

3. (a) Bookwork but should include description of a donor level inside the bandgap with energy gap between the donor level and the conduction band being slightly less than the photon energy to be detected. A good answer would include the need for cooling if the energy gap is less than $\sim kT$ at room temp.

Photoconductive gain is the relative increase of current in the photoconductor relative to the equivalent photocurrent for a given optical incident power.

$$G = \frac{\Delta I_{\text{photoconductor}}}{\Delta I_{\text{photodiode}}} = \frac{\tau_r}{\tau_{\text{set}}} + \frac{\tau_r}{\tau_{\text{pt}}} \quad (\text{derivation not required})$$

recombination time
transit times

$\sim 10^5$ for real photoconductor

- (b) (i) A pin photodiode is relatively simple, and therefore cheap.

It is easy to bias (wide range but a few volts is sufficient).
It doesn't provide any gain.

Noise processes: shot noise due to photocurrent and dark current

An avalanche photodiode has a more complicated structure but can provide multiplicative current gain due to the avalanche process (gains of ~ 200 for Si APD, less for other materials ~ 30 for Ge). The optimum bias voltage is quite critical and can be unstable so complicated bias circuits are needed. Bias voltages can be high (\sim few 100V).

Noise processes: shot noise due to photocurrent & dark current but also excess noise due to avalanche process.

There will also be thermal noise in the input resistor of the amplifier. This noise will dominate the sensitivity of the receiver ~~of~~ for the pin photodiode. However, the APD will be able to have an optimum avalanche

3(b)(ii)

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

$$= \frac{\left(\frac{\eta e \lambda P_{\text{opt}}}{hc} \right)^2}{2e \left(\frac{\eta e \lambda P_{\text{opt}}}{hc} + I_d \right) B + \frac{4kT B}{R}}$$

$$I_{\text{ph}} = \frac{0.85 \times 1.602 \times 10^{-19} \times 1.3 \times 10^{-6} \times 3.16 \times 10^{-6}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 2.817 \text{ pA}$$

$$= \frac{(2.817 \times 10^{-6})^2}{2e(2.817 \times 10^{-6} + 10^{-9}) \times 10^9 + \frac{4 \times 1.38 \times 10^{-23} \times 293 \times 10^9}{250}}$$

$$= \frac{7.9335 \times 10^{-12} \text{ A}^2}{9.024 \times 10^{-16} + 6.469 \times 10^{-14} \text{ A}^2}$$

↑ Note thermal noise limited

$$= 121 \text{ (or } 20.8 \text{ dB)}$$

(iii) For APD

$$\text{SNR} = \frac{(2.817 \times 10^{-6})^2 M^2}{2e(2.817 \times 10^{-6} + 10^{-8}) \times 10^9 M^{2.5} + 6.469 \times 10^{-14}}$$

$$\cong \frac{A M^2}{B M^{2.5} + C}$$

where $A = (2.817 \times 10^{-6})^2 = 7.9335 \times 10^{-12} \text{ A}^2$

$$B = 2e(2.817 \times 10^{-6} + 10^{-8}) \times 10^9 = 9.058 \times 10^{-16} \text{ A}^2$$

$$C = 6.469 \times 10^{-14} \text{ A}^2$$

SNR is max when

$$\frac{d(\text{SNR})}{dM} = 0 = \frac{2AM}{B M^{2.5} + C} - \frac{AM^2 \times 2.5 B M^{1.5}}{(B M^{2.5} + C)^2}$$

$$\Rightarrow \frac{2AM}{(B M^{2.5} + C)} = \frac{2.5 A B M^{2.5}}{(B M^{2.5} + C)^2} \quad (\text{or } M=0)$$

(b)(iii) (cont)

$$2B M^{2.5} + 2C = 2.5B M^{2.5}$$

$$0.5B M^{2.5} = 2C$$

$$M^{2.5} = \frac{4C}{B}$$

Optimum $M = \left(\frac{4C}{B}\right)^{0.4}$

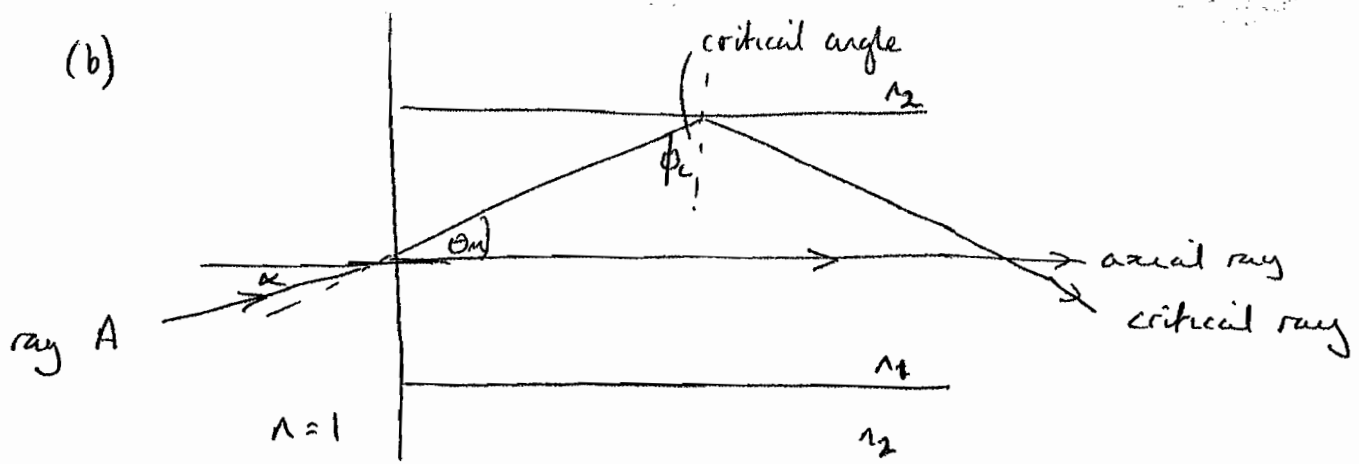
$$= \left(\frac{4 \times 6.469 \times 10^{-14}}{9.058 \times 10^{-16}}\right)^{0.4}$$

$$= (285.7)^{0.4}$$

$$= 9.60$$

$$\begin{aligned} \text{So Max SNR} &= \frac{7.9335 \times 10^{-12} \times 9.60^2}{9.058 \times 10^{-16} \times 9.60^{2.5} + 6.469 \times 10^{-14}} \\ &= 2260 \quad (\text{or } 33.5 \text{ dB}) \end{aligned}$$

4. (a) Bookwork.



From Snell's law $n_1 \sin \phi_i = n_2 \sin \phi_r$

At critical angle
(where all light
reflected at interface
& none refracted)

$$n_1 \sin \phi_c = n_2 \sin 90^\circ$$

$$\Rightarrow \sin \phi_c = \frac{n_2}{n_1} \quad (1)$$

Ray A produces critical ray inside fibre

Snell's law at facet

$$\sin \alpha = n_1 \sin \theta_m = n_1 \cos \phi_c \quad (2)$$

$$\text{Now from (1)} \quad \cos \phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\Rightarrow \sin \alpha = \sqrt{n_1^2 - n_2^2}$$

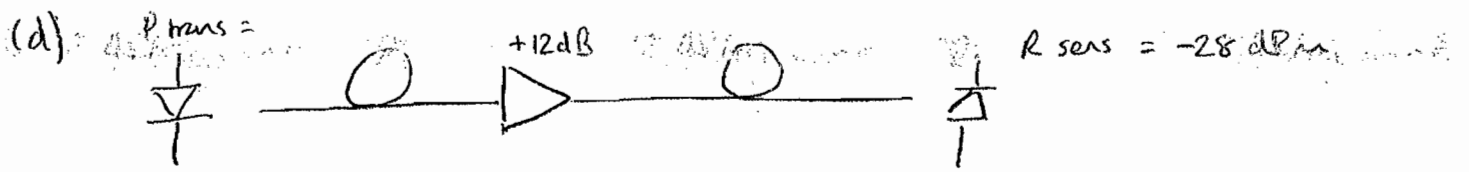
This is the numerical aperture

The numerical aperture gives a metric for the light gathering power of the fibre. For a lambertian source, the coupling efficiency is $(NA)^2$.

(c) Fibre is just single mode when

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = 2.4 \quad (a = \text{core radius})$$

$$\Rightarrow a = \frac{2.4 \lambda}{2\pi \sqrt{n_1^2 - n_2^2}} = 2.47 \mu\text{m} \quad (\text{core diameter} = 4.94 \mu\text{m})$$



Loss/power budget

Transmit power	+ 2 dBm
Fibre loss $h \text{ km} @ 0.3 \text{ dB/km}$	- 0.3 L dB
Splice loss 3 @ 0.2 dB	- 0.6 dB
Connector loss 2 @ 0.4 dB	- 0.8 dB
Amplifier gain	+ 12 dB
System margin	3 dB
Receiver sensitivity	- 28 dBm

Link operates on limit of sensitivity when

$$\text{Transmit Power} - \text{losses} + \text{amp gain} - \text{margin} = \text{Sensitivity}$$

$$+ 2 \text{ dBm} - 0.3L - 0.6 \text{ dB} - 0.8 \text{ dB} + 12 \text{ dB} - 3 \text{ dB} = -28 \text{ dBm}$$

$$0.3L = 37.6 \text{ dB}$$

$$L_{max} = 125.3 \text{ km (attenuation limit)}$$

$$\text{Dispersion limit : Single "1" width} = \frac{1}{10 \times 10^9} = 100 \text{ ps}$$

$$J_{out}^2 = J_{in}^2 + J_{disp}^2$$

So at max_{allowed} dispersion

$$J_{disp}^2 = 300^2 - 100^2$$

$$J_{disp} = 282.8 \text{ ps}$$

$$= D L_{max} \Delta \lambda$$

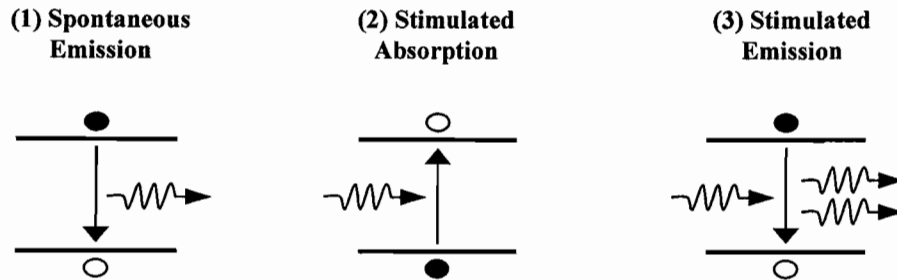
$$(d) \text{ (cont)} \quad h_{\max} = \frac{282.3}{18 \times 0.1} = 157.1 \text{ km}$$

So link is attenuation limited at 125.3 km.

Cribs for 3B6 Exam 2007

- 1 (a) This section would require largely a bookwork answer including:

There are three major types of electron/photon interactions in materials.



(1) Spontaneous Emission:

An electron in a high energy level falls losing energy which is emitted as a photon – the basis of operation of a light emitting diode. A good answer would comment on requirements for efficient operation and dynamics.

(2) Stimulated Absorption:

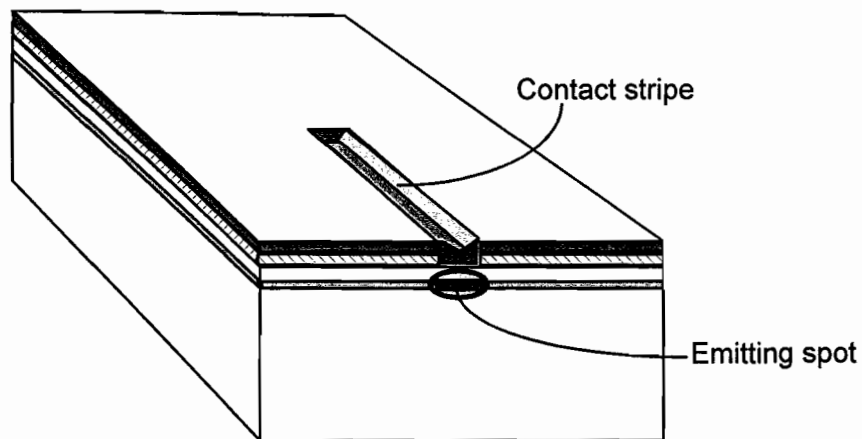
An incident photon is absorbed in a material causing the excitation of an electron to a higher energy level – the basis of operation of a photodiode. A good answer would include comments on the impact of the stimulated aspect of the interaction on device dynamics, spectral properties and causes of inefficiency.

(3) Stimulated Emission:

A photon, incident upon an electron in a higher energy level causes the electron to fall to a lower level thus generating a second photon. This is therefore an amplifying action. Two photons are generated from one and in turn they can cause the generation of two further photons. Using this method high optical powers can be generated and this operation is the basis of lasing action. The generated photon has the same frequency and phase as the incident photon and therefore very pure monochromatic and coherent light is generated. A good answer would include comments on the relevance and importance of population inversion and manners in which this might be achieved.

Q.1 (b) (i) A good answer should provide a detailed description of the operation and structure of a typical LED. Surface or edge emitting LEDs could be considered, it being expected that edge emitting devices being preferred for coupling to single mode fibre. Edge emitting LEDs operate at similar current densities and currents to surface emitters as above, but the emitting spot is much smaller and the use of optical waveguiding increases the maximum brightness available. A good answer should include descriptions of both the optical and electronic design (including heterostructures).

Edge emitting LED



(ii) $E = \frac{hc}{\lambda} = 0.96 \text{ eV}$

(iii) $\frac{1}{\tau_c} = \frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}} \Rightarrow \tau_{rr} = \left(\frac{1}{\tau_c} - \frac{1}{\tau_{nr}} \right)^{-1} = 6 \text{ ns}$

(iv)

$$\eta_{\text{tot}} = \eta_{\text{int}} \eta_{\text{ext}}$$

$$\Rightarrow \eta_{\text{ext}} = \eta_{\text{tot}} \left(\frac{\frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}}}{\frac{1}{\tau_{rr}}} \right) = 9\%$$

(v)

$$P = \eta \frac{hc}{\lambda} \frac{I}{e} = 14.3 \text{ mW}$$

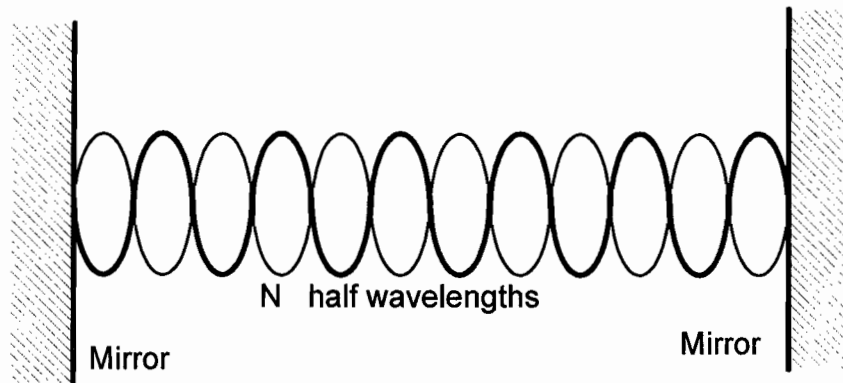
$$\frac{P(T)}{P(T_1)} = \exp \left[-\frac{T - T_1}{T_0} \right]$$

$$\Rightarrow T_0 = -(T - T_1) / \ln \left[\frac{7.3}{14.3} \right] = 90\text{K}$$

- 2 (a) This answer should largely involve bookwork, a good answer including the following:

A Fabry Perot cavity is formed by two reflectors set a distance L apart. This ensures that while light can leave the cavity as a laser beam, it can also be recycled, this maintaining lasing action. The optical filament oscillates at such a wavelength so that nodes occur at both reflectors. As a result a series of different wavelengths λ_m

Fabry Perot Modes

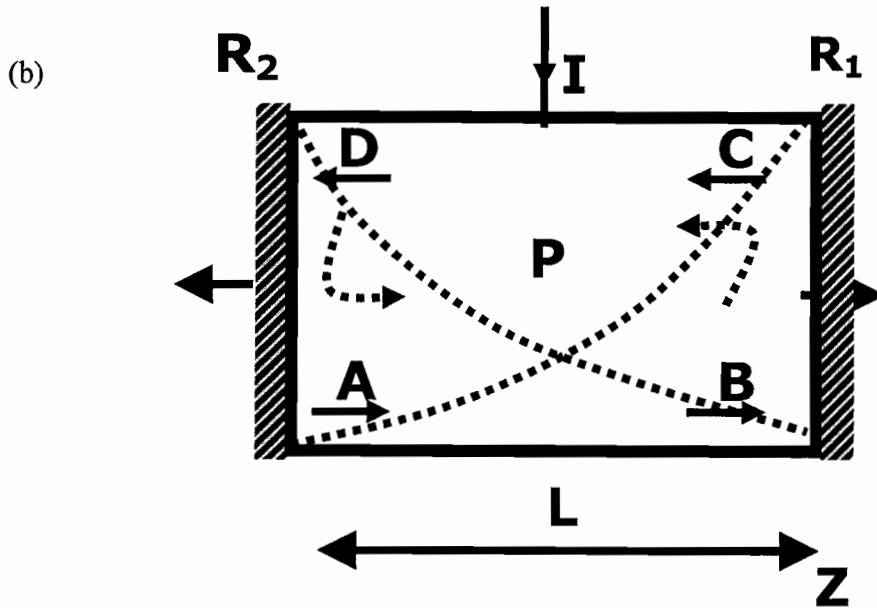


can be supported by such a cavity where

$$\lambda_m = 2L/m.$$

(The wavelength spacing therefore is $\Delta\lambda = \lambda^2/2L$). It should be noted however that wavelengths are only generated if electronic transitions occur with the necessary energy spacing. However a range of optical lasing modes can be generated simultaneously if a range of energy levels are available.

Comments on advantages and disadvantages should include the simplicity and low cost features of the device structure, whereas its multimode nature limits applications, for example in long haul communications where dispersion is important.



The photon lifetime of the laser cavity can be readily determined by considering the amplification of laser light as it propagates along the laser cavity.

Assume that stimulated emission encounters a gain per unit length (due to stimulated amplification), G , and a loss per unit length due to scattering and absorption, α , as it passes along the laser. The gain G in practice creates extra photons to compensate for those photons lost as the signal travels over a distance of unit length.

Therefore the stimulated light A starting at one facet will be incident on the opposite facet with an optical power

$$B = \exp \{ (G - \alpha)L \} A$$

At that point part of the signal is reflected with a coefficient R and the signal then passes back amplified by 1 the same amount as above and again reflected by the initial facet. Lasing action will occur in the nett round trip gain of the signal is unity i.e. if

$$\exp \{ (G - \alpha)L \} \cdot R_1 \exp \{ (G - \alpha)L \} \cdot R_2 = 1$$

$$\Rightarrow G = \alpha + (1/2L) \ln(1/(R_1 R_2))$$

This value of G is equal to the ratio of photons lost as the signal travels a unit length and hence the proportion of photons lost per unit time is simply the gain G times the speed of light in the laser material, v_g . As a result the average time for which one photon will remain in the cavity is given by

$$\tau_p = 1/Gv_g$$

$$= 1/\{v_g \{ \alpha + (1/2L) \ln(1/R_1 R_2) \} \}$$

- (c) Above threshold the differential efficiency is simply the proportion of photons leaving the cavity through the facets over the total number of photons. It is equal therefore to the loss through the facets divided by the total loss; i.e.

$$\eta_D = \frac{1n(1/(R_1 R_2))/(2L)}{\alpha + 1n(1/(R_1 R_2))/(2L)}$$

- (d)

$$\delta\lambda = \frac{\lambda^2}{2nL} \Rightarrow L = \frac{\lambda^2}{2n\delta\lambda} = 310 \mu\text{m}$$

$$\eta = \frac{\frac{1}{L} \ln\left(\frac{1}{R}\right)}{\alpha + \frac{1}{L} \ln\left(\frac{1}{R}\right)} = \frac{1}{1 + \alpha L / \ln \frac{1}{R}}$$

$$\Rightarrow R = \left[\exp \left\{ \alpha L / \left(\frac{1}{\eta} - 1 \right) \right\} \right]^{-1} = 8\% \text{ [power]}$$