ENGINEERING TRIPOS PART IIA

Thursday 11 May 2006 9 to 10.30

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain why intersymbol interference (ISI) may occur in a baseband digital transmission system and what is the effect of ISI on system bit error rate (BER) performance. [15%]

(b) Show that Nyquist's pulse shaping criterion for zero ISI is given by

$$\sum_{k=-\infty}^{\infty} H\left(\omega + \frac{k2\pi}{T}\right) = T \ ,$$

where $H(\omega)$ is the system frequency response and T is the time between successive symbols. [30%]

(c) Suppose the received pulse spectrum in a baseband digital transmission system is

$$H(\omega) = \frac{3T}{2}\text{sinc}^2\left(\frac{3\omega T}{4}\right) \ ,$$

where T is the time between successive symbols.

(i) Show that the corresponding time domain pulse shape for $H(\omega)$ is

$$h(t) = \begin{cases} 1 - \dfrac{2|t|}{3T} & |t| \le \dfrac{3T}{2} \\ 0 & \text{otherwise} \end{cases} \ .$$

(ii) Determine the worst-case BER performance assuming polar line coding, equiprobable binary data, an optimum decision threshold and a noise rms value of 0.1 V .

(iii) Assuming that all system parameters are fixed, how might the BER be improved?

[55%]

Note the Gaussian error integral approximation is

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

2 (a) Define what is meant by a systematic linear binary block code. [20%]

(b) A code of the type specified in (a) has the following generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Determine the minimum Hamming distance for this code and hence its error detecting and correcting abilities. [25%]

(c) Explain the principles of syndrome decoding for linear block codes and show that the error syndrome is independent of the transmitted codeword. [30%]

(d) (i) Show that a parity check matrix H, for the code specified by G in part (b) is

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

(ii) Determine modified versions of H and G that permit syndrome decoding to be performed more easily by letting the error syndrome be the binary address of the bit to be corrected. For example, error vector [1 0 0 0 0 0] could be required to yield error syndrome [0 0 1]. [25%]

3 (a) Write down expressions for the phasor waveforms $p(t)$ of *Binary Phase-Shift Keying* (BPSK) modulation and of *Quadrature Phase-Shift Keying* (QPSK) modulation, when the bit period is T_b and the binary data stream is $b_k = \pm 1$, for all integers k. [25%]

(b) Derive an expression for the power spectral density (or power spectrum) of the phasor waveform of a BPSK signal with bit period T_b, assuming a random binary data stream and a rectangular shaping pulse of amplitude a_0 and width T_b. [25%]

(c) Using your answer to part (b), or otherwise, obtain an expression for the power spectral density of the phasor waveform of a QPSK signal with the same overall data rate. Sketch the power spectra for the BPSK and QPSK cases. [25%]

(d) Describe the key advantage of QPSK over BPSK modulation and explain how this arises. What are the disadvantages of QPSK (if any)? Give an example of an application where QPSK is used. [25%]

4 (a) Explain the meanings of the terms: energy per bit (E_b) , energy per symbol (E_s) and energy of the $g(t)$ shaping pulse (E_g) as they apply to the analysis of multi-level modulation schemes. Describe how the terms relate to each other.

Why is Gray coding often applied in multi-level modulation schemes? [25%]

(b) The mean probability of symbol error for optimally demodulated M^2-level quadrature amplitude modulated (M^2-QAM) signals is given by:

$$P_{SE} = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{2E_g}{N_0}}\right)$$

where N_0 is the noise power spectral density at the receiver input and $Q(x)$ is the Gaussian error integral function.

Hence derive an expression for the probability of bit error P_{BE} as a function of the ratio E_b/N_0 at the receiver input, for the complete M^2-QAM demodulator, if $M = 2^m$ and m is a positive integer. Assume m-bit Gray coding for the M levels is used. [30%]

(c) If the desired maximum value of P_{BE} is 10^{-3} , estimate the increase in E_b (in decibels) required to support 64-QAM , compared with 4-QAM (QPSK). [30%]

(d) Give reasons why 64-QAM has been chosen as the basic modulation scheme for terrestrial digital video broadcasts. [15%]

Note that

$$\sum_{i=0}^{M-1} (2i + 1 - M)^2 = \frac{M(M^2 - 1)}{3}$$

and

$$Q(2.928) = 0.00171$$
$$Q(3.09) = 0.001$$

END OF PAPER

Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2006- Answers

1. Generally well answered. Most candidates had difficulty in showing the result required in part (b). This was surprising since it is a bookwork example.

a) See notes.
b) See notes.
c)
(i)
(ii) $Q(3.33) = 4.3\times10^{-4}$
(iii) Add equalisation. Add Forward Error Correction (FEC).

2. This question was the most popular question and was in general answered very well. A few candidates used an incorrect assumption when determining the minimum Hamming distance in part (b).

a) See notes.
b) $d_{min} = 3$. Max no. of detectable errors = 2. Max no. correctable errors = 1.
c) See notes.
d)
(i)
(ii) $H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ $G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

3. This question was answered quite poorly. Part (a) was generally answered quite well. The evaluation of the power spectrum in part (b) posed most problems to candidates.

a) See notes.
b) See notes.
c) See notes.
d) See notes.

4. This question was generally well answered.

a) See notes.
b) $P_{BE} = \frac{2}{m}\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{3m}{M^2-1}\frac{2E_b}{N_0}}\right)$.
c) 7.98 dB.
d) See notes.

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