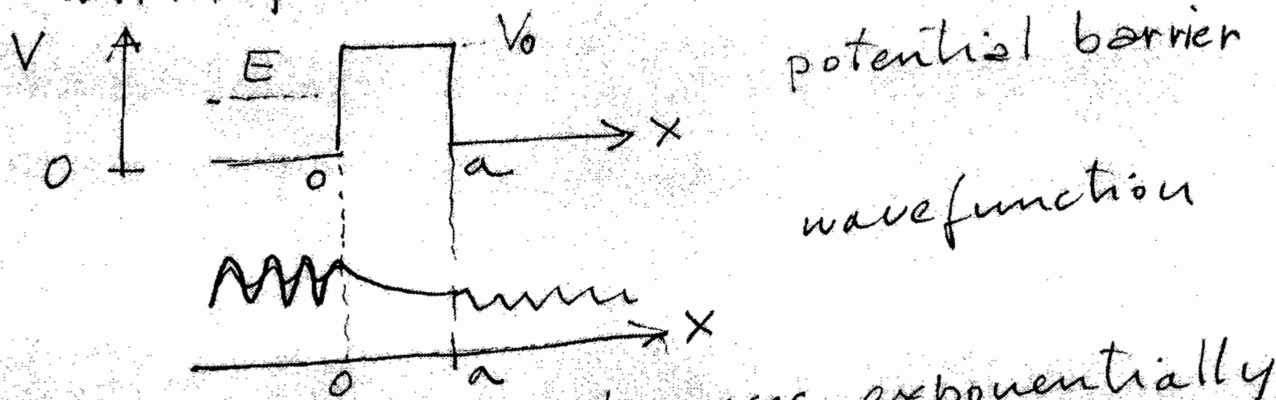
3B5 2007

Q.1

(a) (i) Electron diffraction 10%
Tunnelling etc

(ii) Tunnelling = ability of particles to penetrate potential barriers. With referance to the figure1 below,



the wavefunction ψ decreases exponentially in the region where E < V. The value of ψ(a) determines the amplitude of the transmitted wave. Such value is strongly dependent on $V_0 - E$ and a.

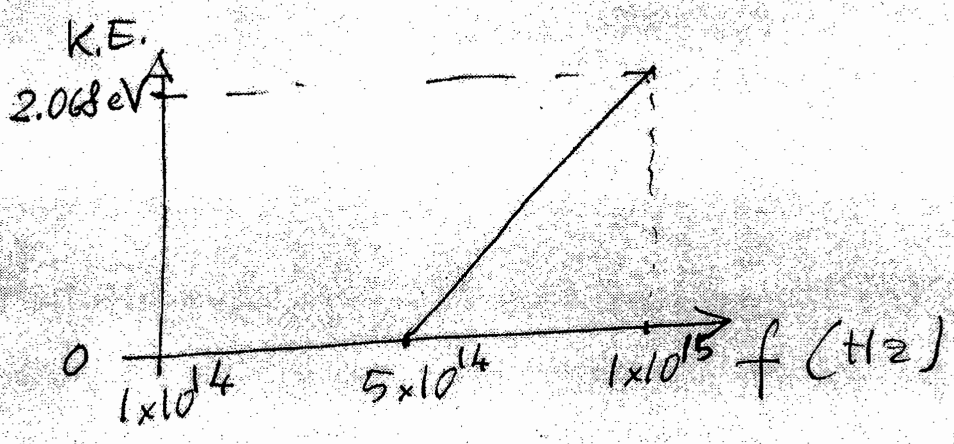
In a scanning tunnelling microscope a conductive tip is scanned across a ~~solid~~ conductive surface. A small gap exists between the tip and the surface and a potential difference is applied between the two. The situation is similar to that of figure 1 above. Due to the dependence of the tunnelling current upon the height and the width of the barrier properties like the chemical composition or the surface roughness can be analysed by measuring the tunnelling current. 30%

(iii) $E = \frac{p^2}{2m} = 100\,eV$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = 1.227 \times 10^{-10}\,m$ 10%

(b) (i) The frepuency threshold $f_{min} = \frac{\phi}{h}$, where ϕ is the metal work function, that is the barrier an electron must overcome to exit the metal. 10%

(ii)



$\phi = h f_{min} = 2.068$

$K.E._{max} = h(f_{max} - f_{min}) = 2.068\,eV$ 20%

(iii) $|e|V_{min} = -\frac{1}{2}mv^2 = -\frac{p^2}{2m} = -h(f - f_{min})$

20%

Q.2

(a)(i) $dP = 4\pi r^2 |\psi|^2 dr$

$P(r) = 4\pi |A|^2 r^2 e^{-2ar}$

$\frac{dP}{dr} = 4\pi |A|^2 \left(2re^{-2ar} - 2ar^2 e^{-2ar}\right) = 0$

25%

$\Longrightarrow r = \frac{1}{a}$

(ii) $\int_0^\infty dP = 4\pi |A|^2 \int_0^\infty e^{-2ar} r^2 dr = 1$

$\int_0^\infty e^{-2ar} r^2 dr = -\frac{1}{2a} e^{-2ar} r^2 \Big|_0^\infty + \frac{1}{a}\int_0^\infty e^{-2ar} r\, dr =$

$= -\frac{1}{2a^2} e^{-2ar} r \Big|_0^\infty + \frac{1}{2a^2}\int_0^\infty e^{-2ar} dr =$

$= \frac{1}{4a^3}$

25%

$\longrightarrow A = \left(\frac{a^3}{\pi}\right)^{1/2}$

(b)(i) Using Linear Combination of Atomic Orbitals:

$\psi(r_1, r_2) = A e^{-a|r-r_1|} + B e^{-a|r-r_2|}$

$|\psi|^2 = |A|^2 e^{-2a|r-r_1|} + 2AB e^{-a(|r-r_1| + |r-r_2|)}$
$+ |B|^2 e^{-2a|r-r_2|}$

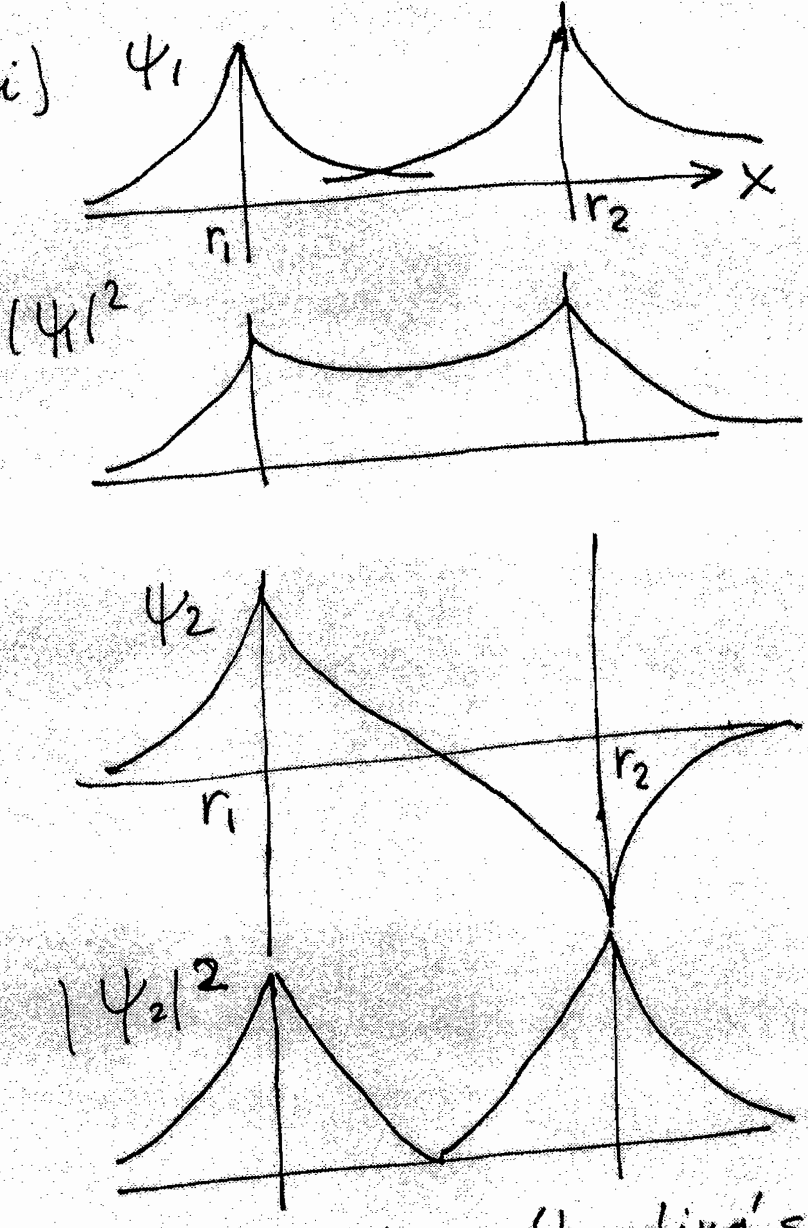
By symmetry $|\psi(r_1, r_2)|^2 = |\psi(r_2, r_1)|^2$

$\longrightarrow A = \pm B$

$$\psi_1 = C\left(e^{-a|r-r_1|} + e^{-a|r-r_2|}\right)$$

$$\psi_2 = C\left(e^{-a|r-r_1|} - e^{-a|r-r_2|}\right)$$ 25%

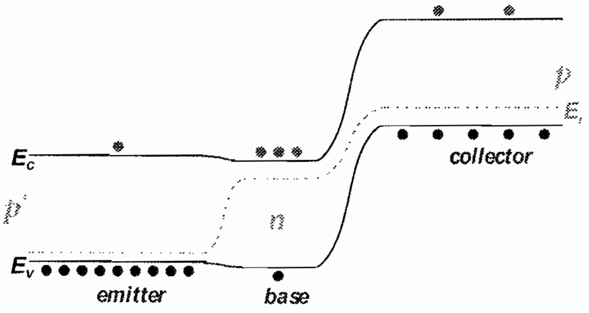
(ii)



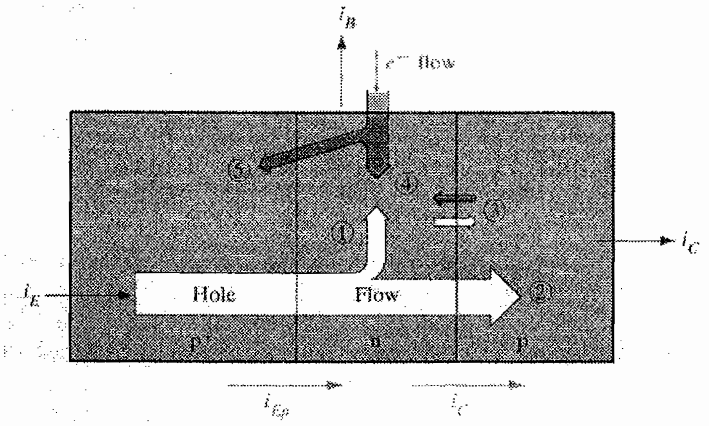
25%

ψ_1 is referred to as 'bonding' state, because the electron density and, therefore, the negative charge between the two protons is higher when the two wave functions add up, than when they are considered separate. Such excess negative charge is the 'glue' cementing the molecule together. On the other hand for ψ_2, the charge in between the protons is zero. So if the electron is excited from E_1 to E. the molecule dissociates.

Q.3 (a) The pnp BJT is operated by applying a positive bias to the emitter with respect to the base and to the base with respect to the collector. This causes the p^+n junction between the emitter and base to be forward biased but the pn junction between the base and the collector to be reverse biased, leading to the following band diagram:



The resulting currents that flow in the device are shown in the following Figure:



By putting the emitter-base junction into forward bias, a large hole current will flow across the junction from the emitter to the base. There will be an electron current in the reverse direction ⑤ supplied by the base, but this will be small in comparison with the hole current because the base is less heavily doped. The injected holes that arrive in the base find themselves in a region of constant potential. As usual, we assume that electric fields only exist inside the depletion region. Inside the base, the holes find themselves in a region where the majority carriers are electrons, and some holes ① will recombine with electrons. An electron current ④ must be supplied by the base to meet this recombination loss.

Holes will cross the base under the action of diffusion and if the width of the undepleted region of the base W_b is much less than the diffusion length of the holes $L_h = \sqrt{(D_h t_h)}$ then most holes will cross the base without recombination.

The base-collector junction is under reverse bias. Normally only a small reverse bias current will flow ③ as there are few electrons on the p-side and holes on the n-side available. However, we have injected a large number of holes into the n-region from the emitter which have crossed the base, and so there are holes on the n-side which we would not normally have. Therefore a hole current flows into the collector ② which is measured as current I_C.

Although perhaps not immediately apparent, it is in fact the small base current that is compensating for recombined electrons which is controlling the current out of the collector. The base must be electrically neutral (there can be no space charge).

Therefore, for a particular base current, only a certain current emitter can flow through the base and into the collector. [35%]

(b) The emitter injection efficiency γ is the fraction of the total emitter current that is due to holes, or expressed as an equation,

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}.$$

The first term in the equation given is due to electrons and the second is due to holes. However, because the base width is much less than the hole diffusion length, this equation becomes

$$I = Ae\left[\underbrace{\left(\frac{D_e}{L_e}\right)\frac{n_i^2}{N_A}}_{\text{electrons}} + \underbrace{\left(\frac{D_h}{W_b}\right)\frac{n_i^2}{N_D}}_{\text{holes}}\right]\left[\exp\left(\frac{eV_{EB}}{kT}\right) - 1\right].$$

By substituting the electron and hole current terms into the expression for γ gives

$$\underline{\gamma = \left[1 + \left(\frac{D_e}{D_h}\right)\left(\frac{W_b}{L_e}\right)\left(\frac{N_D}{N_A}\right)\right]^{-1}}.$$

[30%]

(c) We know from the Einstein relation that the ratio of the electron and hole mobilities in a semiconductor is the same as the ratio of the diffusion coefficients. We can therefore calculate γ as a function of $N_D/N_A \ll 1$.

The current amplification factor can be found to be:

$$\beta = \frac{B\gamma}{1 - B\gamma}.$$

Hence, for $N_D/N_A=0.1$ $\beta= 66$ and for $N_D/N_A=0.01$ $\beta=415$. [15%]

(d) In order to realise a high base to collector current amplification factor, b, it is important very few carriers recombine in the base. To reduce recombination, we must reduce the length of time the holes spend in the base and there are two methods for achieving this:

1) Reduce the width of the base. Ideally we want the width of the base, W_b, to be much less than the hole diffusion length, L_h. However, the base current IB has to pass through contacts at the edge of the base, so if the base is very thin we can get current crowding which reduces β. Interdigitating (ie: changing the geometry of the device) can help to alleviate this problem.

2) Vary the doping concentration across the base. If we heavily n-dope the channel near the emitter. But only lightly dope it near the collector then an internal electric

field will be set up which will help to sweep holes through the base under the influence of drift as well as diffusion. This also improves the frequency response as carriers are swept more quickly out of the base. [20%]

Q4 (a) The pinch-off voltage is the potential difference that has to exist between the gate and the drain ($V_{GS} - V_{DS}$) for the channel in the JFET to be completely depleted at the drain end. [5%]

(b) As the doping density in the n-type channel is much lower than in the p$^+$ gate, we can assume that the depletion region in the gate is negligible. Hence, considering the n-side of the junction, from the Gauss Law of Electrostatics, we know that

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0 \varepsilon_r} = \frac{-eN_D}{\varepsilon_0 \varepsilon_r},$$

where $-eN_D$ is the charge density due to ionised donors on the n-side. Given that V only varies in the x direction across the junction this becomes

$$\varepsilon_0 \varepsilon_r \frac{\mathrm{d}^2 V}{\mathrm{d}x^2} = eN_D .$$

Integrating this and applying the boundary condition that the electric field (and hence dV/dx) is zero at the edge of the depletion region of width w gives

$$\frac{dV}{dX} = \frac{eN_D}{\varepsilon_0 \varepsilon_r}(x - w).$$

Integrating again and applying the boundary condition that $V = 0$ at $x = w$ gives

$$V = \frac{eN_D}{\varepsilon_0 \varepsilon_r}\left(\frac{x^2}{2} - wx + \frac{w^2}{2}\right).$$

Hence, the built-in potential (at $x = 0$) is

$$V_0 = \frac{eN_D w^2}{2\varepsilon_0 \varepsilon_r}.$$

We can rearrange this equation to make w the subject, and allow for the fact that we are also applying a voltage, V, to the gate with respect to the channel,

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r (V_0 - V)}{eN_D}\right)^{1/2}.$$

The channel will just be pinched off when the depletion region width at the drain end of the channel ($x = L$) is half the channel height, h. At this point, the externally applied voltage across the junction will be

$$V_p = V_{GS} - V_{DS},$$

and so we have that

$$\frac{h}{2}=\left(\frac{2\varepsilon_0\varepsilon_r\left(V_0-V_p\right)}{eN_D}\right)^{1/2}.$$

Assuming that $V_0 << V_p$ and rearranging gives

$$\underline{V_p=\frac{-h^2eN_D}{8\varepsilon_0\varepsilon_r}.}$$ [50%]

(c) (i) This solution is only valid for $V_{DS} \le V_{GS} - V_p$. [5%]

(ii) The saturated current may be found by evaluating the equation for I_{DS} when $V_{DS} = V_{GS} - V_p$,

$$I_{DS}(\mathrm{sat})=\frac{dhN_De\mu_eV_p}{L}\left\{\frac{V_{GS}}{V_p}-\frac{2}{3}\left(\frac{V_{GS}}{V_p}\right)^{3/2}-\frac{1}{3}\right\}.$$

The small signal mutual transconductance in the saturated region is then

$$\underline{g_m=\frac{\partial I_{DS}(\mathrm{sat})}{\partial V_{GS}}=\frac{dhN_De\mu_e}{L}\left\{1-\left(\frac{V_{GS}}{V_p}\right)^{1/2}\right\}.}$$ [25%]

(iii) Hence, if g_m is to be maximised we can optimise the geometry of the device by making the channel length short with respect to the channel height and width. We should also maximise the doping density in the channel, although we should be aware that this will increase the pinch-off voltage, which may be undesirable. Finally, we should choose a semiconductor with a high electron mobility. [15%]