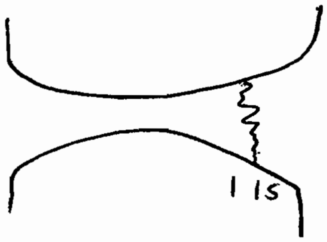
3A3 SOLUTIONS 2007

PART IIA 2007
3A3 Fluid mechanics II (double module)
Dr R Miller

(1) (a) 6.1% DROP IN p_o

$$\frac{p_{oE}}{p_o} = \frac{1-0.061}{1} = 0.939$$



$M_1 = 1.47$

$$\frac{\dot m\sqrt{c_p T_o}}{A_1 p_o} = 1.1077$$

$$\frac{A}{A_*} = \frac{1.281}{1.108} = \underline{\underline{1.156}}$$

(b) $\frac{A_E}{A^*} = 1.2$

$$\frac{\dot m\sqrt{c_p T_o}}{A_E p_{oE}} = \frac{\dot m\sqrt{c_p T_o}}{A^* p_o} \times \frac{p_o}{p_{oE}} \times \frac{A_*}{A_E}$$

$$= 1.281 \times \frac{1}{0.939} \times \frac{1}{1.2}$$

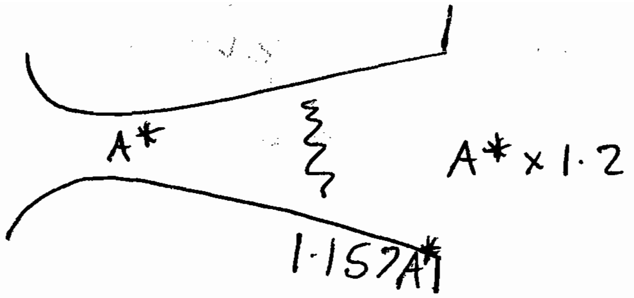
$$= 1.137$$

$$\underline{\underline{M_{EXIT} = 0.660}}$$

$$\frac{p_E}{p_o} = \frac{p_E}{p_{oE}} \times \frac{p_{oE}}{p_o} = 0.747 \times 0.939$$

$$= 0.701$$

1 (c)



$\frac{P_E}{P_0}$ CHANGES AND SHOCK MOVES BY 20% DOWNSTREAM

$$\frac{A_{1NEW}}{A^*} = 1.157 + 0.2 \times 0.2$$
$$= \underline{\underline{1.197}}$$

UPSTREAM SHOCK

$$\frac{\dot{m}\sqrt{C_p T_0}}{A_{1NEW} P_0} = \frac{\dot{m}\sqrt{C_p T_0}}{A^* P_0} \times \frac{A^*}{A_{1NEW}}$$

$$= 1.281 \times \frac{1}{1.197}$$

$$= 1.0702$$

$$M_1 = 1.530$$

$$M_{S1} = 0.691$$

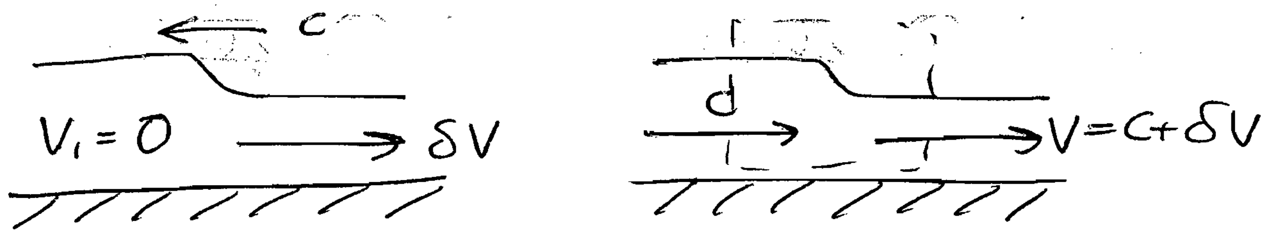
$$\frac{P_{0ENEW}}{P_0} = 0.92 \qquad \frac{\dot{m}\sqrt{C_p T_0}}{A_E P_{0ENEW}} = \frac{\dot{m}\sqrt{C_p T_0}}{A^* P_0} \times \frac{A^*}{A_E} \times \frac{[illegible]}{P_{0ENEW}}$$

$$= 1.281 \times \frac{1}{1.2} \times \frac{1}{0.92} = \underline{\underline{1.160}}$$

$$\frac{P_E}{P_0} = \frac{P_E}{P_{0E}} \times \frac{P_{0E}}{P_0} = 0.727 \times 0.92 = \underline{\underline{0.92}}$$

$$\text{PERC CHANGE} = \frac{0.701 - 0.67}{0.701} = \underline{\underline{4\%}}$$

2 a



$$h_0 g = hg + \frac{v^2}{2}$$

DIFF $$\delta h + \frac{2v\delta v}{2g} = 0$$

$$\delta h + \frac{v\delta v}{g} = 0$$

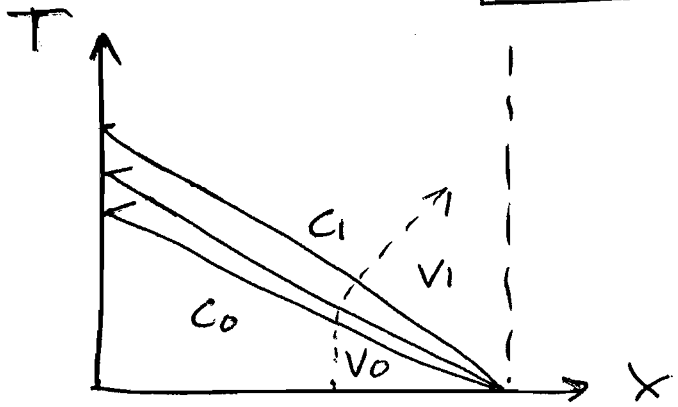
$c = \sqrt{gh}$ DIFF $2c\delta c = g\delta h$

Remove δh

$$2\delta c + \delta v = 0$$

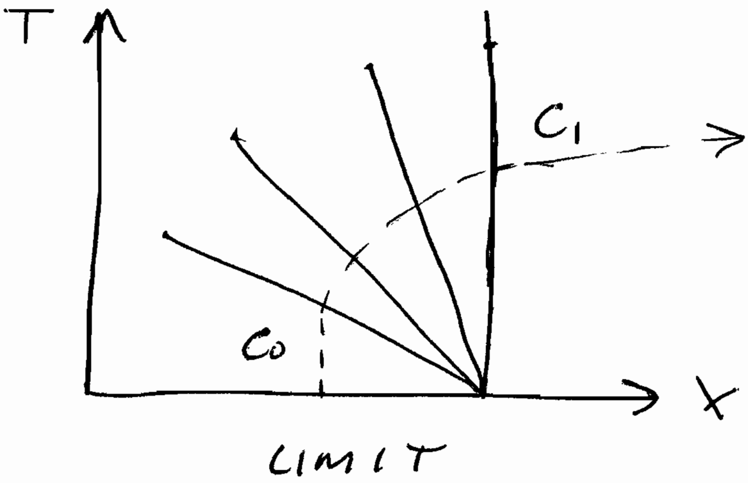
$$\boxed{\delta(v+2c) = 0}$$

2 b



AS $\frac{h_0}{h_1}$ RISES V_1 RISES AND BACK OF WAVE C_1 MOVES SLOWER RELATIVE TO CHANNEL FLOOR.

(2) (b) CONT



$$V_1 = C_1$$

$$V_0 + 2C_0 = V_1 + 2C_1$$

$V_0 = 0$

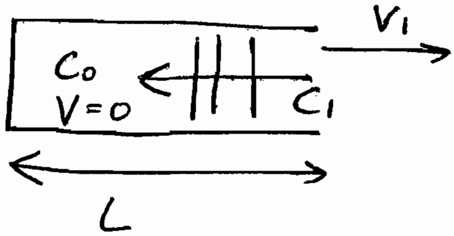
$$2C_0 = 3C_1$$

$$\frac{C_0}{C_1} = \frac{3}{2} \qquad \sqrt{\frac{h_0}{h_1}} = \frac{3}{2}$$

$$\frac{h_0}{h_1} = \left(\frac{3}{2}\right)^2 = 2.25$$

WHEN $\frac{h_0}{h_1} = 2.25$ THE VELOCITY AT EXIT REACHES MAX.

(2) (c)



$$\text{MASS FLOW PER UNIT WIDTH AT EXIT} = V_1 \times h_1 \times \rho$$

$$\text{TOTAL MASS LEAVING PER UNIT WIDTH} = V_1 \times h_1 \times \rho \times T$$

$$\text{INITIAL MASS PER UNIT WIDTH} = L \times h_0 \times \rho$$

$$P = \%\ \text{MASS FLOW EXITING} = \frac{V_1 \times h_1 \times \not{\rho} \times T}{L \times h_0 \times \not{\rho}}$$

$$C_0 = \frac{L}{T}$$

$$P = \frac{V_1 \times h_1}{C_0 \times h_0}$$

$$\overset{=0}{V_0} + 2C_0 = V_1 + 2C_1 \leftarrow C_1 = C_0 \times \sqrt{\frac{h_1}{h_0}}$$

$$\therefore V_1 = 2C_0 - 2C_0\sqrt{\frac{h_1}{h_0}}$$

$$\frac{V_1}{C_0} = 2\left(1 - \sqrt{\frac{h_1}{h_0}}\right)$$

SUB INTO (1)

$$P = 2\left(\frac{h_1}{h_0}\right)\left(1 - \sqrt{\frac{h_1}{h_0}}\right)$$

(3) (a)

CONT $\rho A V = \text{const}$

$$\boxed{\rho\,\delta V + V\,\delta\rho = 0} \quad (1)$$

ENERGY $C_p T + \dfrac{V^2}{2} = \text{const}$

$$\boxed{\delta T + \frac{V\delta V}{C_p} = 0} \quad (2)$$

MOMENTUM $\dfrac{F}{A} = P + \rho V^2$

$$\boxed{\delta\left(\frac{F}{A}\right) = \delta P + V^2\delta\rho + 2V\rho\,\delta V} \quad (3)$$

IDEAL GAS $\dfrac{\delta P}{P} = \dfrac{\delta\rho}{\rho} + \dfrac{\delta T}{T}$

$$\boxed{\delta P = RT\,\delta\rho + \rho R\,\delta T} \quad (4)$$

(1) → (3)

$$\delta\left(\frac{F}{A}\right) = \delta P - V\rho\,\delta V + 2V\rho\,\delta V$$

$$= \delta P + V\rho\,\delta V$$

(4) →

$$= RT\delta\rho + \rho R\,\delta T + V\rho\,\delta V$$

↑ (1) ↑ (2)

$$= -RT\rho\frac{\delta V}{V} - \frac{\rho R}{C_p}V\delta V + V\rho\,\delta V$$

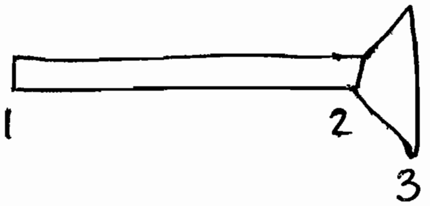
$\dfrac{R}{C_p} = \dfrac{C_p - C_v}{C_p} = 1 - \dfrac{1}{\gamma}$

$$= -P\frac{\delta V}{V} + \rho V\delta V\left(1 - \frac{R}{C_p}\right)$$

$$= -P\frac{\delta V}{V} + \rho V\delta V\left(1 - \frac{R}{C_p}\right)$$

$$= -P\frac{\delta V}{V}\left(M^2 - 1\right)$$

(3)(b) $P_{01} = 4\,bar$



$$M_2 = 0.5 \quad C_f = 0.0025 \quad D = 2m \quad L = 100m$$

$$\frac{4 C_f L_{12}}{D} = 4 \times 0.0025 \times \frac{100}{2} = 0.5$$

$$M_2 = 0.5 \qquad \frac{4 C_f L_2}{D} = 1.0691$$

$$\therefore \frac{4 C_f L_1}{D} = 1.0691 + 0.5 = 1.5691$$

$$M_1 = 0.45$$

$$\frac{\dot{m}\sqrt{C_p T_0}}{A P_{01}} = 0.884$$

$$\frac{P_{02}}{P_{01}} = \frac{\Pi_{m1}}{\Pi_{m2}} = \frac{0.884}{0.956}$$

$$\frac{\dot{m}\sqrt{C_p T_0}}{A P_{02}} = 0.956$$

$$\frac{P_{02}}{P_{03}} = \frac{P_{01}}{P_{03}} \times \frac{P_{02}}{P_{01}} = 4 \times 0.92 = \underline{\underline{3.68}}$$

(c) $\text{POWER} = \dot{m} C_p (T_{02} - T_{03})$

$$= \dot{m} C_p T_{02} \left(1 - \left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right) \eta_s$$

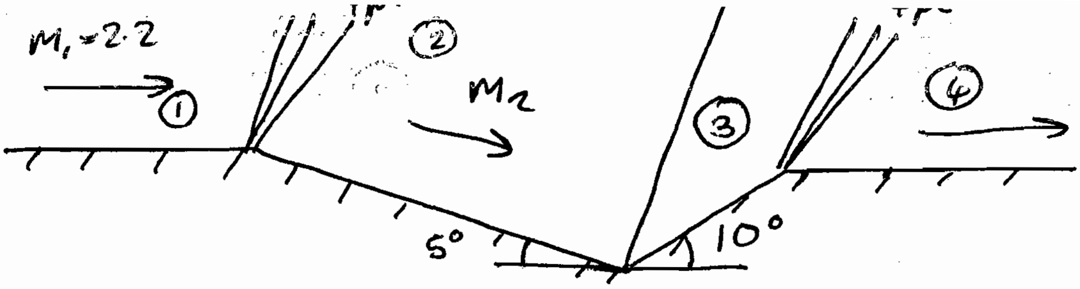
$$\frac{\text{POWER INSTALLED}}{\text{POWER UNINSTALLED}} = \frac{\dot{m}_I}{\dot{m}_{UN}} \times \frac{\left(1 - \left(\frac{1}{PR_I}\right)^{\frac{\gamma-1}{\gamma}}\right)}{\left(1 - \left(\frac{1}{PR_{UN}}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$\frac{\dot{m}_I}{\dot{m}_{UN}} = \frac{3.68}{4}$$

$\text{POWER RATIO} = 0.8764$

<u>12.6% REDUCTION</u>

(4)



(b) $\nu + \theta$ = CONSTANT $+\mu$ P-M CHARACTERISTIC

$\nu_1 = 31.73°$ $\theta = 0°$ $\theta_2 = -5°$ $\nu_2 = 36.73°$

$\underline{M_2 = 2.40}$

SHOCK $\delta = 15°$ $\underline{M_3 = 1.7903}$ $\nu_3 = 20.4$

$\nu + \theta$ = CONST OVER EXPANSION $\Rightarrow \nu_4 = 30.60$

$\underline{M_4 = 2.152}$

(c) DRAG DUE TO PRESSURE

$= (P_3 - P_2) \times 2mm$

$= 0.002 \left(\frac{P_3}{P_2} - 1 \right) \frac{P_2}{P_1} P_1$

$\frac{P_2}{P_1} = 0.7318$ $\frac{P_3}{P_2} = 2.6115$

DRAG $= 0.002 (2.4115 - 1)(0.7318) \times 0.2e^5$

$= 41.3 N/m$

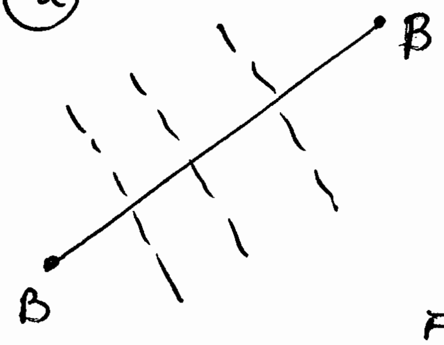
(D) IN CASE (B), THOUGH THE SHOCK IS SLIGHTLY STRENGTHENED BY AN INCREASED M_2 ITS DEFLECTION ANGLE REMAINS UNALTERED. ∴ CHANGES IN THE DEFLECTIONS OF THE TWO EXPANSIONS WILL DOMINATE. THESE REDUCE BOTH P_2 + P_3 BY ROUGHLY THE SAME PROPORTION AND HENCE WE EXPECT THE DRAG DUE TO DENT TO DECREASE.

$M_2 = 2.61$ $P_2 = 0.1 bar$ $\frac{P_3}{P_2} \sim 2.55$ $P_3 = 0.251 bar$

AND DRAG DROPS

5

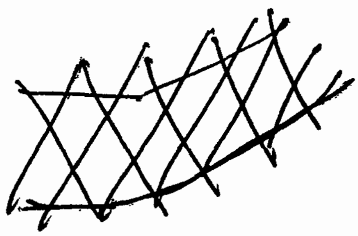
(a)

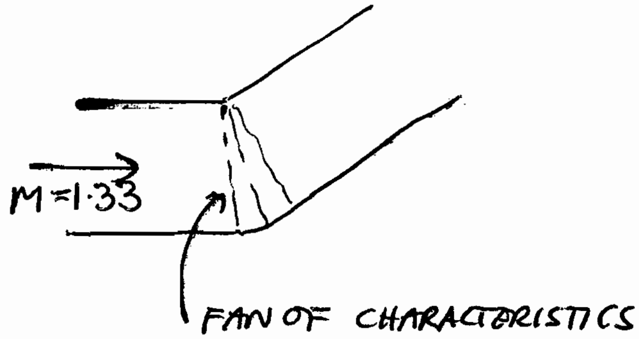


CHARACTERISTIC THROUGH B, BP, IS STRAIGHT IF THE OTHER FAMILY OF CHARACTERISTICS (SHOWN DOTTED) ORIGINATE FROM A REGION OF THE FLOW WHERE FLOW PROPERTIES UNIFORM.

FLOW PROPERTIES UNIFORM ALONG STRAIGHT CHARACTERISTICS.

(b)





ACROSS FAN $\nu - \theta$ = CONST

$$\Rightarrow \nu_{CT} - 6 = \nu(1.33) - 0$$

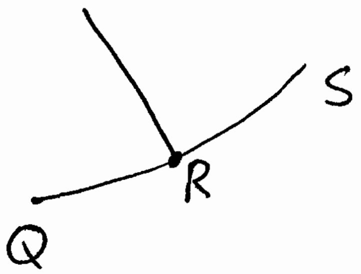
$$\Rightarrow \nu_{CT} = 7 + 6 = 13^{\circ}$$

$$M_{CT} = 1.537$$

FOR CT $$\frac{\dot{m}\sqrt{c_p T_0}}{P_0 A_{CT}} = \frac{\dot{m}\sqrt{c_p T_0}}{P_0 A_{AP}} \cdot \frac{A_{AP}}{A_{CT}}$$

$$\Rightarrow \frac{A_{CT}}{A_{AP}} = \frac{1.1866}{1.0657} = 1.113$$

(c)

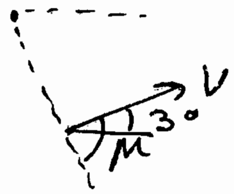


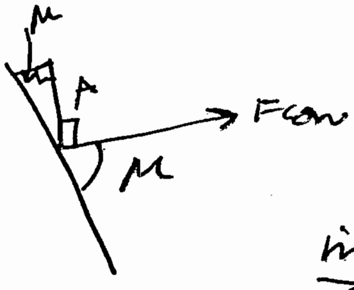
ALONG EACH CH'IC EMINATING FROM B THE FLOW ANGLE IS UNIFORM (AND DIFFERENT) AND DETERMINED BY VALUE OF M ON THAT CH'IC. THE CURVED WALL MUST BE SHAPED TO BE ALIGNED WITH THIS FLOW DIRECTION. THIS WILL ENSURE THE CANCELLATION OF THE CHARACTERISTIC AND PERMIT UNIFORM FLOW AT CT.

(5) (d) ALONG DR $\theta = 3^\circ \Rightarrow \nu = 10^\circ \Rightarrow M = 1.435$

$$\mu = \sin^{-1}\frac{1}{M} = \underline{\underline{44.18^\circ}}$$

$\therefore$ ANGLE FROM B IS $(-)\,41^\circ$





$A_\perp = A$ PERPENDICULAR TO FLOW

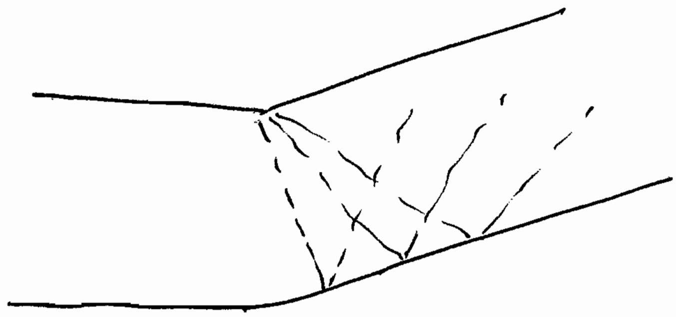
$$\frac{\dot{m}\sqrt{c_p T_0}}{P_0 A_\perp} = f(1.435) = 1.1288$$

AND $\dfrac{A_\perp}{r} = \sin\mu \Rightarrow \dfrac{r}{A_{AP}} = \dfrac{r}{A}\dfrac{A}{A_{AP}} = \dfrac{1}{\sin\mu}\dfrac{\dfrac{\dot{m}\sqrt{c_pT_0}}{P_0A_{AP}}}{\dfrac{\dot{m}\sqrt{c_pT_0}}{P_0A_\perp}}$

$$= \frac{1.435 \times 1.1866}{1.1288} = \underline{\underline{1.5085}}$$

$$R = \underline{\underline{15\text{ cm}}}$$

(e)



NO CANCELLATION