2007 3A1
Prof Britter

(1) a) $F(z) = \frac{-m}{2\pi}\ln(z+a) + \frac{m}{2\pi}\ln(z-a) + Uz$ [20,

b) $\frac{dF}{dz} = u - iv = 0 = \frac{-m}{2\pi(z+a)} + \frac{m}{2\pi(z-a)} + U = 0$

Stag. pts

$$\frac{m}{2\pi}\left(\frac{1}{z-a} - \frac{1}{z+a}\right) = -U$$

$$\frac{z+a-(z-a)}{z^2-a^2} = \frac{-2\pi U}{m}$$

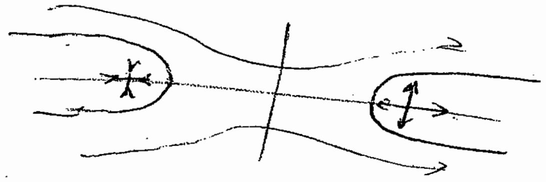
$$\frac{2a}{z^2-a^2} = \frac{-2\pi U}{m}$$

$$z^2 - a^2 = \frac{-ma}{\pi U}$$

$$z^2 = a^2 - \frac{ma}{\pi U}$$

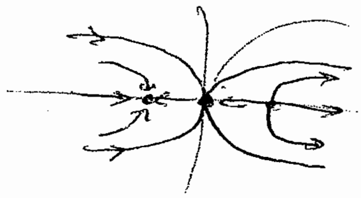
$$\frac{z}{a} = \pm\sqrt{1 - \frac{m}{\pi a U}}$$

$\frac{m}{\pi a U} < 1$ both real stagpts

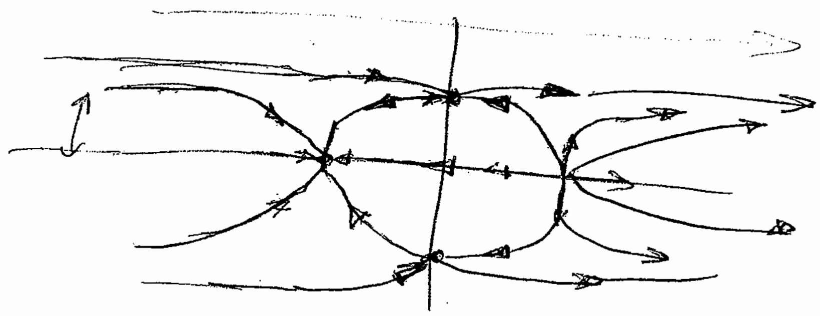


(1) cont'd

If $\frac{m}{\pi a U} = 1$ $\frac{z}{a} = \pm 0$ i.e. both stag pts at the



$\frac{m}{\pi a U} > 1$ two complex stag p



(c) Looking at the patterns then if

$\frac{m}{\pi a U} \leq 1$ there is no fluid from source that reaches the sink

(d) Want streamline that passes through the stagnation point

let $z+a = R_1 e^{i\theta_1}$

$z-a = R_2 e^{i\theta_2}$

$$F(z) = \phi + i\psi = -\frac{m}{2\pi}\ln(R_1) - \frac{im}{2\pi}\theta_1$$

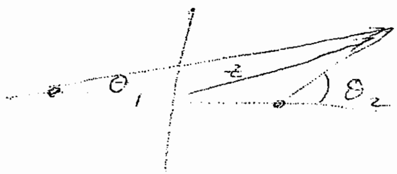
$$+ \frac{m}{2\pi}\ln(R_1) + \frac{im}{2\pi}\theta_2 + Ux + iUy$$

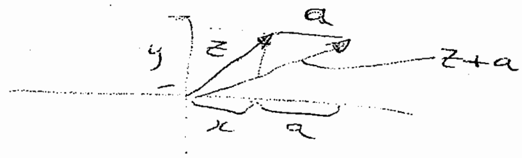
① cont'd

(d) cont'd.

$$\psi = -\frac{m}{2\pi}\theta_1 + \frac{m}{2\pi}\theta_2 + Uy$$

~~tot it pe~~ want the one that passes through the stagnation point





$\theta_1 = \arctan\left(\frac{y}{x+a}\right)$

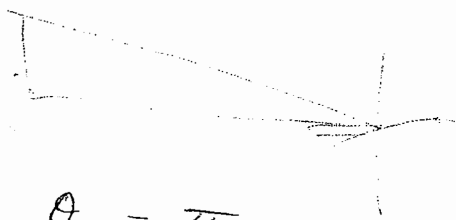
$\theta_2 = \arctan\left(\frac{y}{x-a}\right)$

Stagnation pt is at $y=0$

$\theta_1 = 0 \qquad \theta_2 = \pi \quad y=0$

$\psi = \frac{m}{2}$ is the S/L passing through the stag point

Now go way upstream



$x \to -\infty$

$\theta_1 = \pi, \quad \theta_2 = \pi$

$$\frac{m}{2} = -\frac{m}{2} + \frac{m}{2} + Uy$$

$\frac{m}{\pi a U} < 1$

$Uy = \frac{m}{2} \Rightarrow y = \frac{m}{2U}$

$\frac{m}{U} < \pi a$

$\frac{m}{2U} < \frac{\pi a}{2}$

$(y > \frac{\pi a}{2}$

② (a) $F(z) = \frac{-i\Gamma}{2\pi}\ln(z-ib) + \frac{i\Gamma}{2\pi}\ln(z+ib)$

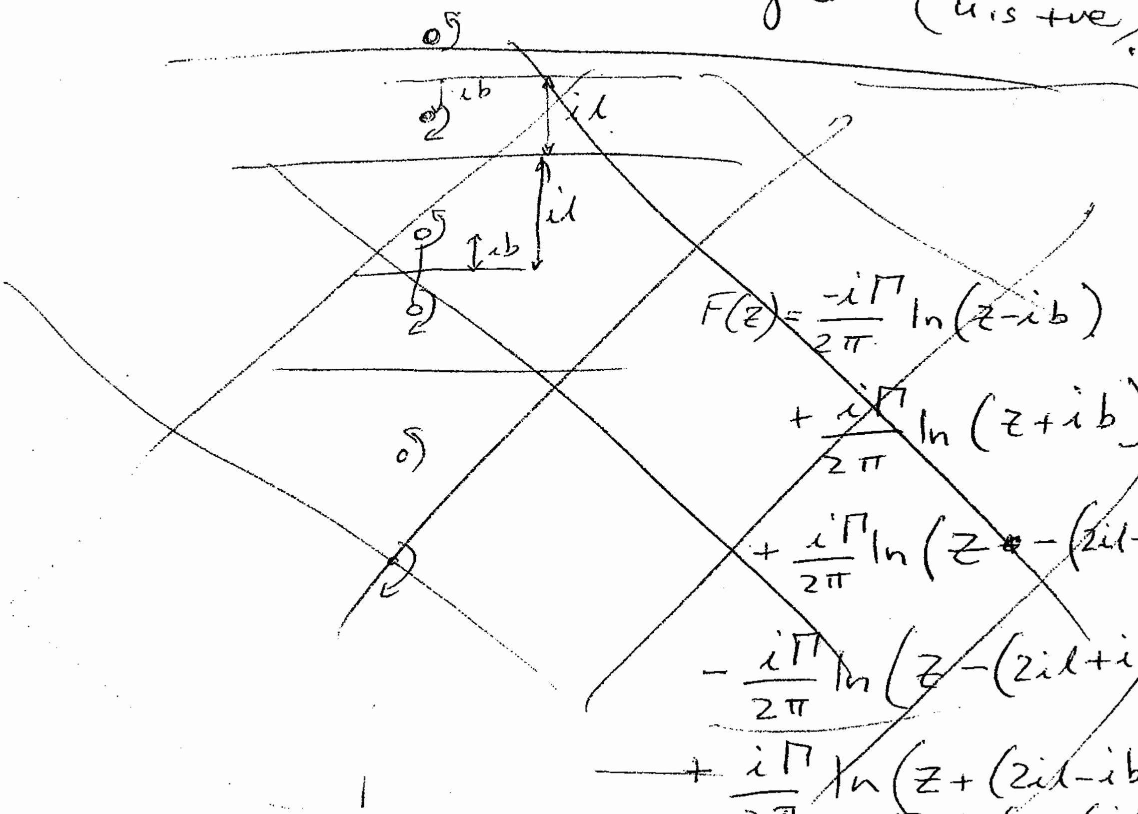
$$\frac{dF}{dz} = \frac{-i\Gamma}{2\pi}\frac{1}{z-ib} + \frac{i\Gamma}{2\pi}\frac{1}{z+ib}$$

look at the point $z = ib$

$$\frac{dF}{dz} = u - iv = \underbrace{\frac{-i\Gamma}{2\pi}\frac{1}{0}}_{\substack{\text{neglect}\\ \text{self-induced but}}} + \frac{i\Gamma}{2\pi}\frac{1}{2ib}$$

$u - iv = \frac{\Gamma}{4\pi b}$ $\quad u = \frac{\Gamma}{4\pi b}$ $\quad v = 0$ as expected

to the right (u is +ve)



$F(z) = \frac{-i\Gamma}{2\pi}\ln(z-ib)$

$+ \frac{i\Gamma}{2\pi}\ln(z+ib)$

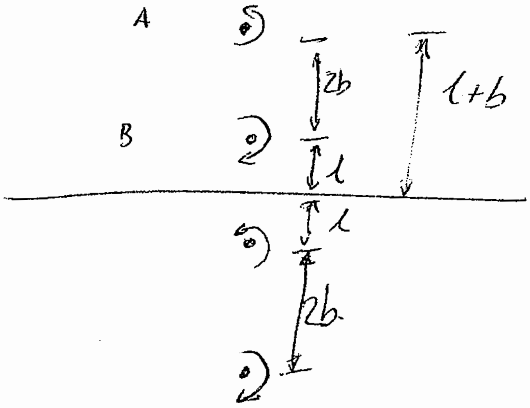
$+ \frac{i\Gamma}{2\pi}\ln(z-(2il-ib)$

$- \frac{i\Gamma}{2\pi}\ln(z-(2il+ib)$

$+ \frac{i\Gamma}{2\pi}\ln(z+(2il-ib))$

$- \frac{i\Gamma}{2\pi}\ln(z-(2il-ib$

(2) (b) Need an image system.



$$F(z) = \frac{-i\Gamma}{2\pi}\ln\left(z - (2b+l)i\right)$$
$$+ \frac{i\Gamma}{2\pi}\ln\left(z - il\right)$$
$$- \frac{i\Gamma}{2\pi}\ln\left(z + il\right)$$
$$+ \frac{i\Gamma}{2\pi}\ln\left(z + (2b+l)i\right)$$

$$\frac{dF}{dz} = \frac{-i\Gamma}{2\pi}\left[\frac{1}{z-(2b+l)i} - \frac{1}{z-il} + \frac{1}{z+il} - \frac{1}{z+(2b+l)i}\right]$$

Look at top vortex $z = (2b+l)i$ (ignore 1st term which is veloc induced on itself)

$$\frac{dF}{dz} = u - iv = \frac{-i\Gamma}{2\pi}\left[-\frac{1}{2bi} + \frac{1}{2(b+l)i} - \frac{1}{(4b+2l)i}\right]$$
$$= \frac{\Gamma}{2\pi}\left[\frac{1}{2b} - \frac{1}{2(b+l)} + \frac{1}{4b+2l}\right]$$

$v = 0$ (cl... c...) (of course)

2 (b) cont'd

dE0

Top vortex moves with speed

$$u_{upp} = \frac{\Gamma}{2\pi}\left[\frac{1}{2b} - \frac{1}{2b+2l} + \frac{1}{4b+2l}\right]$$

Note we could have done this without the complex potential

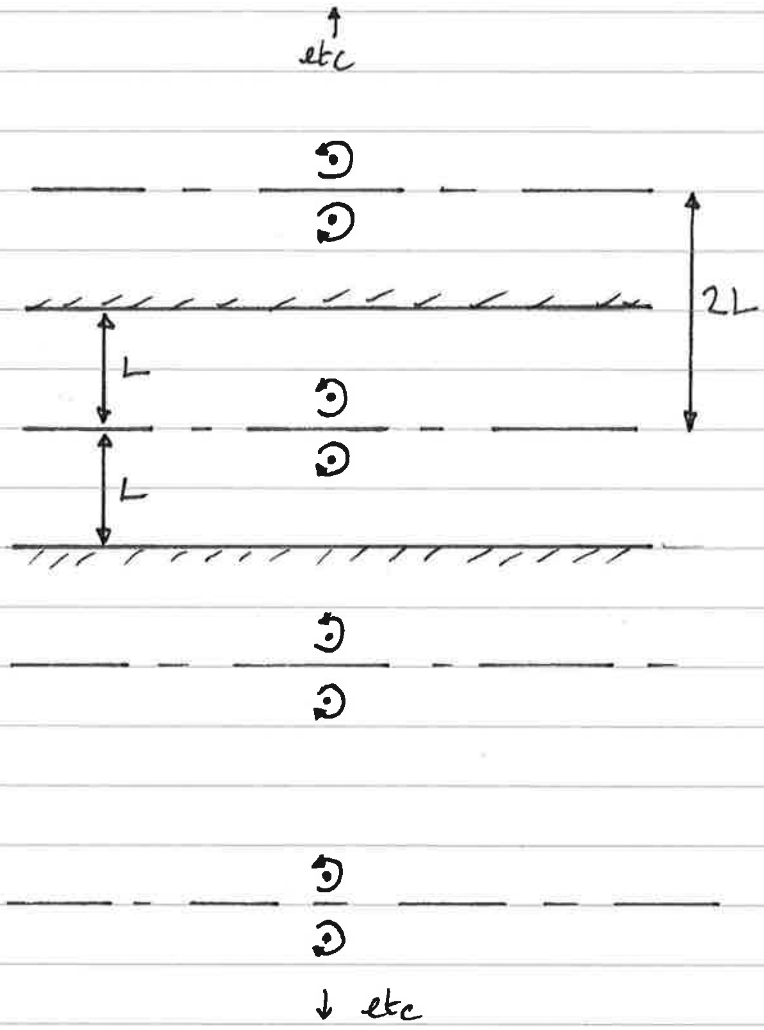
Just based on $\frac{\Gamma}{2\pi r}$

which is to the right

Lower one by same method or simply based on $\frac{\Gamma}{2\pi r}$ gives

$$u_{lower} = \frac{\Gamma}{2\pi}\left[\frac{1}{2b} - \frac{1}{2l} + \frac{1}{2b+2l}\right]$$

c) Now we need an infinite array of images:



The most straightforward approach is to note that this can be viewed as a collection of the original configuration, with its members spaced at 2L.

Original streamfunction (for origin between vortices) can be written

$$F_0(z) = -\frac{i\Gamma}{2\pi} \ln \frac{z-ib}{z+ib}$$

Streamfunction for k'th image:

$$F_k(z) = -\frac{i\Gamma}{2\pi} \ln \frac{z-i2Lk-ib}{z-i2Lk+ib}$$

Hence:

$$F(z) = -\frac{i\Gamma}{2\pi} \sum_{k=-\infty}^{\infty} \ln \frac{z-i2Lk-ib}{z-i2Lk+ib}$$

N.B. Other equivalent forms are acceptable, for example

$$F(z) = -\frac{i\Gamma}{2\pi} \sum_{k=-\infty}^{\infty} \ln \frac{z-i(b+2Lk)}{z+i(b+2Lk)},$$

which follows from counting all vortices with anti-clockwise rotation upwards, and all with clockwise downwards.

3A1 2006-7 Cribs

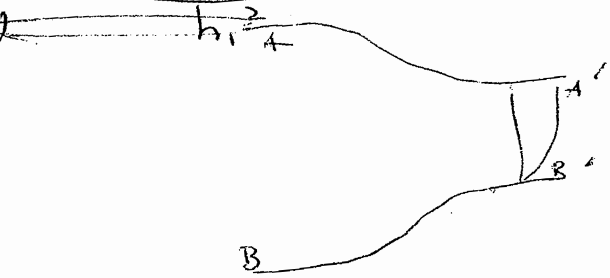
(3)

a) Vorticity $= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ Assume $\frac{\partial v}{\partial x}$ negligible

$$\omega_z = -\frac{\partial U_1}{\partial y} = \frac{-2U_e y}{h_1^2}$$ (20%)

b) To find U_2 use the fact that vorticity is conserved along a streamline

$$\cancel{-\frac{\partial U_2}{\partial y} = \frac{-2U_e y}{h_1^2}}$$



Vorticity at A is $-\frac{\partial U_1}{\partial y} = -\frac{2U_e h_1}{h_1^2}\Big|_{\text{at } y = h_1}$

Should be the same as at A'

$$\omega_z \quad -\frac{\partial U_2}{\partial y} = -\frac{2U_e}{h_1}\Big|_{\text{at } y = h_2}$$

At B vorticity is zero $\omega_z = \frac{-\partial U_1}{\partial y} = \frac{-2U_e \times 0}{h_1^2} = 0$

So at B' $\omega_z = 0$

$$-\frac{\partial U_2}{\partial y} = 0 \Big|_{\text{at } y = h_2}$$

③ cont'd (b) we can compare more points but this is sufficient to see it is also a parabola.

(eg take C/L $y = \frac{h_1}{2}$ $\omega_z = \frac{-2 U_e h_1}{2 h_1^2} = \frac{-U_1}{h_1}$

$$-\frac{\partial U_2}{\partial y} = \frac{-U_e}{h_1}$$

we see it is varying linearly across hence the profile must be parabolic

So

$$U_2 = U_{o2} + U_{e2}\left(\frac{y}{h_2}\right)^2$$

$$\frac{\partial U_2}{\partial y} = \frac{2 U_{e2}\, y}{h_2^2}$$

at $y = h_2$

$$\frac{\partial U_2}{\partial y} = \frac{\cancel{2} U_{e2}}{h_2} = \frac{\cancel{2} U_e}{h_1}$$

$$U_{e2} = U_e \frac{h_2}{h_1}$$

Now continuity.

Volume flux at station ① is

$$Q = \int_0^{h_1} u_1\, dy = \int_0^{h_1} U_o + U_e\left(\frac{y}{h_1}\right)^2 dy$$

$$= \left[U_o y + \tfrac{1}{3} U_e\left(\frac{y^3}{h_1^2}\right)\right]_0^{h_1}$$

$$= U_o h_1 + \tfrac{1}{3} U_e h_1 = h_1\left(U_o + \tfrac{1}{3} U_e\right)$$

Downstream

Flow rate

$$\int_0^{h_2} u_2\,dy = \int_0^{h_2} U_{o2} + U_{e2}\left(\frac{y}{h_2}\right)^2 dy$$

$$= \left[U_{o2}\,y + \tfrac{1}{3}U_{e2}\frac{y^3}{h_2^2}\right]_0^{h_2}$$

$$= \cancel{U_{o2}h_2} \; h_2\left(U_{o2} + \tfrac{1}{3}U_{e2}\right)$$

But $U_{e2} = U_e\frac{h_2}{h_1}$ and flow rate at (2) = flow rate at (1)

$$h_2\left(U_{o2} + \tfrac{1}{3}U_e\frac{h_2}{h_1}\right) = h_1\left(U_o + \tfrac{1}{3}U_e\right)$$

$$U_{o2} + \frac{1}{3}U_e\frac{h_2}{h_1} = \frac{h_1}{h_2}\left(U_o + \tfrac{1}{3}U_e\right)$$

$$U_{o2} = \frac{h_1}{h_2}U_o + \frac{1}{3}U_e\frac{h_1}{h_2} - \frac{1}{3}U_e\frac{h_2}{h_1}$$

$$= \frac{h_1}{h_2}U_o + \frac{1}{3}U_e\left(\frac{h_1^2 - h_2^2}{h_1h_2}\right)$$

(c) $\frac{U_{e2}}{U_{o2}} = 0.01$
$$\frac{U_e\frac{h_2}{h_1}}{\frac{h_1}{h_2}U_o + \frac{1}{3}U_e\left(\frac{h_1^2-h_2^2}{h_1h_2}\right)} = \frac{\frac{U_e}{U_o}\,h_2/h_1}{\frac{h_1}{h_2} + \frac{1}{3}\left(\frac{h_1^2-h_2^2}{h_1h_2}\right)}$$

3 cont'd
(c) cont'd.

let $\frac{h_1}{h_2} = \alpha$

$$\frac{0.1\,\frac{h_2}{h_1}}{\frac{h_1}{h_2} + \frac{1}{3}\left(\frac{h_1}{h_2} - \frac{h_2}{h_1}\right)} = 0.01$$

$$\frac{0.1\,\frac{1}{\alpha}}{\alpha + \frac{1}{3}\left(\alpha - \frac{1}{\alpha}\right)} = 0.01$$

$$\frac{1}{\alpha^2 + \frac{1}{3}\alpha^2 - 1} = 0.1$$

$$\frac{2}{3}\alpha^2 - 1 = 10$$

$$\frac{2}{3}\alpha^2 = 9$$

$$\alpha^2 = \frac{27}{2}$$

$$\alpha = 3.674$$ (to too many places.

QUESTION 4

a) FOR LARGE REYNOLDS NUMBER THE VISCOUSLY INFLUENCED REGION IS VERY THIN COMPARED WITH ~~THE~~ THE BODY DIMENSION. THUS THE COMPLEX FLOW CAN BE BROKEN INTO AN INVISCID FLOW AND A THIN BOUNDARY LAYER, ANALYSED SEPARATELY AND THEN COMBINED TO PROVIDE AN OVERALL SOLUTION. THE TRANSFER PROCESSES ~~FROM~~ BETWEEN SOLID AND FLUID ARE DOMINATED BY THE FLOW AND STRUCTURE OF THE BOUNDARY LAYER

(b) (i) MOMENTUM TRANSFER

— LAMINAR — BY VISCOSITY, DETERMINES SHEAR STRESS AT WALL

— TURBULENT — FLUID PARTICLE EXCHANGE MUCH STRONGER THAN VISCOSITY; LAMINAR/VISCOU SUBLAYER LINKS FLUID TO SOLID

(ii) HEAT TRANSFER — SAME AS FOR MOMENTUM TRANSFER AS $\nu = \alpha$ AND ANALOGY BETWEEN TURBULENT HEAT AND MOMENTUM TRANSFER — THERMAL CONDUCTIVITY PRODUCES THERMAL TRANSFER BETWEEN FLUID & SOLID

(iii) TURBULENT GROWS MUCH FASTER AS INFORMATION TRANSFERRED BY PARTICLE MOTION RATHER THAN MOLECULAR PROCESSES

(iv) MOMENTUM ANALYSIS SHOWS THAT TURBULENT B/L MUST PRODUCE THE LARGER SURFACE SHEAR STRESS DUE TO THE GREATER CHANGE OF MOMENTUM DEFICIT. SIMILAR ~~RES~~ RESULT FOLLOWS FROM ARGUING THAT TURBULENCE PRODUCES A MORE UNIFORM FLOW IN THE BOUNDARY LAYER BY MIXING; BUT THE VELOCITY MUST FALL SHARPLY TO ZERO AT THE WALL PRODUCING A LARGE SURFACE SHEAR STRESS

Q4 CONTINUED

(b) BY SIMILAR ANALOGY THE HEAT FLUX WILL BE ~~FAR~~ FAR LARGER FOR THE TURBULENT CASE.

(c) IN ALL CASES THE TURBULENT FLOW WILL PRODUCE THE LARGER HEAT FLUX. FOR SMALL PRANDTL NUMBERS ν/α THE HEAT FLUX WILL BE LARGER THAN $Pr = 1$ DUE TO LARGER THERMAL DIFFUSIVITY.

SIMILARLY HEAT FLUXES WILL BE REDUCED ~~BY~~ WITH A Pr SMALLER THAN UNITY.

QUESTION 5

a) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \therefore \frac{\partial v}{\partial y} = 0$

$\therefore \; v = -V_0$ everywhere

$\frac{\partial u}{\partial x} = 0 \quad \frac{\partial P}{\partial x} = 0$

$\therefore \; v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$ from Prandtl's B/L approximation

$-V_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$

$-V_0 u = \nu \frac{\partial u}{\partial y} + \text{const}$

as $y \rightarrow \infty \quad \frac{\partial u}{\partial y} = 0$

$-V_0 U_0 = \text{const}$

$\therefore \; -V_0 u = \nu \frac{\partial u}{\partial y} - V_0 U_0$

i.e. $V_0(U_0 - u) = -\nu \frac{\partial}{\partial y}(U_0 - u)$

$(U_0 - u) = \text{const } e^{-\frac{V_0}{\nu} y}$

$y = 0 \quad \text{const} = U_0$

$\therefore \; U_0 - u = U_0 e^{-\frac{V_0}{\nu} y}$

$u = U_0\left(1 - e^{-\frac{V_0}{\nu} y}\right)$

b) $\delta^* = \int_0^\infty \frac{(U_\infty - u)}{U_\infty} dy = \frac{\int_0^\infty U_0 e^{-\frac{V_0}{\nu} y} dy}{U_0}$

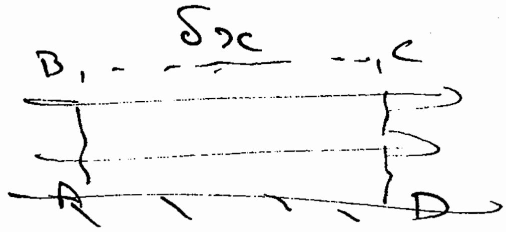
$\delta^* = \int_0^\infty e^{-V_0/\nu\, y} dy = -\nu/V_0 \, e^{-V_0/\nu\, y}\Big|_0^\infty$

$= \nu/V_0$

$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu \frac{U_0 V_0}{\nu} = \rho U_0 V_0$

Q5 CONTINUED

(c)



x - momentum flux across AB and CD are equal $\therefore$ no net ~~mom~~ x - moment. flux ho

x - moment. flux out across BC

$-\rho V_0 U_0$

x - moment. flux out across AD $= 0$

$\therefore$ net efflux of moment $= -\rho V_0 U_0 (\delta x)$ and this must equal the sum of external forces; only external force is

$- \tau_w (\delta x)$

$\therefore \; -\tau_w \cancel{\delta x} = -\rho V_0 U_0 \cancel{\delta x}$

$\tau_w = \rho U_0 V_0$

(d) The result from (c) is independent of whether the flow is laminar or turbulent; this demonstrates the strength of the integral momentum equation

6. (a) The variation of pressure distribution with incidence is shown in Fig. 11.4.2 of Duncan, Thom and Young, 'Mechanics of Fluids', reproduced overleaf.

$\alpha = 1°$

There is a stagnation point very close to the leading edge. The flow then accelerates away, around the lower and upper surfaces, and the pressure drops. The minimum pressure is lower on the upper surface, so the aerofoil generates lift. As the trailing edge is approached, the flow on both surfaces decelerates towards the free-stream value ($c_p = 0$), with associated mild adverse pressure gradients.

$\alpha = 7°$

At this incidence, the stagnation point moves slightly aft on the lower surface, and the position at which the upper surface is horizontal moves forward. The flow in the region between the stagnation point and this location then exhibits greater curvature, and the characteristic 'suction peak' appears. Adverse pressure gradients thus increase on the upper surface; on the lower they disappear.

$\alpha = 12°$

The same factors as for $7°$ are at play; suction peak is more extreme.

$\alpha = 18°$

The afterwards shift of the stagnation point is now clearly visible. There is still strong suction around the leading edge, but then the upper surface flow separates, leading to a flat region and reduced trailing edge pressure.

(b) $\frac{dy_c}{dx} = \frac{h_1}{c}\left(1 - \frac{2x}{c}\right) = -\frac{h_1}{c}\cos\theta$; hence $g_0 = 0$, $g_1 = \frac{2h_1}{c}$ (data card)

$$C_L = 2\pi\alpha + \pi(g_0 + g_1/2) = \pi h_1/c \quad \text{at } \alpha = 0$$

$$\Rightarrow \underline{\underline{h_1 = 0.1c}} \quad \text{for } C_L = \pi/10$$

(c) $$\frac{dy_c}{dx} = \frac{h_2}{c}\left[7 - 30\frac{x}{c} + 24\left(\frac{x}{c}\right)^2\right]$$

$$= \frac{h_2}{c}\left[7 - 15(1+\cos\theta) + 6(1+\cos\theta)^2\right]$$

$$= \frac{h_2}{c}\left[-2 - 3\cos\theta + 6\cos^2\theta\right]$$

$$= \frac{h_2}{c}\left[1 - 3\cos\theta + 3\cos 2\theta\right]$$

By inspection $g_0 = -2\frac{h_2}{c}$, $g_1 = 6\frac{h_2}{c}$, $g_2 = -6\frac{h_2}{c}$

$$C_L = \pi\left(g_0 + \frac{g_1}{2}\right) = \pi\frac{h_2}{c} \Rightarrow h_2 = 0.1c$$

see data card

(d) (i) The g_0 term in the Fourier series for the loading of the second aerofoil corresponds to a suction peak on the lower surface, so there will be an associated adverse pressure gradient subsequently. This will cause a loss of boundary layer momentum and increased drag.

(ii) The suction peak can be removed by increasing the incidence. This will cause an increase in lift coefficient, which can be returned to the desired value (without altering the centre of pressure) by scaling the camber-line.

Calculations (not required): choose $2\alpha + g_0 = 0$

then $C_L = \frac{\pi}{2} g_1 = 3\pi\frac{h_2}{c}$

so new value of h_2 is $c/30$, and $\alpha = 1/30$

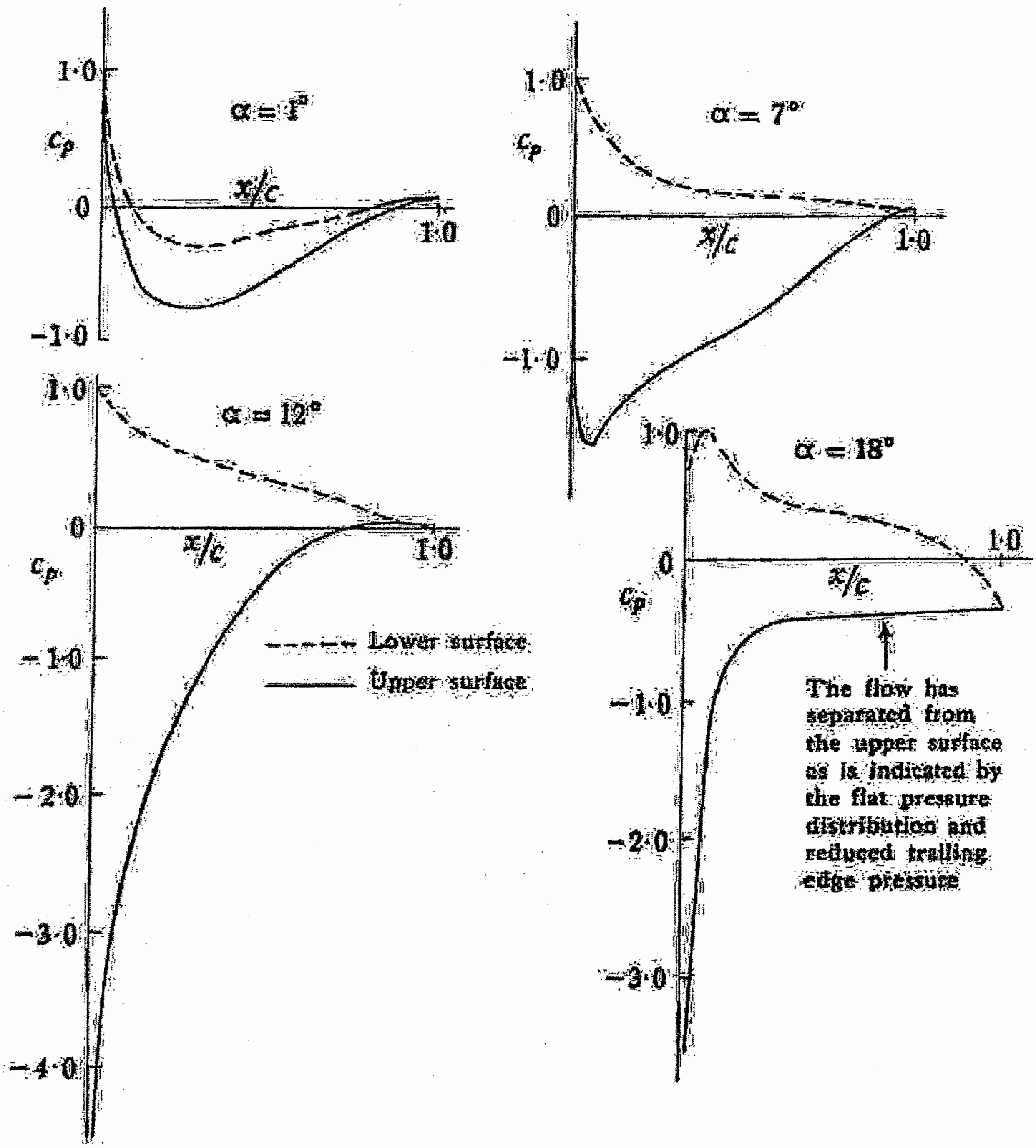


Fig. 11.4.2. Sketches of typical pressure distributions on a

7(a) For elliptical loading, $C_{Di} = \dfrac{C_L^2}{\pi A_R}$, ie $A_R = \dfrac{C_L^2}{\pi C_{Di}}$

Here $C_L = 0.5$, $C_{Di} = 0.01 \Rightarrow A_R = \dfrac{25}{\pi} = \underline{\underline{7.96}}$

(b) (i) Lift $L = \displaystyle\int_{-s}^{s} \rho U \Gamma \, ds = \int_0^{\pi} \rho U \Gamma s \sin\theta \, d\theta$

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 . 2sc} = \frac{1}{Uc}\int_0^{\pi} \Gamma \sin\theta \, d\theta \qquad c: \text{chord}$$

$$= \frac{G_1 s}{c}\int_0^{\pi} \sin^2\theta \, d\theta = \frac{\pi}{4} A_R G_1$$

$$C_L = 0.5 \Rightarrow G_1 = 2/25 = \underline{\underline{0.08}}$$

(ii) $$\alpha_d = \frac{w_d}{U} = -\frac{1}{4\pi U}\int_{-s}^{s} \frac{(-d\Gamma/d\eta)}{y - \eta}\, d\eta$$

Put $\eta = -s\cos\phi$ $\Gamma(\eta) = U s G_1 \sin\phi$, then

$$\alpha_d = \frac{G_1}{4\pi}\int_0^{\pi} \frac{\cos\phi}{\cos\phi - \cos\theta}\, d\phi = \frac{G_1}{4} \quad \text{(data card)}$$

$$= \underline{\underline{0.02}}$$

(iii) Local lift coefficient: $c_l(y) = 2\pi\left[\alpha + \alpha_t(y) - \alpha_d(y)\right]$

Now $c_l(y) = \dfrac{\rho U \Gamma(y)}{\frac{1}{2}\rho U^2 c} = A_R G_1 \sin\theta = \dfrac{2}{\pi}\sin\theta$

so $\dfrac{1}{\pi^2}\sin\theta + 0.02 = \alpha + \alpha_t(y)$

$\alpha_t = 0$ @ $\theta = 0, \pi \Rightarrow \underline{\underline{\alpha = 0.02}}$ $\qquad \underline{\underline{\alpha_t(y) = \frac{1}{\pi^2}\sin\theta}}$

c) Put $\Gamma = \Gamma_D + \delta\Gamma$, where Γ_D is the design distributi[on]

Then $$c_L(y) = \frac{\rho U}{\frac{1}{2}\rho U^2 c}\left[\Gamma_D + \delta\Gamma\right] = \frac{2}{Uc}\left[\Gamma_D + \delta\Gamma\right] = c_{LD} + \frac{2\delta\Gamma}{Uc}$$

and $$\alpha_d = \frac{w_d}{U} = -\frac{1}{4\pi U}\int_{-s}^{s} \frac{-\frac{d}{dy}\left[\Gamma_D + \delta\Gamma\right]}{y - y} \, dy$$

$$= \alpha_{dD} + \frac{1}{4\pi U}\int_{-s}^{s} \frac{\frac{d}{dy}(\delta\Gamma)}{y - y} \, dy$$

Now, in general $$c_L(y) = 2\pi\left[\alpha + \alpha_t(y) - \alpha_d(y)\right]$$

and, at design $$c_{LD}(y) = 2\pi\left[\alpha_D + \alpha_t(y) - \alpha_{dD}(y)\right]$$

The difference is : $$\frac{2}{Uc}\,\delta\Gamma(y) = 2\pi\left[(\alpha - \alpha_D) - \frac{1}{4\pi U}\int_{-s}^{s} \frac{\frac{d}{dy}(\delta\Gamma(y))}{y - y} \, dy\right]$$

which is the lifting-line equation for a rectangular wing without twist. Thus, $\delta\Gamma(y)$ will not be elliptical, and the wing will have sub-optimal induced drag away from the design point.

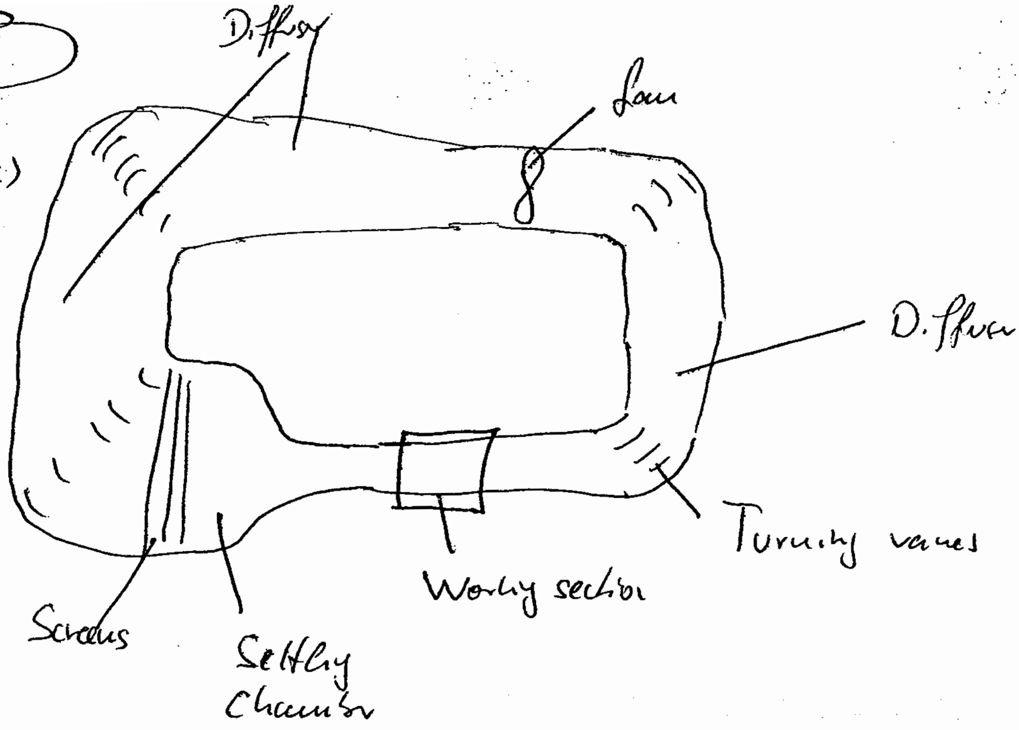
d) Both wings are equivalent at the design point. Off-design, the elliptical wing has the <u>advantage</u> that its induced drag is optimal. However, it has two <u>disadvantages</u>:

(i) its stall behaviour is likely to be less safe, due to its higher tip loading;

(ii) it may be harder to manufacture.

8

a)



b) Power factor : $\lambda = \dfrac{P}{\frac{1}{2}\rho A V^3}$

Because breather is in working section, pressure here is equal to $p_{Atm} \Rightarrow \rho = \rho_{Atm}$

Hence : $\lambda = \dfrac{\cancel{100HP \times 745.7 \frac{W}{HP}}\ 75{,}000}{\frac{1}{2} \times 1.225 \times \cancel{5.5 \times 4 \times (0.305)^2}\ 1.52 \times 1.22 \cdot 60^3} = \cancel{0.275}\ \cancel{0.375}\ 0.306$

30.6

$\Rightarrow$ The motor provides ~~27.5~~ % of the kin. energy in the working section.

This energy is lost as friction along the walls, turning vanes and screens, hence it is converted to heat.

c) Hot wires make use of the fact that the rate of forced convective heat transfer (of a heated wire) depends on the oncoming flow speed (or Re). Hence, if a known heat flux is provided to a wire exposed to the flow, its temperature depends on the flow speed. Even better, in const. temperature hot wire anemometry the temperature of the wire is kept const. and the heat flux (current) is measured. The electric current is related to the flow speed by King's law which is:

$$I^2 \cdot R = A + B\,V^{0.45} \quad (\text{or } I^2 = A + B\sqrt{V})$$

The constants A and B depend on many factors and the probe must therefore be calibrated.

d) The Markham tunnel has no heat exchanger. Therefore it is likely that the temperature inside the flow increases throughout a day's running (see b)). This however affects the constants in Kings law and will lead to errors, unless accounted for by repeated calibrations. Since the traverse starts at the top and ends at the floor, the effect of this error would be an apparent velocity gradient. This can be fixed by moving the probe repeatedly to the same location and performing a continuous adjustment of the calibration constants.