

Module 3F1, April 2007 – SIGNALS AND SYSTEMS – Answers

1 (b) (i) $G(s) = \frac{1}{s(s+1)}$. Both systems are (marginally) unstable.

(ii) $k > 0$.

(c) (i)

$$|z| = \frac{\sqrt{1 + 2\sigma + \sigma^2 + \omega^2}}{\sqrt{1 - 2\sigma + \sigma^2 + \omega^2}}$$

2 (a) (i) $g_k = p^k, k = 0, 1, 2, \dots$

(ii) $y_k = \sum_{i=0}^k u_{k-i} g_i = \sum_{i=0}^k u_{k-i} p^i$

(b) (ii) $r_{XX}(t_1, t_2) = \frac{1}{3} r_{UU}(t_1, t_2)$. The random process is WSS.

3 (b) $\Phi_Y(u) = \Phi_{X_1}(u) \Phi_{X_2}(u)$.

(c) $\Phi_{X_1}(u) = \text{sinc}(ub_1/2), \quad \Phi_{X_2}(u) = \text{sinc}^2(ub_2/2),$
 $\Phi_Y(u) = \text{sinc}(ub_1/2) \text{sinc}^2(ub_2/2).$

4 (a) $P(A) = 0.8, P(B) = 0.1, P(C) = 0.1$.

(b) Efficiency = 91.92%.

(c) $I(X_{n+1}; X_n) = 0.1525$ and $H(X_{n+1}|X_n) = 0.7694$.