Can you foresee any problems in Bangladesh that would be caused by the behaviour you predict? [5%]

(iii) How could a system dynamics perspective help in proposing recommendations for developing countries with high birth rates? [5%]

(TURN OVER

2 (a) You are a drug rehabilitation counselor who is given the choice of two school districts to work in. Both districts show initial stages of drug abuse in the student body. Being a concerned and brave soul, you would like to work for the district that will have the greater number of drug-abusers in ten years if you are not present. The two districts you are presented with are Adams and Barton. Through interviews with school administrators, you have identified the following data:

Adams estimates that 200 students are active drug users. The number of drug users steadily increases because each drug user tends to influence his or her friends to try drugs. Because the student body as a whole comes from wealthy families, cash is not a problem. As a result, every drug user influences approximately 0.2 students to become drug users per year. Without you, the rehabilitation counselor, the school does not expect the number of drug users to decrease.

Barton estimates that 400 students are currently using drugs in the four grades. Fortunately, because the district is comprised mostly of students from lower-income families with budget constraints, every drug user influences only 0.1 additional drug users. Again, the school does not expect the number of drug users to decrease without your aid.

(i) Calculate which school you expect to have more drug users in ten years and explain your logic. [20%]

(ii) Build a model of the system and use the different parameter values to study the number of drug users in each school district. Include the model diagram. [15%]

(iii) Provide the documented equations, and graphs of expected model behaviour. [15%]

(b) You are the culture and tourism director for the City of Cambridge. Whenever you have a particular project in mind, you submit a quarterly funding request to the city Budget Office, and usually 50% of your request is granted. You have just received an excellent architectural proposal from the botanical garden to extend their glasshouses that will cost about £1 million to build.

Remember, when you submit a request you must attach the architect's estimated cost statement and request only the difference between the total cost and the

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amount already granted, so you cannot make requests for absurd amounts of money. For this problem, you should not take interest rates or discounting into consideration.

(i) Build a model to study how much funding you will request in each quarter for building the garden. Include the model diagram. [15%]

(ii) Provide documented equations and graphs of expected model behaviour. [15%]

(iii) How many fiscal quarters will pass before you are granted 95% of the funds needed (assume you can raise the remaining 5% from private donors once you have the other 95%)? [10%]

(iv) Before you submit your first funding request, you suddenly receive an anonymous gift of £500 000 to be used for the botanical garden project. Model this event into the model from part (i). How does this gift affect the number of quarters that will pass before you are granted 95% of the funds needed? [10%]

(TURN OVER

3 (a) A business analyst wishes to test if those who have taken formal training courses perform as well as those who have not, but he has no prior view about whether they are likely to do better or worse. He has two samples, each of size 50: sample A did formal training, sample B did not. He computes the following sample statistics:

	Estimated Mean	Estimated Variance
Sample A	62.6	8.7
Sample B	64.2	9.1

Assuming that both samples are drawn from populations with the same unknown variance

(i) Compute the variance estimate for the pooled sample. [10%]

(ii) Set out the appropriate null hypothesis and test statistic, stating how the test statistic is distributed. [10%]

(iii) Discuss the definitions of Type I and type II errors. [10%]

(iv) Test the null hypothesis using a 5% critical value for the test. [10%]

(v) State the probability-value (p-value) for the computed test statistic [10%]

(b) A study by Kenetic3 Ltd. evaluated the relationship of the time spend on formal training of each employee and their productivity measured by output per hour. The following two lists show how 10 Internet companies ranked in terms of time spent on training and output per hour of the corresponding employee. A positive rank correlation is anticipated because it seems reasonable to expect that an employee with more formal training would be more productive.

(cont.

Employee ID	Time on Training	Output per hour
E1	1	3
E2	2	4
E3	3	1
E4	4	2
E5	5	9
E6	6	5
E7	7	10
E8	8	6
E9	9	7
E10	10	8

(i) Compute the rank correlation between training and output per hour for each employee. [10%]

(ii) Test for a significant positive rank correlation. What is the p-value? [10%]

(iii) Using $\alpha = 0.05$ as the significance level, what is your conclusion about the relationship between the spent on training and output per hour? [10%]

(iv) If we want to find out the direction and extent to which training impact on labour productivity, what method will you use? What are the differences between the suggested method and rank correlation test? What are the limitations of the suggested method in current problem setting? [20%]

(TURN OVER

4 (a) An investigator analysing the relationship between sales, number of employees, net fixed assets and expenditure on R&D in the US using a survey database. The data consists of 51 firms in 8 industries in the US. He computes the following regression.

Dependent Variable		Log(SALES)			
Parameter Estimates					
Variable		Parameter estimate	Standard error	T for H0: parameter=0	Prob >|T|
INTERCEPT		(a)	0.2174	1.257	0.2148
Log(L)		0.1801	(b)	5.754	0.0001
Log(K)		0.8086	0.1010	5.036	0.0001
RD		0.1069	0.0033	32.394	0.0001

SALES = total turnover of firm i
L = total number of employees of firm i
K = net fixed assets of firm i
RD = a dummy variable which equals 1 for those invest in R&D and 0 for the others.

(i) Fill in the missing spaces (a) and (b) in the output sheet. Show your calculations. [10%]

(ii) Give an economic interpretation of the coefficients on log(L), log(K) and RD. Is the sign of the coefficient what you would expect from economic theory? [15%]

(iii) Construct a 99% confidence interval estimate of the estimated coefficient of variable log(L). [10%]

(iv) Test the hypothesis that the slope of the relationship between log(SALES) and log(L) is 0.2, against the alternative that it is not. State in words the meaning of the null hypothesis in the context of this problem. [10%]

(v) State the assumptions about the Error term in the Ordinary Least Squares model. [15%]

(vi) You are now given the following extra information:

$$SST = \Sigma(y_t - \text{mean}(Y))^2 = 0.52876$$
$$SSR = \Sigma(e_t)^2 = 0.14632$$

(cont.

Define the relationship between SST, SSE and SSR. Compute SSE and R^2 for the above regression, and explain the meaning of R^2. [10%]

(vii) For Firm AAA the values of the variables are SALES = 310 and L = 10, K=200, RD=1. Compute the least squares residual. Show your computations. [10%]

(viii) Predict the value of SALES for a firm with L = 75, K=125 and no investment in R&D. [10%]

(ix) Discuss the limitations of this model in the current problem setting. [10%]

END OF PAPER

Statistical Tables

Table 1 **Area Under the Standard Normal Distribution**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Source: This table was generated using the SAS® function PROBNORM.

Table 2 Right-Tail Critical Values for the *t*-distribution

DF	α = .10	α = .05	α = .025	α = .01	α = .005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

Source: This table was generated using the SAS® function TINV.

Table 3 **Right-Tail Critical Values for the *F*-Distribution**

$\upsilon_2 \backslash \upsilon_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
									Upper 5% Points										
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.1	251.14	252.2	253.25	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	301	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.95	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: This table was generated using the SAS® function FINV. υ_1 = numerator degrees of freedom; υ_2 = denominator degrees of freedom.