Engineering Tripos Part IIA, Module 3C2 **May 2007**
Manufacturing Engineering Tripos Part I, Paper P4B

1. (a) (i) The thick plate Rosenthal solution gives the temperature field due to a point heat source moving over the surface of a semi-infinite solid. The heat conduction is three-dimensional in the general thick plate solution, and is used for welds near the surface of a plate with deposits that are small compared to the plate dimensions.

(ii) In the "fast" limit, heat conduction is slow compared to the traverse speed of the weld. There is no heat conduction in the direction of welding – the point source effectively heats a surface line very rapidly, and heat is then conducted laterally (in the y-z plane normal to the welding direction), giving two-dimensional heat flow (for the thick plate case).

The key assumptions used to derive the Rosenthal equations are:
- point source of heat
- no phase changes (neglect melt pool)
- constant thermal properties
- steady-state solution (fixed temperature field relative to source)

(b) (i) $$T = T_o + \frac{(q/v)}{2\pi\lambda t}\exp\left(-\frac{y^2+z^2}{4at}\right)$$

$$\frac{\partial T}{\partial t} = \frac{(q/v)}{2\pi\lambda}\left[-\frac{1}{t^2}\exp\left(-\frac{y^2+z^2}{4at}\right) + \frac{1}{t}\left(-\frac{y^2+z^2}{4at^2}\right)\exp\left(-\frac{y^2+z^2}{4at}\right)\right] = 0$$

$$\Rightarrow t_p = \left(\frac{y^2+z^2}{4a}\right)$$

Substituting back for t: $$T_p - T_o = \frac{(q/v)}{(y^2+z^2)}\left(\frac{2a}{e\pi\lambda}\right)$$

(ii) From (i): $(q/v) \propto (T_p - T_o)$ for given (y^2+z^2)

Hence for original weld: $(q/v) = C_1\,(T_m - T_o)$, with (y^2+z^2) corresponding to the $(\text{radius})^2$ of the fusion zone.

For the pre-heated weld, the ambient temperature T_o is replaced by T_{ph}, the heat input is reduced by a factor α (< 1), and the constant C_1 is unchanged for the same size fusion zone.

Hence: $$\alpha(q/v) = C_1\,(T_m - T_{ph})$$

Dividing the two expressions: $$\alpha = \frac{(T_m - T_{ph})}{(T_m - T_0)} = \frac{(1470-200)}{(1470-20)} = 0.8759$$

(iii) Limit of HAZ is at $T_p = A_1 = 723°\text{C}$.

From (i): $$(T_p - T_o) \propto \frac{(q/v)}{(y^2+z^2)}$$

Hence for original weld: $(A_1 - T_o) = C_2\,\dfrac{(q/v)}{(y^2+z^2)_{HAZ,o}}$ where $(y^2+z^2)_{HAZ,o}$ is the $(\text{radius})^2$ of the original HAZ.

For the preheated weld: $\left(A_1 - T_{ph}\right) = C_2 \dfrac{\alpha(q/v)}{\left(y^2+z^2\right)_{HAZ,ph}}$ where $\left(y^2+z^2\right)_{HAZ,ph}$ is the (radius)^2 of the new HAZ.

Hence ratio of HAZ radii (preheat/original):

$$\sqrt{\frac{\left(y^2+z^2\right)_{HAZ,ph}}{\left(y^2+z^2\right)_{HAZ,o}}} = \sqrt{\alpha\,\frac{\left(A_1 - T_o\right)}{\left(A_1 - T_{ph}\right)}} = \sqrt{0.8759 \times \frac{(723 - 20)}{(723 - 200)}} = 1.085$$

(iv) If T_p is close to the melt temperature, then the time to reach 800 (and lower temperatures) is of order 2-3 t_p. From (i), the temperature solution may be written:

$$T = T_0 + \frac{(q/v)}{2\pi\lambda t}\exp\left(-\frac{t_p}{t}\right)$$

For $t > 3t_p$, the exponential term approaches unity, and hence $\left(T - T_0\right) \approx \dfrac{(q/v)}{2\pi\lambda t}$.

Δt_{8-5} is the cooling time from 800 to 500°C (critical interval for transformations to ferrite/pearlite etc, avoiding martensite formation). Inverting the equation gives the time to reach a specified temperature on the cooling part of the curve: $t \approx \dfrac{(q/v)}{2\pi\lambda\left(T - T_0\right)}$

Thus for the original weld: $\left(\Delta t_{8-5}\right)_o = \dfrac{(q/v)}{2\pi\lambda}\left[\dfrac{1}{\left(500 - T_0\right)} - \dfrac{1}{\left(800 - T_0\right)}\right]$

(noting that t_{500} is the longer time).

And for the preheated weld: $\left(\Delta t_{8-5}\right)_{ph} = \dfrac{\alpha(q/v)}{2\pi\lambda}\left[\dfrac{1}{\left(500 - T_{ph}\right)} - \dfrac{1}{\left(800 - T_{ph}\right)}\right]$

Ratio of the two times (preheat/original):

$$\frac{\left(\Delta t_{8-5}\right)_{ph}}{\left(\Delta t_{8-5}\right)_o} = 0.8759 \times \frac{\left[\dfrac{1}{(500 - 200)} - \dfrac{1}{(800 - 200)}\right]}{\left[\dfrac{1}{(500 - 20)} - \dfrac{1}{(800 - 20)}\right]} = 1.82$$

The cooling time from 800 to 500°C is increased by a factor approaching 2. The increase in background ambient temperature has a stronger influence on the cooling rate than does the drop in heat input (which shortens the cooling time).

<u>Examiner's comments</u>

This is a standard Rosenthal analysis in a slightly unfamiliar context, illustrating quantitatively the role of preheat in slowing cooling rates. The commonest numerical error was to find the reduction in power in (b,ii) but then fail to retain it in (iii, iv). The opening descriptive part (a) was less well-answered than the analytical part (b).

2. (a) Typical input data:

For purely thermal analysis:
- thermal properties (specific heat, thermal conductivity; can be functions of temperature)
- density
- thermal boundary conditions (heat transfer to tooling, backing plates; specified heat inputs or temperatures)

If also a deformation process:
- elastic-plastic response of material (modulus as f(T); flow stress as f(T, strain, strain-rate))
- mechanical boundary conditions (friction between tool and workpiece)

Predicted outputs:
- temperature histories
- deformation histories (strain, strain-rate)
- forming loads
- empirically predicted final strength (and perhaps grain size)
- empirical quality measures (e.g. likelihood of cracking)
- distortion and residual stress

Other essential data required for the models to be of any use industrially: *validation* data from experiments (e.g. for temperature, loads, strength, grains size, shape, residual stress). Also important to know the *cost* (of model and data).

Microstructural models potentially add value by enabling:
- physically-based input to constitutive deformation laws used in forming analyses
- "through-process modelling", tracking microstructure evolution through multi-stage processing (from casting , homogenisation, forming, heat treatment, welding and into service)
- physically-based prediction of quality measures (e.g. cracking, downstream formability)
- prediction of more difficult material properties that are microstructure sensitive (toughness, fatigue, corrosion)

(i) In cogging: repeated deformation of long cylindrical billets at high temperature. Hot work increases dislocation density, subsequent recrystallisation produces refined grain structure compared to as-cast. Empirical grain size model, dependent on hot deformation temperature, strain and strain-rate predicted over billet cross-section.

(ii) Fatigue failure in oil rig collapse: based on Alexander Kielland disaster (but a generic problem in steel welding for marine applications). Thermal cycle input to microstructural models for:
- dissolution of precipitates that should pin grain boundaries in microalloyed steel;
- grain growth in austenite, once pinning precipitates have gone;
- prediction of phases formed on cooling, notably % martensite that is responsible for cold cracking and embrittlement, initiating fatigue crack.

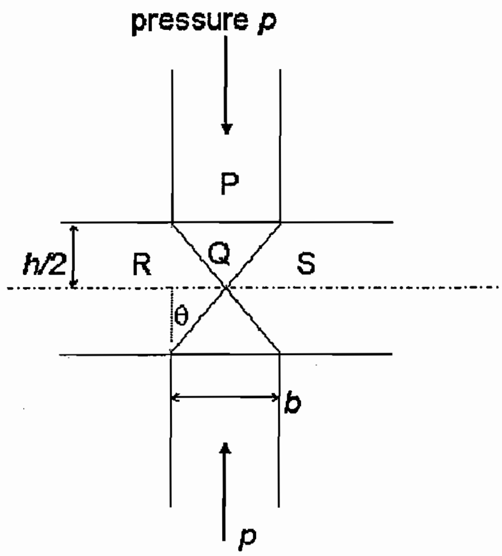
First two use microstructure evolution "internal state variable" models, based on underlying physics; martensite prediction obtained empirically from weld simulations tests on steel.

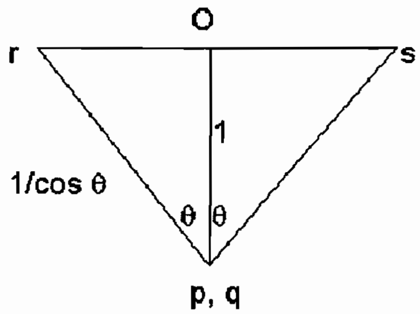
(b) (i) Residual stresses in weld, arising from thermal contraction following welding (useful to sketch typical longitudinal residual stress pattern, with tension along weld line). Flame cutting provides additional thermal stress, and changes the stress distribution in the welded structure. Stresses may be high enough that pre-existing cracks may become unstable. Added complication may be if prior welding procedure was not appropriate for the steel, leaving some brittle martensite in the HAZ (but residual stress alone might be expected to cause this to fail). Remedies: stress-relieve weld before cutting; cut by machining rather than flame; post-weld heat treat (if possibility of martensite formation during welding).

(ii) Assume correctly welded initially (i.e. not embrittled). Weld may be inspected for pre-existing cracks (X-ray, C-scan, visual), though safest to assume cracks will be present anyway. Grind surface smooth to remove weld bead, with machining direction normal to the weld (parallel to load direction). This removes stress concentrations from section changes; machining direction chosen so that its surface features don't initiate cracks and failure. Peen surface at weld, to put weld area into compression. Makes crack initiation more difficult, so improves fatigue lifetime.

<u>Examiner's comments</u>
The two parts of the question were unconnected. In both parts, many students who clearly didn't know the relevant material nonetheless offered extensive answers to the questions they wish they had been asked. For example, the benefits of microstructure modelling were explained without reference to anything microstructural. Needless to say, this did not attract many marks.

3. (a) Consider unit depth into paper, with plane of symmetry stationary (vertically). Let dies approach with unit velocity.





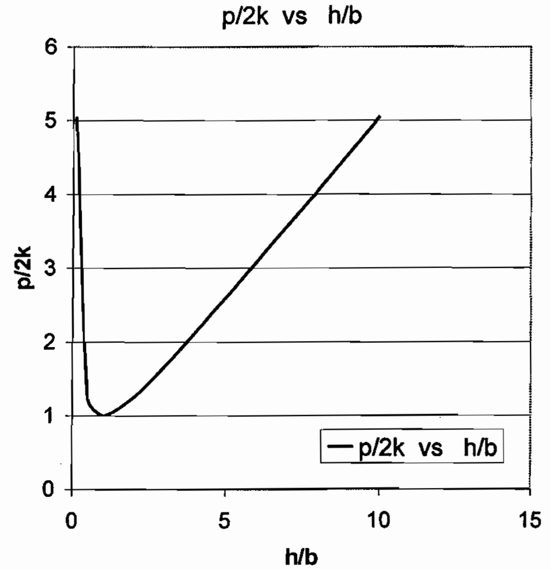
$$\tan\theta = \frac{b}{h}, \quad \text{and} \quad \cos\theta = \frac{h}{\sqrt{h^2+b^2}}$$

External work done by pressure (per unit width): $2 \times (p \times b \times 1)$
Internal work dissipation:

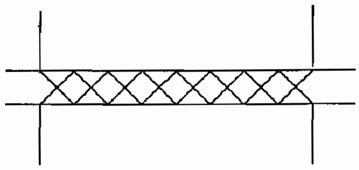
$$4 \times k \times \frac{\sqrt{h^2+b^2}}{2} \times \frac{1}{\cos\theta} = 2k\sqrt{h^2+b^2} \times \frac{\sqrt{h^2+b^2}}{h} = 2k\,\frac{(h^2+b^2)}{h}$$

Hence: $2pb = 2k\,\dfrac{(h^2+b^2)}{h} \Rightarrow \dfrac{p}{2k} = \dfrac{(h^2+b^2)}{bh} = \dfrac{1}{2}\left(\dfrac{h}{b} + \dfrac{b}{h}\right)$

(b) Sketch of function:

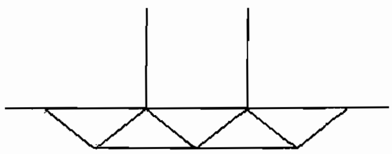


For $b >> h$ (i.e. $h/b << 1$), the steep rise in pressure does not occur as an alternative deformation pattern takes over (lower work done). The number of slip surfaces increases to maintain slip planes around 45°.



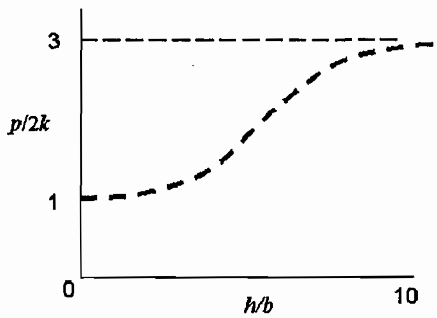
Deformation pattern for (frictionless) plane strain compression, for which $p = 2k$, i.e. $p/2k = 1$.

For $b << h$ (i.e. $h/b >> 1$), the rise in pressure is again truncated, as a lower work-rate is required to indent the surfaces (the plastic zones do not meet).



Deformation pattern for indentation of a semi-infinite solid, for which $p/2k \approx 3$.

Overall expected variation in pressure:



(c) Given: $k_{die} = 1.5\ k_{plate}$ and $b = 10$ mm

Dies just yield when pressure on their ends $p = 2\ k_{die} = 3\ k_{plate}$

Assuming the situation falls in the region for which the result in (a) is valid, pressure to yield plate is:

$$\frac{p}{2k_{plate}} = \frac{3k_{plate}}{2k_{plate}} = \frac{1}{2}\left\{\frac{h}{b} + \frac{b}{h}\right\} \quad \Rightarrow 3 = \left\{\frac{h}{b} + \frac{b}{h}\right\}$$

Let $\frac{h}{b} = x$ then $3 = x + \frac{1}{x}$

$$3x = x^2 + 1$$

$$x^2 - 3x + 1 = 0$$

$$\therefore x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2} = 2.618 \text{ or } 0.382$$

Thus $\frac{h}{b} = 2.62$, so reasonable to use result in (a). Maximum thickness = 26.2 mm.

Examiner's comments

Straightforward upper bound geometry, but with variable aspect ratio. Some students found the lengths in terms of the aspect ratio (h/b) but then assumed a 45° geometry for the hodograph. Many only plotted the function for h/b = 1–10, and thus missed the steep rise predicted as h/b tends to zero. Very few students were able to suggest sensible values for p/2k for large or small h/b. In spite of recognising that the equation derived would no longer be valid, most tried to take its mathematical limit and ended up with answers that depended on b or h (i.e. were not dimensionless).

4 (a) Differential expansion and contraction between the continuous glass fibres and the epoxy matrix will produce internal stresses. Epoxy will be brittle at all temperatures. There will be progressive cracking in the composite, and when cracks link significantly then the vessel will leak.
Cracking will tend to occur in tension because all components are brittle. Fibres go into tension in the hot cycle, and cracking across fibres (tensile failure) as well as at the fibre-matrix interface (shear failure) will occur. This will lead to progressive weakening of the structure, so the rate of damage accumulation will accelerate.
Once fibre-matrix cracking has begun, there will be the possibility of wicking water along into the interior of the structure. Glass is corroded by water (leaching); epoxy may be plasticised (swelling, reduction in strength). Both will tend to accelerate damage accumulation.

(b) Crevice corrosion: differential aeration of water trapped between steel plates.

$Fe \Rightarrow Fe^{2+} + 2e^-$ (Anode)

$O_2 + 2H_2O + 4e^- \Rightarrow 4(OH)^-$ (Cathode)

Anode dissolves, causing the problem.

$Fe^{2+} + 2(OH) \Rightarrow Fe(OH)_2$ Rust.

(Dissimilar metal contact between container and scrap metal contents may also cause localised galvanic corrosion – not an effect on the welds, but a contribution to failure).

Improvements:
Seal joints between plates to prevent water ingress, and improve joint geometry – perhaps by using fillet welds rather than spot welds.
Galvanise the container (extends lifetime by preventing corrosion while the zinc coating lasts).
Make the container walls thicker, so it takes longer for corrosion to penetrate right through.
(Changing to stainless steel is not a realistic option for storing scrap – too expensive).

(c) Stress corrosion cracking (SCC): specific material and environment, with some plastic strain, are the ingredients of SCC. The mechanism is that localised plastic deformation disrupts protective surface films, allowing the material to be attacked by chlorine from the marine environment. Many deep, narrow cracks are expected to form. Uncracked austenitic steels can show a stress limit, below which SCC does not occur; this is the explanation of the stress limit seen in the tests here. The design criterion will not be safe however: bolts have threads which are likely to act as crack nucleation sites. Cracked structures do not show stress limits. Note that the tests were static, so fatigue loading of the bolts is not relevant here.

(d) High-strength steel can show hydrogen cracking if exposed to free hydrogen. The origin of the hydrogen here will be the acid treatment:

$H^+ + e^- \Rightarrow [H]$

Hydrogen embrittles the structure by a number of possible mechanisms, including promoting microslip at crack tips and formation of bubbles.

Prevention: Bake the vessel to allow the hydrogen to diffuse out; use a lower strength steel (hydrogen embrittlement usually only a problem when yield strengths are above 700MPa); don't pickle the vessel – clean with a different agent that does not introduce free hydrogen.

Examiner's comments

Most popular question on the paper, but this was not reflected in the marks. Another question in which detailed accurate answers were given to problems other than those set, often scrambling the answers to (b, c and d). Some candidates blamed absolutely everything in steel welding on martensite formation (in both Q2 and 4). In (b), the geometry of spot welding was clearly unknown to some candidates.

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Numerical answers

1. (b) (ii) Ratio of heat inputs = 0.876
   (iii) Ratio of HAZ radii = 1.085
   (iv) Ratio of Δt_{8-5} values = 1.82

3. (b) $b >> h$: $p/2k = 1$; $b << h$: $p/2k \approx 3$.
   (c) Maximum thickness = 26.2mm.