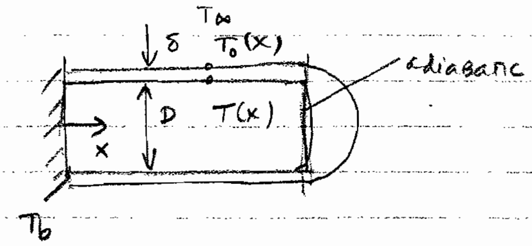
PART IIA 2007
3A6 Heat and mass transfer
Prof S Hochgreb

3A6 FINAL 2007

1

(a)



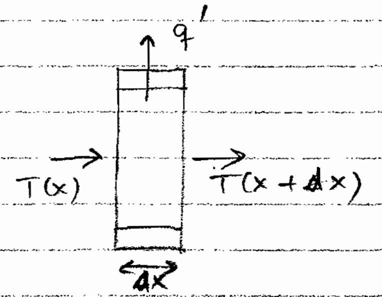
THE OVERALL RESISTANCE TO HEAT TRANSFER IS:

$$T(x) \quad \frac{1}{2\pi k_2}\ln\left(\frac{D+2\delta}{D}\right) \quad T_0(x) \quad \frac{1}{h\pi(D+2\delta)} \quad T_\infty$$

THE CORRESPONDING HEAT FLUX IS:

$$q' = \frac{T(x) - T_\infty}{\frac{1}{2\pi k_2}\ln\left(1+\frac{2\delta}{D}\right) + \frac{1}{h\pi(D+2\delta)}}$$

(b)



$$A = \frac{\pi D^2}{4}$$

$$+k_1 A\frac{dT}{dx} - k_1 A\left.\frac{dT}{dx}\right|_{x+dx} = -q'$$

$$k_1 A\frac{dT}{dx} - k_1 A\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right) = -q'$$

$$k_1 A\frac{d^2T}{dx^2} = \frac{T(x) - T_\infty}{R} \qquad R = \frac{1}{2\pi k_2}\ln\left(1+\frac{2\delta}{D}\right) + \frac{1}{h\pi(D+2\delta)}$$

THE GENERAL SOLUTION TO THE EQUATION

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

IS $\theta = C_1\exp(-mx) + C_2\exp(mx)$

USING $\theta = T(x) - T_\infty$, WE HAVE $m^2 = \dfrac{1}{Rk_1A}$

$$\theta(0) = T_b - T_\infty$$

ADIABATIC TIP:

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$\theta(0) = C_1 + C_2 = \theta_b$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = -m\, C_1 \exp(-mL) + m C_2 \exp(mL) = 0$$

$$-C_1 \exp(-mL) + (\theta_b - C_1)\exp(mL) = 0$$

$$C_1\left[\exp(-mL) + \exp(mL)\right] = \theta_b \exp(mL)$$

$$C_1 = \frac{\theta_b \exp(mL)}{\exp(-mL) + \exp(mL)}$$

$$C_2 = \theta_b\left[\frac{\exp(-mL)}{\exp(-mL) + \exp(mL)}\right]$$

$$\theta(x) = \theta_b\left[\frac{\exp(m(L-x)) + \exp(-m(L-x))}{\exp(-mL) + \exp(mL)}\right] = \theta_b \frac{\cosh(m(L-x))}{\cosh(mL)}$$

$$\boxed{T(x) - T_\infty = (T_b - T_\infty)\frac{\cosh(m(L-x))}{\cosh(mL)}}$$

(c) $$q = \int_0^L q'\, dx = \frac{1}{R}\frac{(T_b - T_\infty)}{2\cosh(mL)}\int_0^L\left[\exp(mL)\exp(-mx) + \exp(-mL)\exp(mx)\right]dx$$

$$= \frac{1}{R}\frac{T_b - T_\infty}{2\cosh(mL)}\left[\left.\frac{\exp mL \exp(-mx)}{-m}\right|_0^L + \left.\frac{\exp(-mL)\exp(mx)}{m}\right|_0^L\right]$$

$$= \frac{1}{R}\frac{T_b - T_\infty}{2m\cosh(mL)}\left[-\left(\cancel{1} - \exp(mL)\right) + \left(\cancel{1} - \exp(-mL)\right)\right]$$

$$\boxed{q = \frac{T_b - T_\infty}{mR}\tanh(mL)}$$

(d) $\dfrac{q_{nc}}{q_c} = \dfrac{m_c R_c \tanh(m_{nc} L)}{m_{nc} R_{nc} \tanh(m_c L)}$ nc - no cladding, c - cladding.

BUT

$$\frac{m_c}{m_{nc}} = \left(\frac{R_{nc}}{R_c}\right)^{1/2}$$

$$\frac{m_c}{m_{nc}} \cdot \frac{R_c}{R_{nc}} = \left(\frac{R_{nc}}{R_c}\right)^{1/2} \frac{R_c}{R_{nc}} = \left(\frac{R_c}{R_{nc}}\right)^{1/2}$$

$$\frac{q_{nc}}{q_c} = \left(\frac{R_c}{R_{nc}}\right)^{1/2} \frac{\tanh(m_{nc} L)}{\tanh(m_c L)}$$

FOR THE values Given:

$h = 100\ W/m^2 K$

$\delta/D = 0.1$

$D = 5\times10^{-3}\ m$ —— $A = \pi D^2/4 = 1.96\times10^{-5}\ m^2$

$L = 2\times10^{-2}\ m$

$k_1 = 400\ W/mK$

$k_2 = 1\ W/mK$

$$R_{nc} = \frac{1}{\pi h D} = \frac{1}{\pi(100\ W/mK)(5\times10^{-3}\ m)} = 0.637\ \frac{mK}{W}$$

$$R_c = \frac{1}{\pi h D\left(1+\frac{2\delta}{D}\right)} + \frac{1}{2\pi k_2} \ln\left(1+\frac{2\delta}{D}\right) =$$

$$\underbrace{\frac{1}{\pi\left(100\frac{W}{m^2K}\right)(5\times10^{-3}\ m)(1+2(0.1))}}_{0.531\ \frac{mK}{W}} + \underbrace{\frac{1}{2\pi(1\ W/mK)} \ln(1+2(0.1))}_{0.029\ \frac{mK}{W}} = 0.560\ \frac{mK}{W}$$

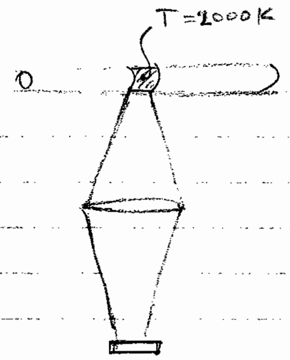
$$m_{nc} = \left(\frac{1}{R_{nc} k_1 A}\right)^{1/2} = \left(\frac{1}{0.637\frac{mK}{W} \cdot 400\frac{W}{mK} \cdot 1.96\times10^{-5} m^2}\right)^{1/2} = 14.1\ m^{-1}$$

$$m_c = m_{nc}\left(\frac{R_{nc}}{R_c}\right)^{1/2} = 14.1\,m^{-1}\left(\frac{0.637}{0.560}\right)^{1/2} = 15.1\ m^{-1}$$

$$\frac{q_{nc}}{q_c} = \left(\frac{0.560\ mK/W}{0.637\ mK/W}\right)^{1/2} \frac{\tanh(14.1\ m^{-1}\ 0.02\,m)}{\tanh(15.1\ m^{-1}\ 0.02\,m)} = \underline{0.88}$$

THEREFORE, MORE HEAT IS Conducted with THAN without CLADDING, A COUNTER INTUITIVE RESULT. THIS IS BECAUSE H.T. IS DOMINATED BY convection, THUS AN AREA INCREASE IS MORE IMPORTANT THAN THE ADDITIONAL RESISTANCE.

(a)



$$E = \varepsilon E_b = \varepsilon \sigma T^4 = (0.8)\left(5.67\times10^{-8}\frac{W}{m^2K}\right)(2000K)^4$$

$$E = 7.26\times10^{5}\ \frac{W}{m^2}$$

(b) THE TOTAL ENERGY EMITTED BY THE WIRE WITHIN THE RANGE OF DETECTION IS:

$$Q_T = \varepsilon \int_{\lambda_1}^{\lambda_2} E_b(\lambda)\, d\lambda = \varepsilon E_b\left(F_{0-\lambda_2} - F_{0-\lambda_2}\right)$$

$$\lambda_1 T = (0.4\ \mu m \cdot 2000\ K) = 800\ \mu m\cdot K \qquad F_{0-\lambda_1} = 0.000016$$

$$\lambda_2 T = (0.8\ \mu m \cdot 2000\ K) = 1600\ \mu m\cdot K \qquad F_{0-\lambda_2} = 0.019718$$

$$Q_T = \left(7.26\times10^{5}\ \frac{W}{m^2}\right)\underbrace{\left(0.019718 - 0.000016\right)}_{0.019702} = 14298\ \frac{W}{M^2}$$

THE FRACTION OF THE RADIATION THAT REACHES THE DETECTOR IS

$$Q = Q_T A = Q_T \frac{\pi D}{2}\Delta x = \left(14298\ \frac{W}{m^2}\right)\left(\pi\,\frac{1.5\times10^{-3}\ m}{2}\right)\left(1\times10^{-3}\ m\right)$$

$$\underline{Q = 3.36\times10^{-2}\ W}$$

(c)

$$\frac{Q}{Q_0} = \frac{E_b}{E_{b0}}\,\frac{\left(F_{0-\lambda_1} - F_{0-\lambda_2}\right)}{\left(F_{0-\lambda_1} - F_{0-\lambda_2}\right)_0} = \left(\frac{T}{T_0}\right)^4 \frac{\Delta F}{\Delta F_0}$$

ASSUME FIRST THAT $\Delta F/\Delta F_0$ VARIES LITTLE COMPARED TO $(T/T_0)^4$

$$\left(\frac{T}{T_0}\right) \cong \left(\frac{Q}{Q_0}\right)^{1/4} = 1.778 \qquad T = (2000\ K)(1.778) = 3557\ K.$$

$$\lambda_1 T = (0.4\ \mu m)(3557\ K) = 1423\ \mu m\cdot K \longrightarrow F_{0-\lambda_1} = 9.162\times10^{-3}$$

$$\lambda_2 T = (0.8\ \mu m)(3557\ K) = 2845\ \mu m\cdot K \longrightarrow F_{0-\lambda_2} = 2.381\times10^{-1}$$

$$\Delta F = 2.289\times10^{-1}$$

THE REVISED ESTIMATE IS :

$$\left(\frac{T}{T_0}\right) = \left[\frac{Q}{Q_0}\ \frac{\Delta F_0}{\Delta F}\right]^{1/4} = \left[10\cdot\frac{6.671\times10^{-2}}{2.289\times10^{-1}}\right]^{1/4} = 1.307 \qquad T = 2613\ K$$

NOW ITERATE WITH NEW VALUE OF T:

$$\lambda_1 T = 1045\ \mu m \cdot K \longrightarrow F_{0-\lambda_1} = 3.618 \times 10^{-3}$$
$$\lambda_2 T = 2091\ \mu m \cdot K \longrightarrow F_{0-\lambda_2} = 8.227 \times 10^{-2}$$
$$\Delta F = 7.865 \times 10^{-2}$$

$$\left(\frac{T}{T_0}\right) = \left[10 \cdot \frac{6.671 \times 10^{-2}}{7.865 \times 10^{-2}}\right]^{1/4} = 1.707 \qquad T = 3413\ K$$

$$\lambda_1 T = 1365\ \mu m K \longrightarrow F_{0-\lambda_1} \cong 6.464 \times 10^{-2}$$
$$\lambda_2 T = 2730\ \mu m K \longrightarrow F_{0-\lambda_2} = 2.122 \times 10^{-1}$$
$$\Delta F = 0.1476$$

$$\left(\frac{T}{T_0}\right) = \left[10 \cdot \frac{6.671 \times 10^{-2}}{0.1476}\right]^{1/4} = 1.458 \qquad T = 2916\ K.$$

$$\lambda_1 T = 1166\ \mu m K \longrightarrow F_{0-\lambda_1} = 1.826 \times 10^{-3}$$
$$\lambda_2 T = 2333\ \mu m K \longrightarrow F_{0-\lambda_2} = 0.1271$$
$$\Delta F = 0.1252$$

$$\left(\frac{T}{T_0}\right) = \left[10 \quad \frac{6.671 \times 10^{-2}}{0.1476}\right]^{1/4} = 1.519 \qquad T = 3038\ K$$

$$\lambda_1 T = 1215\ \mu m\ K \longrightarrow F_{0-\lambda_1} = 2.558 \times 10^{-3}$$
$$\lambda_2 T = 2430\ \mu m \cdot K \longrightarrow F_{0-\lambda_2} = 0.1467$$
$$\Delta F = 0.1441$$

$$\left(\frac{T}{T_0}\right) = \left[10 \quad \frac{6.671 \times 10^{-2}}{0.1441}\right]^{1/4} = 1.467 \qquad T = 2934\ K.$$

$$\lambda_1 T = 1173\ \mu m K \qquad F_{0-\lambda_1} = 2.026 \times 10^{-3}$$
$$\lambda_2 T = 2437\ \mu m K \qquad F_{0-\lambda_2} = 0.1482$$
$$\Delta F = 0.1412$$

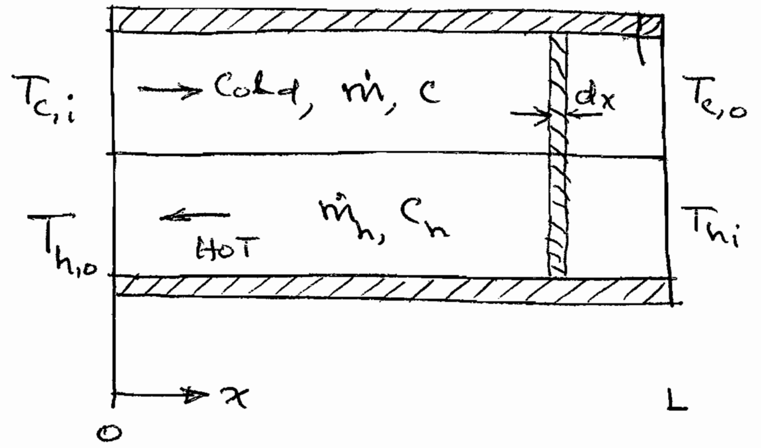
$$\left(\frac{T}{T_0}\right) = \left[10 \quad \frac{6.671 \times 10^{-2}}{0.1412}\right]^{1/4} = 1.474 \qquad T = 2949\ K.$$

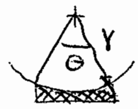
THEREFORE, WE CAN SEE THAT THE TEMPERATURE LIES BETWEEN

$2934 K < T < 2949 K.$ WITHIN THE REQUIRED PRECISION.

FURTHER INTERPOLATION CAN YIELD THE TEMPERATURE TO ARBITRARY PRECISION.

Q3)

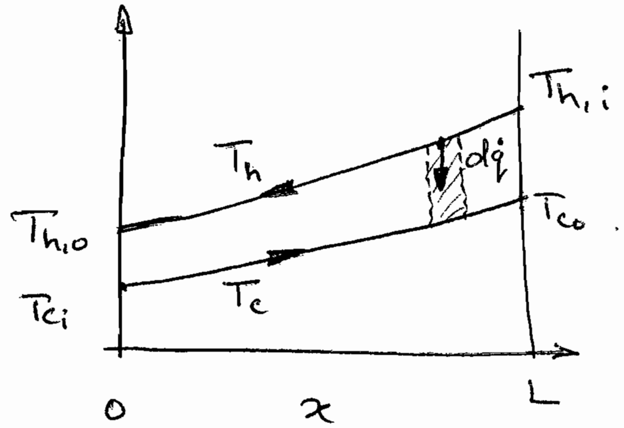




$$\dot{m}_h = 1.5\dot{m} \; ; \qquad c_h = 0.45c$$

$$h_h = 0.5h \implies U = \frac{h_h h}{h + h_h} = \boxed{\frac{h}{3} = U}$$

a)



b) Energy balance for the heat exchange

$$\dot{m}_h c_h \, dT_h = \dot{m} \cdot c \, dT_c = d\dot{q}$$

$$\implies dT_h = \frac{d\dot{q}}{0.675\dot{m}c} \; ; \qquad dT_c = \frac{d\dot{q}}{\dot{m}c}$$

Also

$$d\dot{q} = U(T_h - T_c)\, dA \; ; \qquad dA = r\theta \, dx$$

$$= \frac{h}{3}(T_h - T_c)\, r\theta \, dx$$

Define $\Delta T = T_h - T_c$

$$d(\Delta T) = dT_h - dT_c = \frac{d\dot{q}}{\dot{m}c}\left(\frac{1}{0.675} - 1\right)$$

$$= 0.481 \frac{hr\theta}{3\dot{m}c} \Delta T \, dx$$

$$\frac{d(\Delta T)}{\Delta T} = 0.16 \frac{hr\theta}{\dot{m}c} dx = B\,dx$$

integrate $\displaystyle\int_0^x \frac{d(\Delta T)}{\Delta T} = \int_0^x B\,dx$

$$\Rightarrow \boxed{(T_h - T_c) = (T_{h,o} - T_{c,i}) \exp(\mathcal{B} x)}$$

where $\displaystyle \mathcal{B} \equiv 0.16 \frac{hr\theta}{\dot{m}c}$.

c) $\displaystyle d\dot{q} = \frac{hr\theta}{3}(T_h - T_c)\,dx$

$$\therefore \dot{Q} = \int_0^L d\dot{q} = \frac{hr\theta}{3}\int_0^L (T_h - T_c)\,dx$$

Using the solution to (b)

$$\dot{Q} = \frac{hr\theta}{3}(T_{h,o} - T_{c,i})\int_0^L e^{Bx}\,dx$$

$$\boxed{\dot{Q} = 2.08\,\dot{m}c\,(T_{h,o} - T_{c,i})\left(e^{BL} - 1\right)} \text{ W.}$$

To maximize $\dot{Q}$:

$$\dot{Q} = \int d\dot{q} \quad ; \quad d\dot{q} = U \Delta T \, dA$$

To maximize $\dot{Q}$;

1) maximize ΔT
2) maximize A - Area for heat transfer.

$A = P * L \Rightarrow$ maximize $P \Rightarrow$ maximize Θ.

By increasing Θ; β increases $\Rightarrow$ ΔT also incre

if the given parameters are constant.

This can be achieved by having 'Co-axial' tubes. thus the whole of the circumferential area of the inner tube is available for heat transfer.

d) $$\frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}} = \exp(\beta L)$$

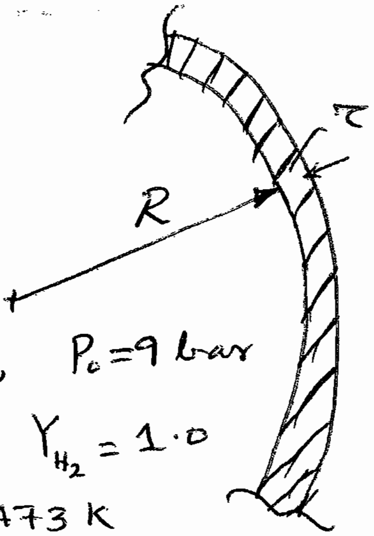
$$\beta = 0.16 * \frac{1950 * 0.01 * \pi}{6 * 0.02 * 4175} = 0.0196 \ m^{-1}$$

$$\therefore L = \frac{1}{\beta} \ln\left(\frac{360 - 324}{310 - 290}\right) = 29.99 \approx \underline{30 m}.$$

Note! By having Co-axial tubes, L becomes $\underline{2.5 m}$.

(4)

$\boxed{R \gg \tau}$



H_2, $P_o = 9$ bar

$Y_{H_2} = 1.0$

$T = 473$ K

$P = 1$ atm

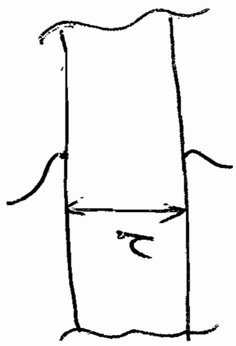
$$D = 1.65 \times 10^{-6} \exp\left(-\frac{3267}{T}\right) \; m^2/s$$

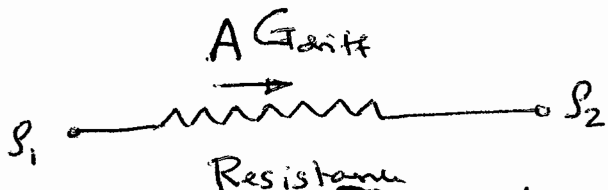
$$S = \mathcal{S} / \sqrt{P_{H_2}} \quad kg/m^3\text{-bar}$$

Sol.

a) $R \gg \tau \Rightarrow$ tank wall can be treated as plane wall.

$\Rightarrow$





$R_J = \frac{\tau}{D}$; $A = 4\pi R^2$

$$G_{H_2} = \text{Diffusion rate} = \boxed{A\, G_{diff} = \frac{(\rho_1 - \rho_2) A D}{\tau}} \; kg/s$$

b) $\rho_1 = \rho Y_1 = S\, P_{H_2} = \mathcal{S}\, P_{H_2}^{1/2} \; kg/m^3$

when ρ - mixture density (H_2 + Steel is the mixture)

$P_{H_2,1} = 9$ bar, $P_{H_2,2} = 0$.

Since H_2 is diffusing via the wall

$\left(\text{Rate of change of } H_2 \text{ mass in the tank}\right) = \text{Net Diffusion rate}$

b)
$$\frac{dm}{dt} = - A\, G_{diff} = - \frac{A\mathcal{D}}{\tau} S_i \quad (\text{from (a)})$$

$$= - \frac{A\mathcal{D}S}{\tau} p_{H_2}^{1/2}$$

But from ideal gas law

$$pV = mRT \times 10^{-5}$$

p - in bar, T - in K
V - in m^3
m - in kg
R - J/kg-K

$$\therefore m = \frac{pV}{R^*T}$$

$$\Rightarrow \frac{dm}{dt} = \frac{V}{R^*T} \frac{dp}{dt}$$

$$\therefore \frac{dp}{dt} = - \frac{A\mathcal{D}S R^*T}{V\tau} p^{1/2}$$

$$\Rightarrow \frac{dp}{\sqrt{p}} = - B\, dt \qquad B \equiv \frac{A\mathcal{D}S R^*T}{V\tau} \left(B \text{ in } \frac{\sqrt{bar}}{Sec.}\right)$$

$$\Rightarrow 2\left(\sqrt{P_0} - \sqrt{P_1}\right) = Bt$$

$$\Rightarrow \boxed{t = \frac{2}{B}\left(\sqrt{P_0} - \sqrt{P_1}\right)} \quad \text{Sec.}$$

(Required result)

c) $S = 4.6 \times 10^{-3} / \sqrt{P_{H_2}}$ $kg/m^3 - bar$

$\cancel{S} = 4.6 \times 10^{-3}$ $kg/m^3 - \sqrt{bar}$

$\mathcal{D} = 1.65 \times 10^{-6} \exp\left(-\frac{3267}{473}\right) = 1.65 \times 10^{-9}$ $m^2/sec.$

$A = 4\pi R^2 \Rightarrow R = ?$

$V = 1 \times 10^{-3}\ m^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 6.204 \times 10^{-2}\ m.$

$\therefore A = 4\pi \times (6.204 \times 10^{-2})^2 = 0.0484\ m^2$

$R^* = 4.157 \times 10^{-2}$

$T = 473\ K$; $\tau = 2 \times 10^{-3}\ m$

$$\therefore B = \frac{0.0484 \times 1.65 \times 10^{-9} \times 4.6 \times 10^{-3} \times 4.157 \times 10^{-2} \times 4.73 \times 10^{2}}{2 \times 10^{-3} \times 1 \times 10^{-3}}$$

$= 3.6116 \times 10^{-6}$ $\sqrt{bar}/sec.$

$$\therefore t = \frac{2}{3.6116 \times 10^{-6}} \times (\sqrt{9} - \sqrt{4}) = 6.41 \text{ days.}$$

$$\boxed{t = 6.41 \text{ days}}$$