Engineering Tripos Part IIA 2007

Answers

1. (c) $\dot{\phi} = \dfrac{mga}{C\omega} = \sqrt{\dfrac{mga}{A}} = \dfrac{C\omega}{A}$ (d) 120°

2. (a) $m(a^2+L^2)/12,\ m(a^2+L^2)/12,\ ma^2/6$
   (b) $\boldsymbol{\omega} = 0\,\boldsymbol{i} - \Omega\sin\theta\,\boldsymbol{j} + \Omega\cos\theta\,\boldsymbol{k}$
   (i) $\boldsymbol{h} = -A\Omega\sin\theta\,\boldsymbol{j} + C\Omega\cos\theta\,\boldsymbol{k}$
   (ii) $\boldsymbol{h} = (C-A)\Omega\cos\theta\sin\theta\,\boldsymbol{J} + (A\Omega\sin^2\theta + C\Omega\cos^2\theta)\,\boldsymbol{K}$
   product of inertia $= -(C-A)\cos\theta\sin\theta$
   (c) $Q_1 = (A-C)\Omega^2\cos\theta\sin\theta$
   (d) $L = a$

3. (a) $\boldsymbol{Q} = -mga\cos\theta\,\boldsymbol{j}$ (b) $2\sqrt{\dfrac{g}{a\tan\theta}}$ (c) $\sqrt{\dfrac{g\theta^3}{a}}$

4. (a) $V = \tfrac{1}{2}k\theta^2 - m_1gx - m_2g(x - L(1-\cos\theta))$
   (b) $(m_1+m_2)\ddot{x} - m_2L\ddot{\theta}\sin\theta - m_2L\dot{\theta}^2\sin\theta\cos\theta + kx - (m_1+m_2)g = 0$
   $L\ddot{\theta} - \ddot{x}\sin\theta + g\sin\theta = 0$
   (c) $(m_1+m_2)\ddot{x} + kx = (m_1+m_2)g$
   $L\ddot{\theta} + g\theta = 0$
   (d) $M = \begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2L^2 \end{bmatrix} \quad K = \begin{bmatrix} k & 0 \\ 0 & m_2gL \end{bmatrix}$

5. (b) $\dfrac{I\ddot{x}}{L^2-x^2} + \dfrac{I\dot{x}^2x}{(L^2-x^2)^2} = \dfrac{F}{2} - \dfrac{Px}{2\sqrt{L^2-x^2}}$
   (c) $T = \tfrac{1}{2}I\dot{\theta}^2 \qquad Q = \dfrac{FL\cos\theta}{2} - \dfrac{PL\sin\theta}{2}$

HEMH May 2007