Engineering Tripos IIA

3C4 Machine Design - Transmissions    CRIB 06/07

M SUTCLIFFE

1(a) Need gears to match power of source and load
Epicyclics have good $\eta$ and reliability
and are conducive to automatic transmission
as elements can be controlled by brake bands
without the need to engage gears eg with clutches. [15%]

b(i) Linkages: $C_1 = A_2$, $C_2 = A_1 = A_3$, $\omega_i = S_2 = S_3$, $\omega_o = C_3$, $S_1 = 0$

Epicyclic speed rules    Put everything in terms of $A_2$ and $A_3$

(1) $\overset{0}{\cancel{S_1}} = (1+R_1)\overset{A_2}{\cancel{C_1}} - R_1 \overset{A_3}{\cancel{A_1}} \Rightarrow A_3 = A_2 (1+R_1)/R_1$

(2) $\overset{\omega_i}{\cancel{S_2}} = (1+R_2)\overset{A_3}{\cancel{C_2}} - R_2 A_2 \Rightarrow \omega_i = (1+R_2)A_3 - \dfrac{R_2 R_1 A_3}{1+R_1}$

$$= A_3 \frac{(1+R_1+R_2)}{1+R_1}$$

(3) $\overset{\omega_i}{\cancel{S_3}} = (1+R_3)\overset{\omega_o}{\cancel{C_3}} - R_3 A_3$

$$\Rightarrow \omega_i = (1+R_3)\,\omega_o - \frac{\omega_i R_3 (1+R_1)}{1+R_1+R_2}$$

$$\Rightarrow \frac{\omega_i}{\omega_o} = \frac{1+R_3}{1 + R_3(1+R_1)/(1+R_1+R_2)}$$

For $R_1 = 2.5$, $R_2 = 3.5$, $R_3 = 3$    $\dfrac{\omega_i}{\omega_o} = \dfrac{4}{1+3.3.5/7} = \underline{1.6}$    [50%]

PART IIA 2007
3C4 Machine design – transmissions
Dr M Sutcliffe

1 (b) (ii) $$\frac{P_{s3}}{\text{Total Power}} = \frac{T_{s3}\,\omega_{s3}}{T_{c3}\,\omega_{c3}}$$

But from (b)(i) $$\frac{\omega_{s3}}{\omega_{c3}} = \frac{\omega_i}{\omega_o} = 1.6$$

To get torques use virtual power

$$\omega_s' T_s + \omega_c' T_c + \omega_A' T_A = 0 \quad \text{for third epicyclic}$$

$$\Rightarrow \frac{T_s}{T_c} = -\left.\frac{\omega_c'}{\omega_s'}\right|_{\omega_A'=0} = \frac{-1}{1+R_3} \quad \text{using epicyclic rule}$$

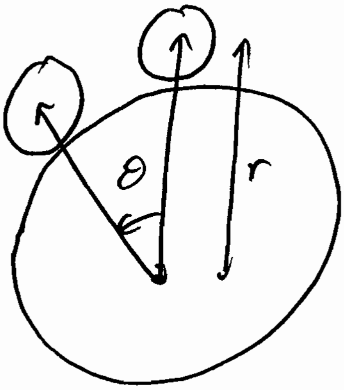
$$\Rightarrow \frac{P_{sc}}{\text{Total}} = -1.6 \cdot \frac{1}{4} = 60\%$$

The minus sign arises because power flows out of $C_2$ [25%]

(iii) Could include more gears.
If the system includes a variable speed unit (which in itself is inefficient) the overall $\eta$ might be improved by better power matching to operate at the engine's optimum operating point. [10%]

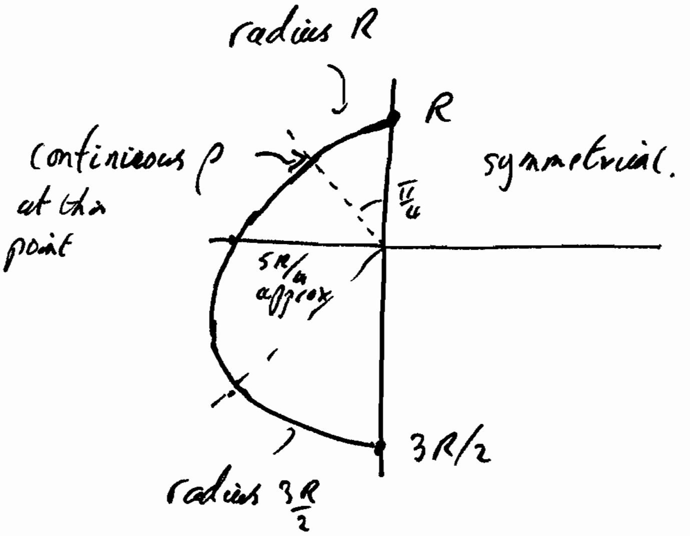
[Generally well answered, though many students got lost in the algebra of (b)(i).]

2 (a)

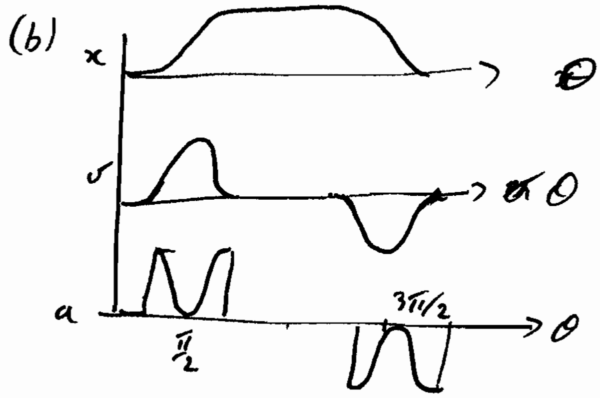



Easiest to consider rotating the follower around the cam to derive a pitch circle of radius $r(\theta)$. This pitch circle is equivalent to the lift profile, and the cam profile is given by the envelope of follower positions.

Here the base circle has radius $R$, there is a lift of $R/2$ to give contact on a circle radius $3R/2$ and there is a transition between. [35%]




(b)

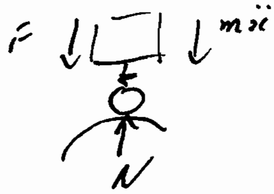



Need to find max accn and min accn.

$$\frac{dx}{dt} = \frac{R}{4} 2\dot{\theta} \cos(2\theta - \pi) \Rightarrow \frac{d^2x}{dt^2} = -R\dot{\theta}^2 \sin(2\theta - \pi)$$

which is max. at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ and min. at $\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

To maintain contact $F + m\ddot{x} \geq 0 \Rightarrow F \geq -\min(m\ddot{x})$




$F \geq mR\omega^2$

[35%]

2 (c) ~~R~~ The maximum Hertzian pressure

$$p_0 = \left(\frac{W' E^*}{\pi R'}\right)^{\frac{1}{2}}$$

is given when $W'/R$ is maximum.

Here the maximum force $F + mR\omega^2$ at $\theta = \frac{\pi}{4}$ seems to correspond to the minimum cam radius of $R$.

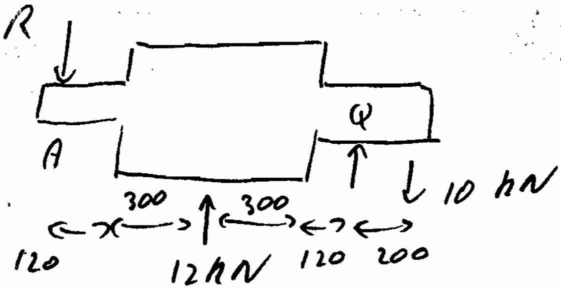
So $$p_0 = \left(\frac{(F + mR\omega^2)E^*}{\pi \, l \, R/3}\right)^{\frac{1}{2}}$$ where the effective radius $R' = \left(\frac{1}{R} + \frac{2}{R}\right)^{-1}$

Note that the spring and acceleration forces are perpendicular to the cam face.

[30%]

[Many good answers. The sketch in (a) was the weak part of the answers.]

3 (a)




Find bearing forces: $12 + Q = 10 + R$

Moments at A: $12 \times 420 + Q \times 840 = 10 \times 1040$

$\Rightarrow Q = 6.38$ kN, $R = 8.38$ kN

$a = 45$ mm $\quad L = a_1 a_{23} \left(C/P\right)^p$ with $p = 3$

For 95% survival $a_1 = 0.62$

$$L = \frac{10{,}000 \times 60 \times 350}{10^6} = 210$$

Supposing $1 < a_{23} < 2 \simeq 1.5$ then for the higher loaded A $\quad 210 = 0.62 \times 1.5 \times \left(\frac{C}{8.38}\right)^3$

$\Rightarrow C = 51020$ N

So try #6309, $D = 100$ mm, $C = 52700$

$d_m = 72.5$ mm, $\nu_1 = 40\ mm^2 s^{-1} \Rightarrow \frac{\nu}{\nu_1} = \frac{100}{40} = 2.5 \Rightarrow a_{23} = 1.8$

$$L = 0.62 \times 2 \times \left(\frac{52700}{8380}\right)^3 = 277 \quad \text{So OK}$$

Choose #6309

[45%]

3(b) Axial load raises effective load at A to 9.6kN.

Still need axial load at A so deep groove ball bearing OK – could switch to taper roller bearing.

For #6309 $L = 0.62 \times 1.8 \times \left(\frac{52700}{9600}\right)^3 = 185$ so below limit of 210.

Try #6409 $C = 76100$, $d_m = 82mm$ $\nu_1 = 35$, $a_{23} = 2$

$\Rightarrow L = 0.62 \times 2 \times \left(\frac{76.1}{9.6}\right)^3 = 620$ so OK.

Choose #6409 for A

For B no axial load so replace by roller bearing.

Try NU 1009 EC – smallest 45 internal diameter

$d_m = 60$ mm $\nu_1 = 45 \, mms^{-2}$, $a_{23} = 1.7$

$$L = 0.62 \times 1.7 \times \left(\frac{44606}{6380}\right)^{10/3 \leftarrow \text{roller}} = 688 > 210 \text{ so ok}$$

Choose #NU 1009 EC for B

[45%]

(c) Consider bending moment and stresses in shaft. Check contact stress associated with point load (non uniform crushing) and fatigue associated with variable loading. Also vibration issues.

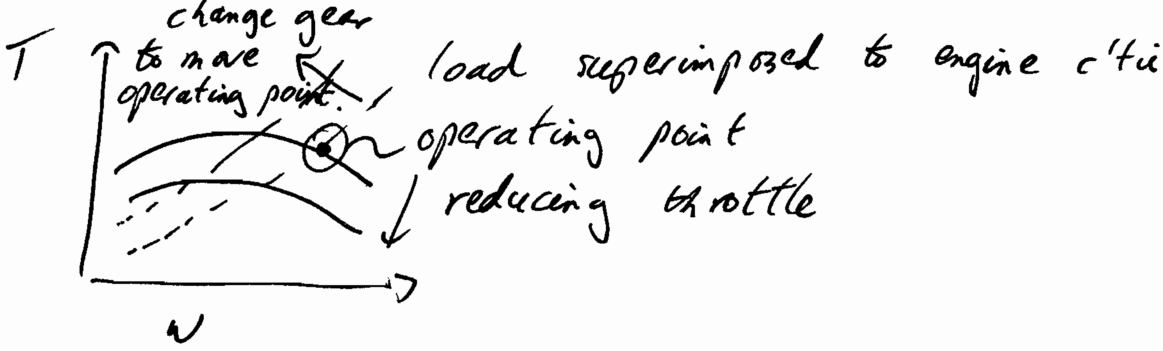
Take care over lubrication / seals in dirty environment.

[10%]

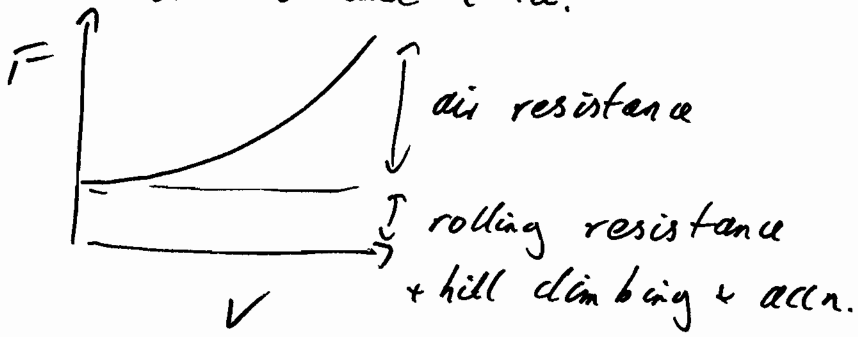
[Well answered. Many people didn't consider carefully which bearings should be used for part (b)]

4 (a) Typically we want to use the power source at optimum efficiency or perhaps power (for acceleration). As the load demand changes, this would imply changing the source operating point for a fixed gear coupling. Power matching uses e.g. gears to match the source to the load.

eg. a IC car engine characteristics are:




and the load c'tic:




By superimposing c'tics via a gear ratio G the operating point is found. Use this to find appropriate engine c'tics or G

[25 %]

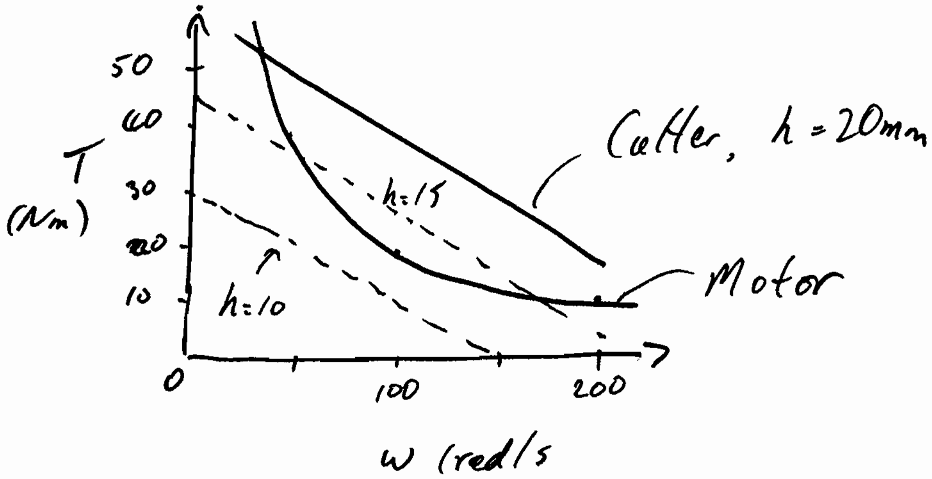
[Longer answer than required]

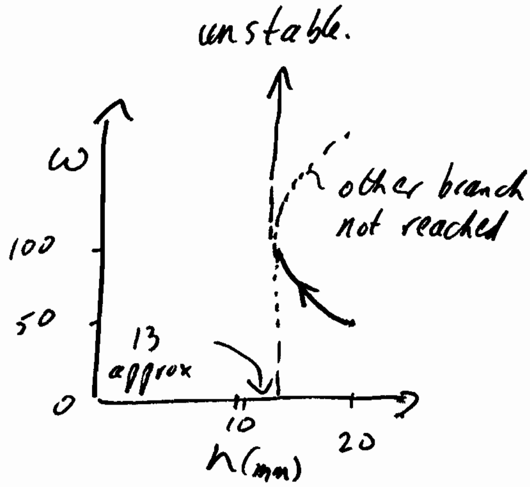
6 (b) (i) This could be achieved by connecting a hydrostatic pump and motor

Input $\xrightarrow{T\omega}$ | Pump | $\frac{p}{Q}$ | Motor | $\xrightarrow{T\omega}$ Output

One component should be variable speed to allow variation of output $T$ and $\omega$.
Advantages, easy to control, reasonable $\eta$ and reasonably light.
See the ~~noted~~ notes for a description of an appropriate hydrostatic pump / motor – eg a swash plate pump. [25 %]

(ii) Cutter: $T_c = 3h - \omega/5$ Motor $T_m = 2000/\omega$
Put $h = 20$, $\omega = 38 \cdot 2$ : $T_c = 52 \cdot 36 = T_m$ ✓ [25 %]




(iii) Add additional cutter c'tics on to plot above to find operating point as a function of $h$. At $h \approx 13$mm there is no stable operating point. The motor power needs to be reduced to avoid overspeeding. [25 %]

[Less popular question, perhaps due to descriptions. (b)(iii) not generally well answered unless characteristics were added to the sketch of (b) (ii)]

**Engineering Tripos Part IIA: Module 3C4**
**Machine Design - Transmissions**
**Numerical solutions - 2006/7**

1 (b) (i) 1.6, (ii) 40%

2. (b) $F \geq mR\omega^2$

3. (a) #6309, (b) #6409 for A (or a taper roller bearing), NU 1009 EC for B